

## GENERALIZED RECURRENT KAEHLERIAN WEYL SPACES

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### ABSTRACT

*In the present paper we have studied generalized recurrent Kaehlerian Weyl spaces. The properties of generalized Weyl Concircular, Conformal and Projective curvature tensor is shown and the relation between them is established.*

**Key Words:** Kaehlerian Weyl space, generalized recurrent Weyl space.

**Mathematics Subject Classification:** 53C25, 53C15

### 1. Introduction

An-n-dimensional differentiable manifold  $W_n$  is said to be Weyl space if it has a symmetric connection  $\nabla^*$  and a symmetric conformal metric tensor  $g_{ij}$  preserved by  $\nabla^*$ . Accordingly, in local coordinates there exists a covariant vector field  $T_k$ (complementary vector field) satisfying the condition [1],[2], and [3]

$$\nabla^*_k g_{ij} - 2T_k g_{ij} = 0. \quad (1.1)$$

The above equation can be extended to

$$\partial_k g_{ij} - g_{hj} \Gamma_{jk}^h - 2T_k g_{ij} = 0, \quad (1.2)$$

where  $\Gamma_{jk}^h$  are the connection coefficients of the symmetric connection  $\nabla$  and are defined as

$$\Gamma_{jk}^h = \left\{ \begin{matrix} h \\ jk \end{matrix} \right\} - g^{hm} (g_{mj} T_k + g_{mk} T_j - g_{jk} T_m), \quad (1.3)$$

$\left\{ \begin{matrix} h \\ jk \end{matrix} \right\}$  being the coefficient of the metric connection defined by

$$\left\{ \begin{matrix} h \\ jk \end{matrix} \right\} = \frac{1}{2} g^{hm} \{ \partial_j g_{mk} + \partial_k g_{jm} - \partial_m g_{ji} \}.$$

Moreover, under the renormalization condition

$$\tilde{g}_{ij} = \lambda^2 g_{ij}, \quad (1.4)$$

of the metric tensor  $g_{ij}$ , the covariant vector field  $T_k$  is transformed by the law

$$\tilde{T}_k = T_k + \partial_k \ln \lambda, \quad (1.5)$$

where  $\lambda$  is a scalar function defined on  $W_n$ . We denote such a Weyl space by  $W_n(\Gamma_{jk}^h, g_{ij}, T_k)$  or  $W_n(g, T)$ .

An  $n$ -dimensional differential manifold having an anti-symmetric connection  $\nabla$  and anti-symmetric metric tensor  $g_{ij}$  preserved by  $\nabla$  is called generalized Weyl space [4]. It is denoted by  $GW_n(g, T)$ .

For such a space, in local co-ordinate system, the compatibility condition is

$$\nabla_k g_{ij} - 2T_k g_{ij} = 0, \quad (1.6)$$

where  $T_k$  are the components of a covariant vector field, called the complementary vector field of the  $GW_n(g, T)$  space. Using the concept of covariant differentiation ([5],[6]), the compatibility condition of (1.6) can be written as

$$\partial_k g_{ik} - g_{hj} L_{ik}^h - g_{ih} L_{jk}^h - 2T_k g_{ij} = 0, \quad (1.7)$$

where  $L_{ik}^h$  are the connection coefficient of the anti-symmetric connection  $\nabla$  and are obtain from the compatibility condition as

$$L_{ij}^h = \Gamma_{ij}^h + \frac{1}{2} [\chi_{im}^h g_{jh} + \chi_{mj}^h g_{ih} + \chi_{ij}^h g_{hm}] g^{mi} \quad (1.8)$$

Now putting

$$\chi_{ij}^h = \frac{1}{2} [\chi_{im}^h g_{jh} + \chi_{mj}^h g_{ih} + \chi_{ij}^h g_{hm}] g^{mi}, \quad (1.9)$$

we obtain

$$L_{ij}^h = \Gamma_{ij}^h + \chi_{ij}^h, \quad (1.10)$$

where  $\Gamma_{ij}^h$  and  $\chi_{ij}^h$  are respectively the coefficient of a Weyl connection and the torsion tensor of  $GW_n(g, T)$  space and are expressed as

$$\Gamma_{ij}^h = \frac{1}{2} [L_{ij}^h + L_{ji}^h] = L_{ij}^h, \quad (1.11)$$

and

$$\chi_{kl}^h = \frac{1}{2} [L_{kl}^h - L_{lk}^h] = L_{[kl]}^h, \quad (1.12)$$

where square bracket stands for anti-symmetry.

