International Journal of Advanced Scientific and Technical Research Issue 3 volume 1, January-February 2013
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

Intermittency route to chaos in the Logistic Map

[1]Hemanta Kr. Sarmah,[2] Tapan Kr. Baishya,[3]DebasishBhattacharjee
[1]Department of Mathematics, Gauhati University, Assam
[2]Department of Mathematics, Debraj Roy College, Golaghat, Assam
[3]Department of Mathematics,B.BorooahCollege,Guwahati,Assam

Abstract :
Intermittency is sporadic switching between two qualitatively different behaviours.The intermittent transition to turbulence was first discussed byPomeau and Manneville[15] in connection with Lorentz model. The type I intermittency is associated with a saddle-node bifurcation or tangent bifurcation in one dimensional maps.The aim of this paper is to examinethe laminar length given by the power law establishedby Pomeauand Manneville [15, 18] in case of the Logistic map .

Key words : Saddle node bifurcation,Chaos,Intermittency.

1. Introduction:

Intermittency denotes a type of behaviour where the dynamics varies chaotically between two different phases of motion. One of these phases is regular (close to stationary, periodic or quasiperiodic motion) and is called laminar phase. The laminar phase is interrupted by turbulent bursts which corresponds to some irregular phases of motion. In chaotic systems, there exist different types of intermittencies. Pomeau and Manneville[15,18]described three different types of intermittency. Each type of intermittencyis related to a different kind of bifurcation. For example type I intermittency occurs when the system is close to a saddle node bifurcation, type II is due to the Hopf bifurcation, and type III due to the reverse period doubling bifurcation. All these types of intermittency yield chaotic behaviour when a system's parameter is varied. This is manifested by more and more frequent turbulent bursts. The mean time between the appearance of bursts becomes shorter and changes according to certain scaling laws which are characteristic to the different types of intermittency.

In crisis-induced intermittency[7,8] a chaotic attractor suffers a crisis, where two or more attractors cross the boundaries of each other's basin of attraction. As an orbit moves through the first attractor it can cross over the boundary and become attracted to the second attractor, where it will stay until its dynamics moves it across the boundary again.

Experimentally, the type I intermittency has been observed in turbulent fluids [3], nonlinear oscillators [12], chemical reactions [19] and Josephson junctions [23]. An excellent introduction to the intermittency route to chaos is given in Schuster [21] and Ott [16].

Manffra, Caldas, Viana and Kalinowski have detected Type I intermittency and crisis induced intermittency in a Semiconductor Laser under Injection Current Modulation[14].

As already mentioned above, intermittency is characterized by rather long laminar phases of the dynamics in the neighbourhood of some former stable fixed point or periodic orbit, interrupted by turbulent bursts. After the bursts, the dynamics come close to the fixed point again. Hence, on one hand we need the presence of a saddle-node bifurcation, which ensures a dynamics corresponding to the laminar phase. Beyond the saddle-node bifurcation the trajectory travels through some small

"channel" in the neighbourhood of the former fixed points. On the other hand a certain re-injection mechanism is necessary which models the turbulent bursts and makes sure that the trajectory comes close to the channel again to repeat the laminar phase.
The logistic map given by

$$f(x) = \mu x(1-x)$$

where μ, the control parameter is the "fertility" or "growth rate" of a population with limited resources, $0 \le \mu \le 4$ and $x \in [0,1]$, fulfils these conditions. It displays the type I intermittency during the transition from the periodic to chaotic state near the period three window [9, 11].

2. Saddle node bifurcation or tangent bifurcation or Fold bifurcation in one dimensional maps:

The saddle-node bifurcation occurs when a stable fixed point and an unstable fixed point appear simultaneously as the parameter μ passes through a critical value,sayμ_t. Reversing the direction of μ, we can see a stable and unstable fixed point merge in phase space and then both disappear.

At a tangent bifurcation, we have

$$\frac{\mathrm{d}f^{(n)}(x^*)}{\mathrm{d}x} = 1$$

Where x^* is a fixed point of the n^{th} iterate of the map function. i.e., the function $f_{\mu_t}^{(n)}(x)$ is tangent to the diagonal line i.e.$y = x$.

In order to explain the phenomena of saddle node bifurcation we consider an one dimensional map $g(x) = \mu - x^2$. This map can be transformed by a change of variables to the logistic map, $x_{n+1} = \mu x_n(1 - x_n)$. Note however that the logistic map does not possesses a tangent bifurcation analogous to that of the equation $g(x) = \mu - x^2$ at $\mu = -\frac{1}{4}$ due to its non-genericbehaviour[8]. On the contrary, for the transformed logistic map, we get a transcritical bifurcation at $\mu = 1$ which corresponds to $\mu = -\frac{1}{4}$ for the map $g(x) = \mu - x^2$.

For themap$g(x) = \mu - x^2$, $x \in \mathbb{R}$,when $\mu < -1/4$ there are no fixed points. At $\mu = -1/4$, the graph of f is tangent to the line $y = x$. Using the condition of tangent bifurcation stated above, the fixed point is found to be $x = -1/2$. As μ increases beyond $-1/4$, the graph of g crosses the line $y = x$ in two points, giving rise to two fixed points of g viz, $x_1 = \frac{-1+\sqrt{1+4\mu}}{2}$ and $x_2 = \frac{-1-\sqrt{1+4\mu}}{2}$which are shown in the following figure.

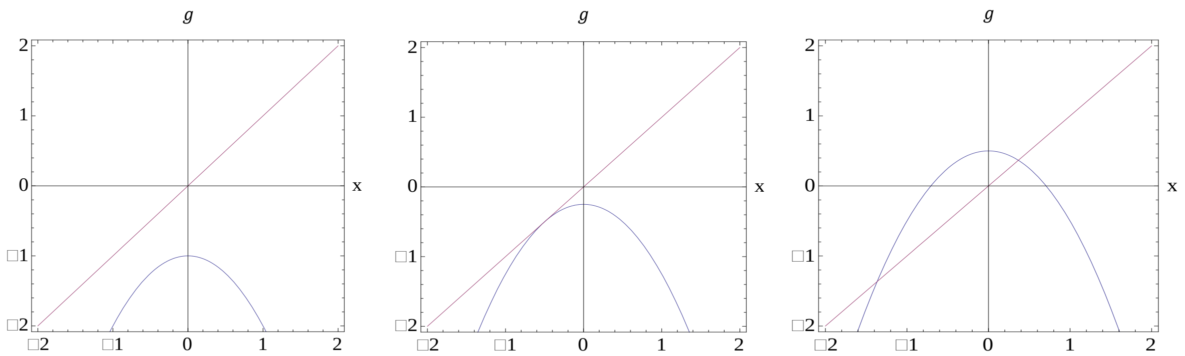


Figure 1: The graph of $g(x) = \mu - x^2$ before, at and following a saddle node bifurcation is shown. (a) At $\mu = -1$, the graph does not intersect the line $y = x$. (b) At $\mu = -\frac{1}{4}$, the graph and the line $y = x$ intersect in one point, the point of tangency; (c) for $\mu > -\frac{1}{4}$, they intersect in two points for e.g. at $\mu = \frac{1}{2}$, g has a repelling fixed point and an attracting fixed point which is shown in the third figure.

To verify the nature of stability of the above fixed points, we observe that

$$\text{For } \mu > -\frac{1}{4}, \quad \left.\frac{dg}{dx}\right|_{x=\frac{-1-\sqrt{1+4\mu}}{2}} = 1 + \sqrt{1+4\mu} > 1$$

$$\left.\frac{dg}{dx}\right|_{x=\frac{-1+\sqrt{1+4\mu}}{2}} = 1 - \sqrt{1+4\mu} < 1$$

Which ascertains that the fixed point$x_1 = \frac{-1+\sqrt{1+4\mu}}{2}$ is an attracting fixed (taking $\mu < 0.75$) point and $x_2 = \frac{-1-\sqrt{1+4\mu}}{2}$ is a repelling fixed point.

If we start with $\mu < -\frac{1}{4}$ and let it increase, then we find that a bifurcation takes place at $\mu = -\frac{1}{4}$. Atthat value of the parameter we have $\frac{dg}{dx} = 1$, i.e. at $\mu = -\frac{1}{4}$, we have a saddle point as shown in figure 2. As μ increases further we have a pair of stable and unstable fixed points.

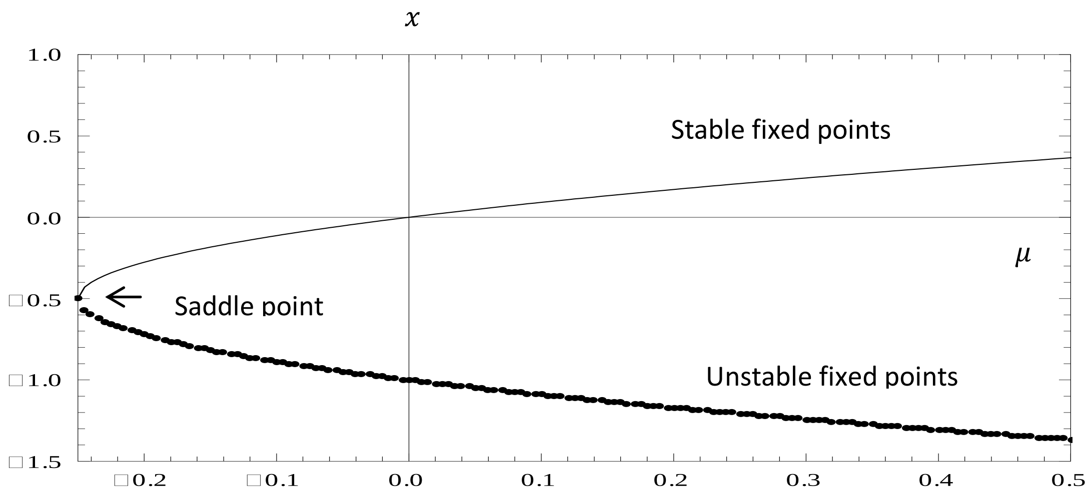


Figure: 2

In reverse direction as μ decreases, the two stable and unstable fixed points collide at $\mu = -\frac{1}{4}$ and annihilate themselves.