The components of mixed curvature tensor and Ricci tensor of  $GW_n(g, T)$  are respectively

$$L_{jki}^h = \partial_k L_{ji}^h - \partial_i L_{jk}^h + L_{lk}^h L_{ji}^l - L_{li}^h L_{jk}^l, \quad (1.13)$$

$$L_{ij} = L_{ija}^a. \quad (1.14)$$

On the other hand scalar curvature of  $GW_n(g, T)$  is defined by

$$L = g^{ij} L_{ij}. \quad (1.15)$$

It is easy to see that curvature tensor  $L_{jki}^h$  of  $GW_n(g, T)$  can be written as

$$L_{jki}^h = B_{jki}^h + \chi_{jkl}^h, \quad (1.16)$$

where the tensors  $B_{jki}^h$  and  $\chi_{jkl}^h$  are defined respectively as

$$B_{jki}^h = \partial_k \Gamma_{ji}^h - \partial_i \Gamma_{jk}^h + \Gamma_{lk}^h \Gamma_{ji}^l - \Gamma_{li}^h \Gamma_{jk}^l, \quad (1.17)$$

$$\chi_{jki}^h = \nabla_k \chi_{ji}^h - \nabla_i \chi_{jk}^h + \chi_{li}^h \chi_{jk}^l - \chi_{lk}^h \chi_{ji}^l - 2\chi_{jl}^h \chi_{ki}^l. \quad (1.18)$$

The curvature tensor of  $GW_n(g, T)$  satisfies the relation [6].

$$L_{jkl}^h + L_{jlk}^h = 0, \quad (1.19)$$

$$L_{hkl}^j + L_{hkl}^j + L_{klh}^j = 2[\nabla_k \chi_{lh}^j + \nabla_h \chi_{kl}^j + 2\chi_{lm}^j \chi_{hk}^m + 2\chi_{hm}^j \chi_{kl}^m + 2\chi_{km}^j \chi_{lh}^m], \quad (1.20)$$

$$\nabla_m L_{jkl}^i + \nabla_k L_{jlm}^i + \nabla_l L_{jmk}^i = 2[L_{jpl}^i \chi_{mk}^p + L_{jpk}^i \chi_{lm}^p + L_{jpm}^i \chi_{kl}^p] \quad (1.21)$$

A Kaehlerian Weyl space denoted by  $KW_n$  is an  $n$ -dimensional ( $n=2m$ ) space with an almost complex structure  $F_j^i$  satisfying

$$F_j^i F_i^k = -\delta_j^k, \quad g_{ij} F_h^i F_k^j = g_{hk} \quad (1.22)$$

$$\dot{\nabla}_k F_j^i = 0 \quad (\text{for all } i, j, k) \quad (1.23)$$

$$F_{ij} = g_{jk} F_i^k = -F_{ji} \quad (1.24)$$

$$F^{ij} = g^{ih} F_h^j = -F^{ji} \quad (1.25)$$

the tensors  $F_{ij}$  and  $F^{ij}$  are of weight 2 and  $-2$  respectively [4].

The mixed curvature tensor  $R_{ijk}^h$  and the covariant curvature tensor  $R_{hijk}$  of  $W_n(g, T)$  are given respectively

$$R_{ijk}^h = \frac{\partial}{\partial x^k} \Gamma_{ij}^h - \frac{\partial}{\partial x^j} \Gamma_{ik}^h + \Gamma_{lk}^h \Gamma_{ij}^l - \Gamma_{lj}^h \Gamma_{ik}^l$$

$$\text{and } R_{ijkl} = g_{ih} R_{jkl}^h.$$

The Ricci tensor and the scalar curvature of  $W_n(g, T)$  are defined by

$$R_{ij}^h = R_{ij} \text{ and } R = g^{ij} R_{ij}$$

Also, it can be seen that the anti-symmetric part of the Ricci tensor satisfies

$$R_{[ij]} = n \nabla_{[i} T_{j]}$$

Let

$$G_{ij} = \frac{1}{2} R_{ijkl} F^{kl}, \quad H_{ij} = g_{ki} R_j^k$$

where  $R_{ijkl} = g_{ih} R_{jkl}^h$ ,  $R_j^i = R_{jl}^h g^{kl}$ . Then the following relations hold

$$H_{ij} = \frac{n-2}{n} R_{ij} + \frac{2}{n} R_{ji} = R_{ij} + \frac{2}{n} \{R_{ji} - R_{ij}\}$$

$$G_{ij} = -H_{hi} F_i^h = H_{ih} F_j^h$$

$$G_{hi} F_j^h = -G_{jh} F_i^h = H_{ji}$$

$$G_{hi} F^{hi} = -M_{hi} g^{hi} = -R$$

$$R_{ijkl} + R_{jikl} = 4\nabla_{[k}T_{l]}g_{ij}$$

$$G_{ij} + G_{ji} = 0$$

## 2. Generalized Recurrent Kaehlerian Weyl Space

An  $n$ -dimensional Weyl space is called generalized recurrent Weyl space if its curvature tensor  $R_{hijk}$  satisfies

$$\dot{\nabla}_m R_{hijk} = K_m R_{hijk} + L_m g_{hijk} \quad (2.1)$$

where

$$g_{hijk} = g_{hk}g_{ij} - g_{hj}g_{ik}. \quad (2.2)$$

$K_m$  and  $L_m$  are associate vectors of recurrence.

Multiplying (2.1) by  $g^{hk}$  we get

$$\dot{\nabla}_m R_{ij} = K_m R_{ij} + L_m (n-1)g_{ij} \quad (2.3)$$

Transvecting (2.3) by  $g^{ij}$  we have

$$\dot{\nabla}_m R = K_m R + L_m n(n-1)g_{ij} \quad (2.4)$$

Eliminating  $L_m$  from (2.1) and (2.4) we have

$$\dot{\nabla}_m S_{hijk} = K_m S_{hijk}$$

where

$$S_{hijk} = R_{hijk} - \frac{R}{n(n-1)}g_{hijk} \quad (2.5)$$

Multiplying (2.5) by  $g^{hk}$  we get

$$S_{ij} = R_{ij} - \frac{R}{n}g_{ij} \quad (2.6)$$

Such a space is denoted by  $W_n$ .

The Weyl Conformal curvature  $C_{hijk}$ , Weyl Concircular curvature tensor  $Z_{hijk}$  and Weyl Projective curvature tensor  $W_{hijk}$  in  $W_n(g, T)$  is given by

$$C_{hijk} = R_{hijk} - \frac{1}{n-2}\{g_{ij}R_{hk} - g_{ik}R_{hj} + g_{hk}R_{ij} - g_{hj}R_{ik}\} + \frac{R}{(n-1)(n-2)}\{g_{ij}g_{hk} - g_{ik}g_{hj}\} \quad (2.7)$$

$$Z_{hijk} = R_{hijk} - \frac{R}{n(n-1)}\{g_{ij}g_{hk} - g_{ik}g_{hj}\} \quad (2.8)$$

$$W_{hijk} = R_{hijk} + \frac{1}{n-1}\{R_{ij}g_{hk} - R_{ik}g_{hj}\} \quad (2.9)$$

From (2.8) and (2.9) we have

$$W_{hijk} = Z_{hijk} + \frac{R}{n(n-1)}\{g_{ij}g_{hk} - g_{ik}g_{hj}\} + \frac{1}{n-1}\{R_{ij}g_{hk} - R_{ik}g_{hj}\} \quad (2.10)$$

From (2.7) and (2.8) we have

$$C_{hijk} = Z_{hijk} - \frac{1}{n-2} \{g_{ij}R_{hk} - g_{ik}R_{hj} + g_{hk}R_{ij} - g_{hj}R_{ik}\} + \frac{2R}{n(n-2)} \{g_{ij}g_{hk} - g_{ik}g_{hj}\} \quad (2.11)$$

From (2.7) and (2.9) we have

$$C_{hijk} = W_{hijk} + \frac{R}{(n-1)(n-2)} \{g_{ij}g_{hk} - g_{ik}g_{hj}\} - \frac{1}{n-2} \{g_{ij}R_{hk} - g_{ik}R_{hj}\} - \frac{2n-3}{(n-1)(n-2)} \{g_{hk}R_{ij} - g_{hj}R_{ik}\} \quad (2.12)$$

**Definition (2.1).** If the curvature tensor  $C_{hijk}$  of  $GRKW_n$  satisfies the condition

$$\dot{\nabla}_m C_{hijk} = K_m C_{hijk} + L_m g_{hijk}, \quad (2.13)$$

where  $K_m$  and  $L_m$  are associate vectors of recurrence then  $GRKW_n$  is called generalized recurrent Kaehlerian Weyl space with generalized recurrent Weyl Conformal curvature tensor. We denote such a space by  $C^* - GRKW_n$ .