In the nonlinear dynamics literature, the bifurcation just described above is called a saddle node bifurcation, tangent bifurcation, or a fold bifurcation.

Thus the saddle node bifurcation is the basic mechanism by which fixed points are created and destroyed. As a parameter is varied, two fixed points move toward each other, collide and annihilate each other.

3. Periodic window and tangent bifurcation in logistic map:

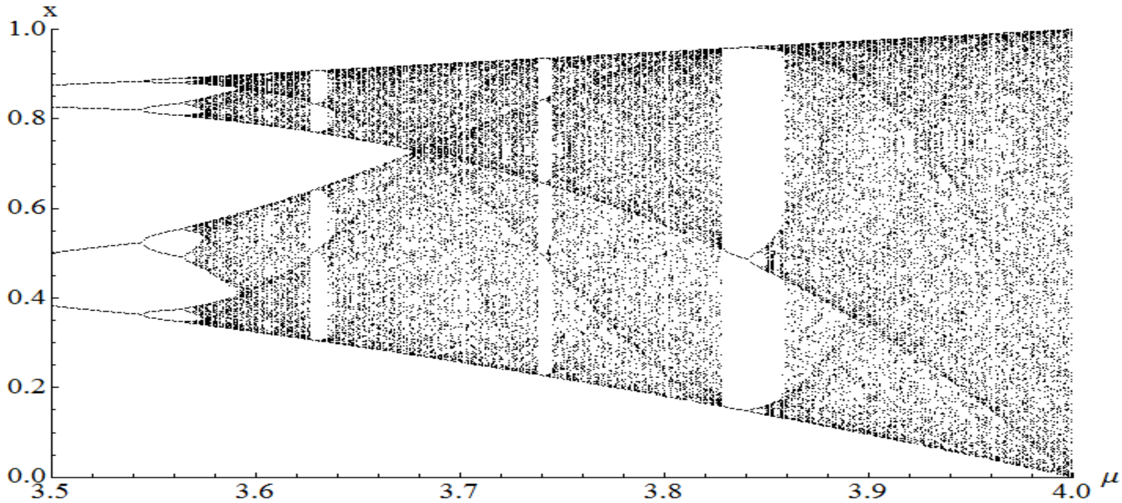


Figure 3: Bifurcation diagram of the logistic map for $3.5 \leq \mu \leq 4$

One of the most interesting features of the orbit diagram or bifurcation diagram (Figure 3) is the occurrence of periodic windows which are marked by the white spaces.These periodic windows appear for $\mu > \mu_\infty = 3.5699456 \ldots$,where μ_∞ is the critical value of the parameter after which chaos creeps into the dynamics given by the logistic map after an infinite series of period doubling bifurcations.The most prominent and largest period 3-window occurs in the interval $3.8284 \ldots \leq \mu \leq 3.8415 \ldots$where at 3.8415… we again visualise period doubling bifurcation[6].

The third-iterate map $f^3(x)$ of the logistic map $f(x) = \mu x(1-x)$ is the key to understanding the birth of the period-3 cycle. Any point p in a period-3 cycle repeats every three iterates, by definition, so such points satisfy $p = f^3(p)$ and are therefore fixed points of the third-iterate map. Since $f^3(x)$ is an eighth-degree polynomial, we cannot solve for the fixed points explicitly. Therefore, we take the help of graph. Figure 4 shows a plot of$f^3(x)$for $=$ 3.835.

The solutions of the equation $f^3(x) = x$ is given by the intersections between the graph of $f^3(x)$and the diagonal line $y = x$. There are eight solutions, six solutions are marked with s_i , $i = 1,2,3$ and u_i , $i = 1,2,3$ and the other two solutions are not the genuine period-3 solutions; they are actually fixed points or period-1 points for which $f(x) = x$. In figure 4, the points marked with s_i , $i = 1,2,3$correspond to a stable period-3 cycle as $\left|\frac{df^3}{dx}\right| < 1$ at these points. Again the points marked with u_i , $i = 1,2,3$ correspond to an unstable period-3 cycle as $\left|\frac{df^3}{dx}\right| > 1$ at these points which are shown in table 1.

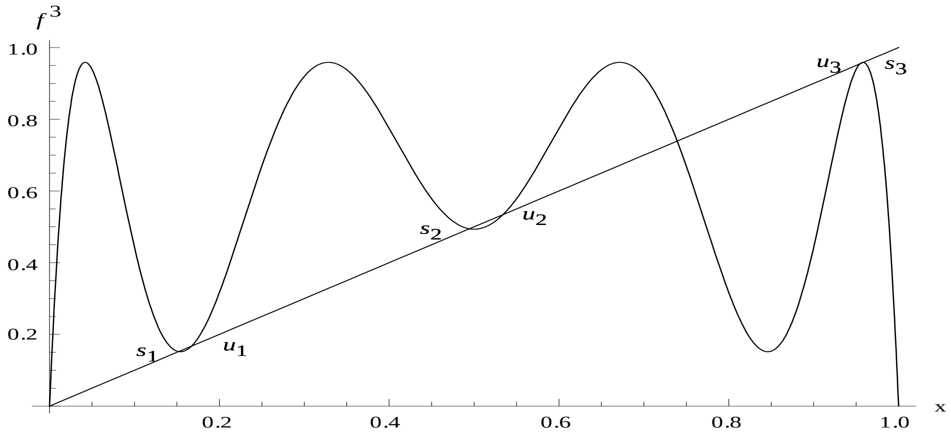


Figure 4. Graphical iteration for f^3 when $\mu = 3.835$

Table 1:

Fixed Point	Value of the derivative
s_1 =0.152074…	-0.394972…
u_1 =0.167205…	2.32052…
s_2 =0.494514…	-0.394972…
u_2 =0.534015…	2.32052…
u_3 =0.954313…	2.32052…
S_3 =0.958635…	-0.394972…

Now, if we decrease μ towards the chaotic regime i.e. to the left of $\mu = 1+\sqrt{8} =$ 3.828427124746 ....where the periodic window starts. Figure 5 shows that when $\mu =$ 3.8, the six marked intersections have vanished. The curve therefore moves away from the diagonal.

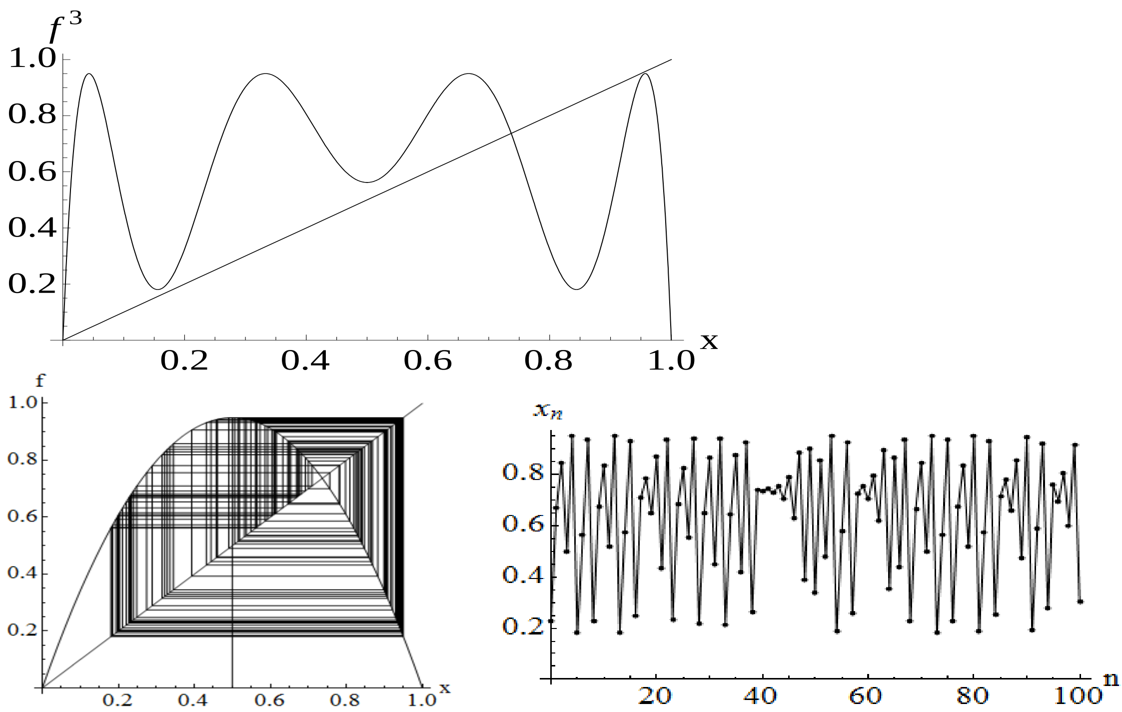


Figure 5: Graph of f^3, Cobweb diagram and time series plot for $\mu = 3.8$ with initial point 0.5

Hence, for some intermediate value between $\mu = 3.8$and $\mu = 3.835$, the graph of $f^3(x)$ must have become tangent to the diagonal. At this critical value ofμ, the stable and unstable period-3 cycles coalesce and annihilate in a tangent bifurcation. This transition defines the beginning of the period three window.