**Definition (2.2).** If the curvature tensor  $Z_{hijk}$  of  $GRKW_n$  satisfies the condition

$$\dot{\nabla}_m Z_{hijk} = K_m Z_{hijk} + L_m g_{hijk}, \quad (2.14)$$

where  $K_m$  and  $L_m$  are associate vectors of recurrence then  $GRKW_n$  is called generalized recurrent Kaehlerian Weyl space with generalized recurrent Weyl Concircular curvature tensor. We denote such a space by  $Z^* - GRKW_n$ .

**Definition (2.3).** If the curvature tensor  $W_{hijk}$  of  $GRKW_n$  satisfies the condition

$$\dot{\nabla}_m W_{hijk} = K_m W_{hijk} + L_m g_{hijk}, \quad (2.15)$$

where  $K_m$  and  $L_m$  are associate vectors of recurrence then  $GRKW_n$  is called generalized recurrent Kaehlerian Weyl space with generalized recurrent Weyl Projective curvature tensor. We denote such a space by  $W^* - GRKW_n$ .

**Theorem (2.1) :** A Kaehlerian Weyl space is generalized recurrent if and only if it is Projective generalized recurrent.

**Proof:** Let the space be Kaehlerian Weyl generalized recurrent then (2.1) is satisfied.

Taking covariant derivative of (2.9) we have

$$\dot{\nabla}_m W_{hijk} = \dot{\nabla}_m R_{hijk} + \frac{1}{n-1} \{\dot{\nabla}_m R_{ij}g_{hk} - \dot{\nabla}_m R_{ik}g_{hj}\}, \quad (2.16)$$

using (2.1) and (2.3) above equation reduces to

$$\dot{\nabla}_m W_{hijk} = K_m R_{hijk} + L_m g_{hijk} + \frac{1}{n-1} \{K_m R_{ij}g_{hk} + L_m(n-1)g_{ij}g_{hk} - K_m R_{ij}g_{hj} - L_m(n-1)g_{ik}g_{hj}\},$$

which in view of (2.9) gives

$$\dot{\nabla}_m W_{hijk} = K_m W_{hijk} + L_m g_{hijk}. \quad (2.17)$$

Conversely, let us assume that (2.17) holds then using (2.9) we obtain

$$K_m \left[ R_{hijk} + \frac{1}{n-1} \{ R_{ij} g_{hk} - R_{ik} g_{hj} \} \right] + L_m g_{hijk} = \dot{\nabla}_m R_{hijk} + \frac{1}{n-1} \{ \dot{\nabla}_m R_{ij} g_{hk} - \dot{\nabla}_m R_{ik} g_{hj} \}, \quad (2.18)$$

multiplying (2.18) by  $g^{hk}$  we get

$$\dot{\nabla}_m R_{ij} = K_m R_{ij} + \frac{L_m}{2} g_{ij}. \quad (2.19)$$

Equation (2.18) in view of (2.19) reduces to

$$\dot{\nabla}_m R_{hijk} = K_m R_{hijk} + \frac{2n-3}{2(n-1)} L_m g_{hijk}.$$

which completes the proof.

**Theorem (2.2):** The necessary and sufficient condition for a  $GRKW_n$  to be  $W^* - GRKW_n$  is that it should be  $Z^* - GRKW_n$ .