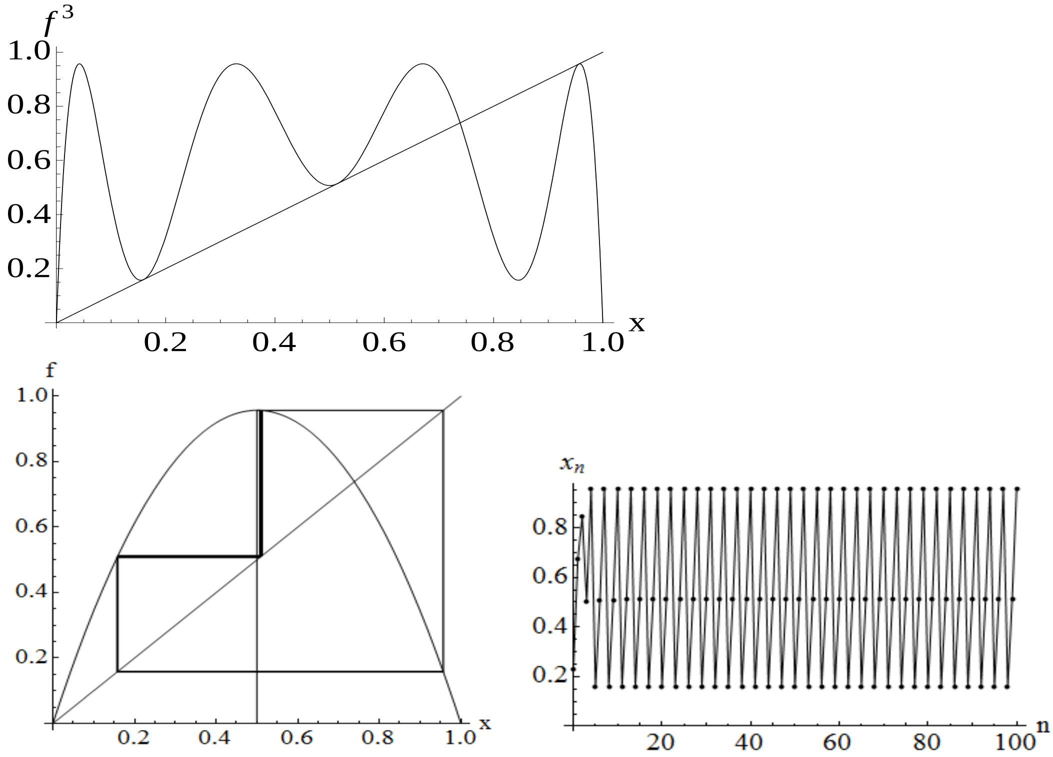


Figure 6: The corresponding cobweb diagram and time series plot at $\mu = 1 + \sqrt{8} =$ 3.828427124746 …. where tangent bifurcation occur

It is already established in [2, 6, 20] that the value of μ at the tangent bifurcation for the period three window in logistic map isexactly given by$1 + 2\sqrt{2} = 3.828427124746 \ldots$.

In figure 6, we have drawn the graph of $f^3(x)$ and corresponding cobweb diagram and time series plot for $\mu = 1 + \sqrt{8} = 3.828427124746 \ldots$.for verification of the above fact.

4. Intermittency and intermittency routes to chaos in logistic Map:

The phenomenon of Intermittency is a routeto chaos in nonlinear systems.The period *n*-behaviour of a iterated map function is determined by the fixed points of the *n*-thiterate of the function. The disappearance of the fixed points is the root cause for the occurrence of intermittency.

We have already discussed the mechanism of how the period 3 window occur at $\mu_t = 1 + \sqrt{8} = 3.8284 \ldots$ inside the chaotic region. Inside the periodic window for $\mu \geq \mu_t = 3.8284 \ldots$ the behaviour is completely periodic. If we decrease μ inside the periodic window, at μ_t the two stable and unstable fixed points collide and annihilate each other in a tangent bifurcation. In contrast to period doubling, the previously stable fixed points are not replaced by new stable fixed points. Hence the motion becomes aperiodic. As μ decreases from μ_t, the aperiodic behaviour increases and periodic behaviour decreases.

Figure 7 and figure8 shows the time series plot and cobweb diagram with its blown up area inside the box of Logistic map function for the parameter value $\mu = 3.82$ and $\mu = 3.827$with initial point 0.227 and 0.5 respectively.

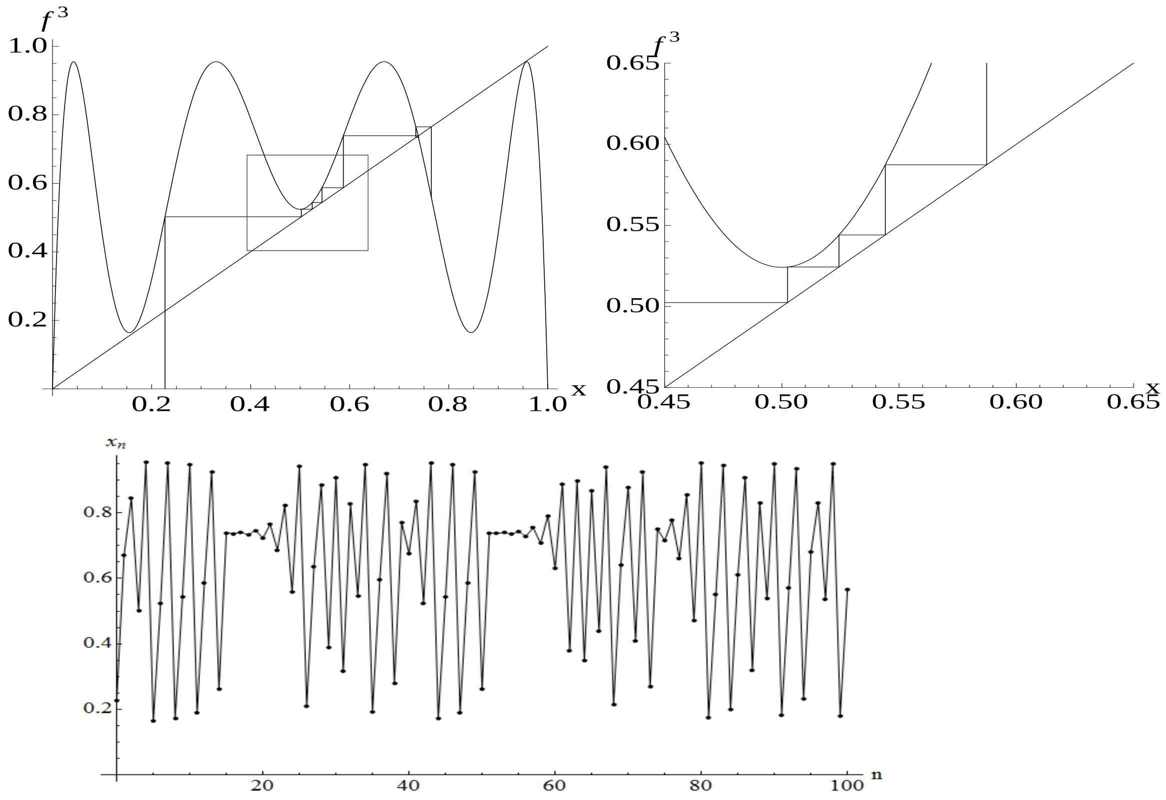


Figure 7

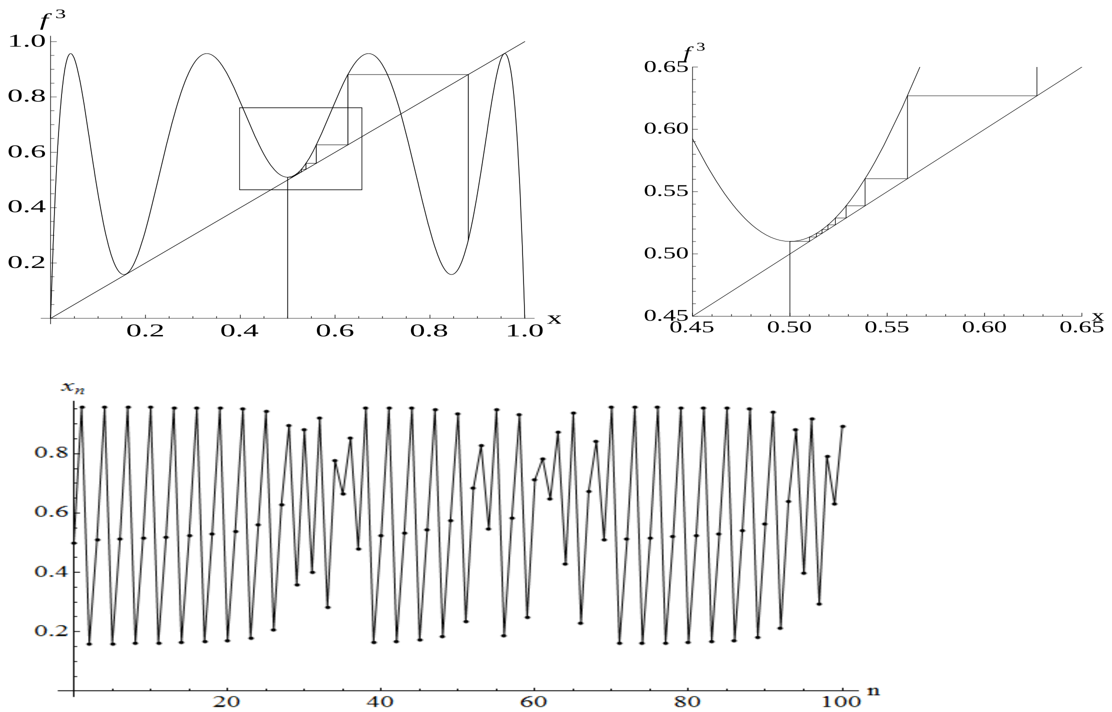


Figure 8:

For parameter value $\mu = 3.82$, the figure 7 shows that there are three narrow gaps between the curve $f^3(x)$ and the diagonal. We focus on the middle gap shown in the small box. The graphic iteration technique shows that a trajectory spends a significant amount of time (many successive iterations) nearly previously stable period -3 fixed points which was attained for the parameter value $\mu = 1 + \sqrt{8}$=3.8284271…. The amount of time (number of iterations) which the map spends in the narrow channel is known as the laminar region. Eventually, however, the trajectory is repelled from this region and wanders off to another region of state space. In figure 8we observe a similar type of behaviour but at the same time it throws light on the fact that as the gap size decreases, the trajectory spends more time in the gap region. During the passage through the narrow channel$f^3(x_n) \approx x_n$ and so the orbit looks like a 3-cycle.This explains why we see the apparent periodic behaviour. This is qualitative evidence that more time is spent in "periodic" behaviour as the parameter approaches the parameter value μ_t, at which point the gap between the curve and the line $y = x$vanishes and the behaviour becomes exactly periodic.

5. Length of the Laminar Region:

In the pioneering studies [15, 18], it was found that the number of iterations followed an $\varepsilon^{-1/2}$(where $= \mu_t - \mu$)dependence for the logistic map. We further recall that μ_t is the parameter value where the tangent bifurcation occurs and μ is any parameter value prior to μ_t , of course within a certain specific intervals. In the work by [11], an expression for the number of iterations spent inside the channel was developed which we have verified in the following way.