**Proof:** Let  $GRKW_n$  satisfies the relation (2.15), then (2.15) in view of (2.10) gives

$$K_m W_{hijk} + L_m g_{hijk} = \dot{\nabla}_m Z_{hijk} + \frac{\dot{\nabla}_m R}{n(n-1)} \{ g_{ij} g_{hk} - g_{ik} g_{hj} \} + \frac{1}{n-1} \{ \dot{\nabla}_m R_{ij} g_{hk} - \dot{\nabla}_m R_{ik} g_{hj} \}, \quad (2.20)$$

equation (2.20) in view of (2.3) and (2.4) reduces to

$$\dot{\nabla}_m Z_{hijk} = K_m \left[ R_{hijk} - \frac{R}{n(n-1)} \{ g_{ij} g_{hk} - g_{ik} g_{hj} \} \right] + L'_m g_{hijk},$$

which in view of (2.8) reduces to (2.14).

Conversely, let us assume that (2.14) is satisfied, taking covariant derivative of (2.10) gives

$$\dot{\nabla}_m W_{hijk} = \dot{\nabla}_m Z_{hijk} + \frac{\dot{\nabla}_m R}{n(n-1)} \{ g_{ij} g_{hk} - g_{ik} g_{hj} \} + \frac{1}{n-1} \{ \dot{\nabla}_m R_{ij} g_{hk} - \dot{\nabla}_m R_{ik} g_{hj} \}. \quad (2.21)$$

Using (2.3) and (2.4), equation (2.21) reduces to

$$\dot{\nabla}_m W_{hijk} = K_m [R_{hijk} + \frac{1}{n-1} \{ R_{ij} g_{hk} - R_{ik} g_{hj} \}] + 3L_m g_{hijk},$$

which on using (2.9) gives

$$\dot{\nabla}_m W_{hijk} = K_m W_{hijk} + 3L_m g_{hijk}.$$

Therefore the proof is completed.

**Theorem (2.3):** The necessary and sufficient condition for a  $GRKW_n$  to be  $C^* - GRKW_n$  is that it should be  $Z^* - GRKW_n$ .

**Proof:** Let  $GRKW_n$  satisfies the relation (2.14), then (2.13) in view of (2.14) gives

$$\dot{\nabla}_m C_{hijk} = \dot{\nabla}_m Z_{hijk} - \frac{1}{n-2} \{ g_{ij} \dot{\nabla}_m R_{hk} - g_{ik} \dot{\nabla}_m R_{hj} + g_{hk} \dot{\nabla}_m R_{ij} - g_{hj} \dot{\nabla}_m R_{ik} \} + \frac{2\dot{\nabla}_m R}{n(n-2)} \{ g_{ij} g_{hk} - g_{ik} g_{hj} \} \quad (2.22)$$

Using (2.3), (2.4) and (2.14), equation (2.22) reduces to

$$\dot{V}_m C_{hijk} = K_m Z_{hijk} + L_m g_{hijk} - \frac{1}{n-2} \{K_m (g_{ij} R_{hk} - g_{ik} R_{hj} + g_{hk} R_{ij} - g_{hj} R_{ik}) + 2L_m (n-1)(g_{ij} g_{hk} - g_{ik} g_{hj})\} + \frac{2\{K_m R + L_m n(n-1)\}}{n(n-1)} \{g_{ij} g_{hk} - g_{ik} g_{hj}\}, \quad (2.23)$$

using (2.8), (2.23) reduces to

$$\dot{V}_m C_{hijk} = K_m C_{hijk} + L_m g_{hijk}.$$

Conversely, let  $GRKW_n$  satisfies the relation (2.13), then (2.11) in view of (2.13) gives (2.23). Now from (2.7) and (2.23) we have

$$\begin{aligned} K_m \left[ R_{hijk} - \frac{1}{n-2} \{g_{ij} R_{hk} - g_{ik} R_{hj} + g_{hk} R_{ij} - g_{hj} R_{ik}\} + \frac{R}{(n-1)(n-2)} \{g_{ij} g_{hk} - g_{ik} g_{hj}\} \right] + L_m g_{hijk} \\ = \dot{V}_m Z_{hijk} - \frac{1}{n-2} \{K_m (g_{ij} R_{hk} - g_{ik} R_{hj} + g_{hk} R_{ij} - g_{hj} R_{ik}) + 2L_m (n-1)(g_{ij} g_{hk} - g_{ik} g_{hj})\} \\ + \frac{2\{K_m R + L_m n(n-1)\}}{n(n-1)} \{g_{ij} g_{hk} - g_{ik} g_{hj}\}, \end{aligned}$$

which in view of (2.8) reduces to

$$\dot{V}_m Z_{hijk} = K_m Z_{hijk} + L_m g_{hijk}.$$

Therefore the proof is completed.