The third iterate of the Logistic Map is given by

$$f^3(x,\mu) = \mu^3 x - \mu^3 x^2 - \mu^4 x^2 - \mu^5 x^2 + 2\mu^4 x^3 + 2\mu^5 x^3 + 2\mu^6 x^3 - \mu^4 x^4 - \mu^5 x^4 - 6\mu^6 x^4 + 6\mu x^5 + 4\mu^7 x^5 - 2\mu^6 x^6 - 6\mu^7 x^6 + 4\mu^7 x^7 - \mu^7 x^8$$

At $\mu_t = 1 + \sqrt{8}$, where the period three window begins through a saddle node bifurcation.

$$\frac{d}{dx} f^3(x_t, \mu_t) = 1, f^3(x_t, \mu_t) = x_t$$

Expanding $f^3(x,\mu)$ around (x_t, μ_t) in Taylor's series, where x_t is one of the three fixed points,$f^3(x,\mu) = f^3(x_t,\mu_t) + (x - x_t)\frac{\partial}{\partial x} f^3(x_t,\, \mu_t) + (\mu - \mu_t)\frac{\partial}{\partial \mu} f^3(x_t, \mu_t)$

$$+\frac{1}{2}(x - x_t)^2 \frac{\partial^2}{\partial x^2} f^3(x_t,\, \mu_t) + higher\ order\ terms$$

$$= x_t + (x - x_t).1 + (\mu - \mu_t)\frac{\partial}{\partial \mu} f^3(x_t, \mu_t) + \frac{1}{2}(x - x_t)^2 \frac{\partial^2}{\partial x^2} f^3(x_t,\, \mu_t) + higher\ order\ terms$$

$$= x + \varepsilon\, b_t + (x - x_t)^2 a_t \qquad (1)$$

where$\varepsilon = \mu_t - \mu$ and $b_t = -\frac{\partial}{\partial \mu} f^3(x_t, \mu_t)$,$a_t = \frac{1}{2}\frac{\partial^2}{\partial x^2} f^3(x_t,\, \mu_t)$. Below in the table we have seen that although the value of a_t and b_t are different for different fixed point their products are equal.

Table 2:

	x_t(fixed point)	a_t	b_t	$a = a_t b_t$
1	0.1599288109970451 ...	88.91013710422149 ...	0.7793990594531017...	69.29647723487655 ...
2	0.5143552630501662 ...	34.145309970391004 ..	2.0294579396691006...	69.29647042187253 ...
3	0.9563178296531996...	−310.648221390486 ...	−0.2230702123943047 ..	69.29636472548876 ...