**Theorem (2.4):** The necessary and sufficient condition for a  $GRKW_n$  to be  $C^* - GRKW_n$  is that it should be  $W^* - GRKW_n$ .

**Proof:** Let  $GRKW_n$  satisfies the relation (2.14), then (2.13) in view of (2.14) gives

$$\begin{aligned} \dot{V}_m C_{hijk} = \dot{V}_m W_{hijk} + \frac{\dot{V}_m R}{(n-1)(n-2)} \{g_{ij} g_{hk} - g_{ik} g_{hj}\} - \frac{1}{n-2} \{g_{ij} \dot{V}_m R_{hk} - g_{ik} \dot{V}_m R_{hj}\} \\ - \frac{2n-3}{(n-1)(n-2)} \{g_{hk} \dot{V}_m R_{ij} - g_{hj} \dot{V}_m R_{ik}\}, \end{aligned} \quad (2.24)$$

using (2.3), (2.4) and (2.15), equation (2.24) reduces to

$$\begin{aligned} \dot{V}_m C_{hijk} = K_m W_{hijk} + L_m g_{hijk} + \frac{[K_m R + L_m n(n-1)]}{(n-1)(n-2)} \{g_{ij} g_{hk} - g_{ik} g_{hj}\} \\ - \frac{1}{n-2} \{K_m (g_{ij} R_{hk} - g_{ik} R_{hj}) + L_m n(n-1)(g_{ij} g_{hk} - g_{ik} g_{hj})\} \\ - \frac{2n-3}{(n-1)(n-2)} \{K_m (g_{hk} R_{ij} - g_{hj} R_{ik}) + L_m n(n-1)(g_{hk} g_{ij} - g_{hj} g_{ik})\}, \end{aligned} \quad (2.25)$$

using (2.14), (2.25) reduces to

$$\dot{V}_m C_{hijk} = K_m C_{hijk} + L_m g_{hijk}.$$

Conversely, let  $GRKW_n$  satisfies the relation (2.13), then (2.12) in view of (2.13) gives

$$\dot{V}_m C_{hijk} = \dot{V}_m W_{hijk} + \frac{[K_m R + L_m n(n-1)]}{(n-1)(n-2)} \{g_{ij} g_{hk} - g_{ik} g_{hj}\}$$

$$\begin{aligned}
 & -\frac{1}{n-2}\{K_m(g_{ij}R_{hk} - g_{ik}R_{hj}) + L_m n(n-1)(g_{ij}g_{hk} - g_{ik}g_{hj})\} \\
 & -\frac{2n-3}{(n-1)(n-2)}\{K_m(g_{hk}R_{ij} - g_{hj}R_{ik}) + L_m n(n-1)(g_{hk}g_{ij} - g_{hj}g_{ik})\},
 \end{aligned} \tag{2.26}$$

using (2.7), (2.26) reduces to

$$\begin{aligned}
 & K_m \left[ R_{hijk} - \frac{1}{n-2}\{g_{ij}R_{hk} - g_{ik}R_{hj} + g_{hk}R_{ij} - g_{hj}R_{ik}\} + \frac{R}{(n-1)(n-2)}\{g_{ij}g_{hk} - g_{ik}g_{hj}\} \right] + L_m g_{hijk} \\
 & = \dot{\nabla}_m W_{hijk} + \frac{[K_m R + L_m n(n-1)]}{(n-1)(n-2)}\{g_{ij}g_{hk} - g_{ik}g_{hj}\} \\
 & -\frac{1}{n-2}\{K_m(g_{ij}R_{hk} - g_{ik}R_{hj}) + L_m n(n-1)(g_{ij}g_{hk} - g_{ik}g_{hj})\} \\
 & -\frac{2n-3}{(n-1)(n-2)}\{K_m(g_{hk}R_{ij} - g_{hj}R_{ik}) + L_m n(n-1)(g_{hk}g_{ij} - g_{hj}g_{ik})\},
 \end{aligned}$$

using (2.9) above equation reduces to

$$\dot{\nabla}_m W_{hijk} = K_m W_{hijk} + \frac{2n-3}{n-2} g_{hijk}.$$

Therefore the proof is completed.

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