Since each step is very small one may approximate $x_{n+1} - x_n$ by dx and since the number of steps through the channel is very large we may write $dn = 1$. These assumptions transform equation (1), into an integrable differential equation:

$$\frac{dx}{dn} = a_t(x - x_t)^2 + \varepsilon\, b_t$$

Hence we get $\int_{x_{in}}^{x_{out}} \frac{dx}{a_t(x-x_t)^2+\varepsilon\, b_t} = \int_0^N dn$ where $x_{in} \le x_t \le x_{out}$ and N is the number of iterations inside the narrow channel.

$$\Rightarrow \left[\frac{1}{\sqrt{\varepsilon\, a_t\, b_t}} \tan^{-1}\left((x - x_t)\sqrt{\frac{a_t}{\varepsilon\, b_t}}\right)\right]_{x_{in}}^{x_{out}} = [n]_0^N$$

$$\Rightarrow \frac{1}{\sqrt{\varepsilon\, a_t\, b_t}}\left[\tan^{-1}\left((x_{out} - x_t)\sqrt{\frac{a_t}{\varepsilon\, b_t}}\right) - \tan^{-1}\left((x_{in} - x_t)\sqrt{\frac{a_t}{\varepsilon\, b_t}}\right)\right] = N \qquad (2)$$

"x_{in}" is the entrance to the tangency channel and "x_{out}" is the exit value.

Hirsch, Huberman and Scalapino [11] have considered a transformation $y = \frac{x - x_t}{b_t}$. Applying this transformation the equation reduces to

$$N(y_{in}) = \frac{1}{\sqrt{a\varepsilon}}\left[\tan^{-1}\left(y_{out}\sqrt{\frac{a}{\varepsilon}}\right) - \tan^{-1}\left(y_{in}\sqrt{\frac{a}{\varepsilon}}\right)\right] \qquad (3)$$

"y_{in}" is the entrance to the tangency channel and "y_{out}" is the exit value and one has that
$-y_{out} \le y_{in} \le y_{out}$.

This yields the number of iterations to travel the channel is approximately

$$N \equiv \frac{2}{\sqrt{a\varepsilon}} \tan^{-1}\left(\frac{y_{out}}{\sqrt{\frac{\varepsilon}{a}}}\right) \qquad (4)$$

For $\sqrt{\frac{\varepsilon}{a}} \ll y_{out}$, $N = \pi . \frac{1}{\sqrt{a\varepsilon}} = \frac{k}{\sqrt{\varepsilon}}$,

where $k = \frac{\pi}{\sqrt{a}}$ and its value for logistic map is approximately *0.37739341268365634* ....

Thus the length of the laminar phase ie.the total number of iterations inside the channel N varies as

$$N \propto \varepsilon^{-1/2}$$

where $\varepsilon = \mu_t - \mu$.

6. Verification of the above theoretical value of the constant k:

Below in the table 3 we have shown the number of iterations inside the channel considering the first laminar region taking 0.5 as an initial point for the middle fixed point0.514355… of the period three window of the logistic map for $\mu = 1 + \sqrt{8}$. We have considered $x_{in} = 0.504$..and $x_{out} = 0.525$.

Table 3:

a	$\mu_a = \mu_t - 10^{-a}$	$1/\sqrt{\mu_t - \mu}$	$N(\mu_a)$, i.e. number of iterations within the narrow channelwhich is calculated on the basis of points used for generations of the graphs in figure 10	$k = N(\mu_a)\sqrt{\mu_t - \mu_a}$which is the observed value of k calculated on the basis of observed value of$N(\mu_a)$ of the previous column.
1	$3.7284271247461...$	$\sqrt{10^1}$	0	0
2	$3.8184271247461...$	$\sqrt{10^2}$	1	0.1
3	$3.8274271247461...$	$\sqrt{10^3}$	7	0.221359
4	$3.8283271247461...$	$\sqrt{10^4}$	32	0.32
5	$3.8284171247461...$	$\sqrt{10^5}$	113	$0.3573373755990...$
6	$3.8284261247461...$	$\sqrt{10^6}$	371	0.371
7	$3.8284270247461...$	$\sqrt{10^7}$	1187	$0.3753623582619...$
8	$3.8284271147461...$	$\sqrt{10^8}$	3768	0.3768
9	$3.8284271237461...$	$\sqrt{10^9}$	11928	$0.3771964793048...$
10	$3.8284271246461...$	$\sqrt{10^{10}}$	37733	0.37733
11	$3.8284271247361...$	$\sqrt{10^{11}}$	119337	0.3773767291315…
12	$3.8284271247452...$	$\sqrt{10^{12}}$	377407	0.377407
13	$3.8284271247461...$	$\sqrt{10^{13}}$	1195148	$0.3779389820994...$
μ_t	$1 + \sqrt{8}$		∞	

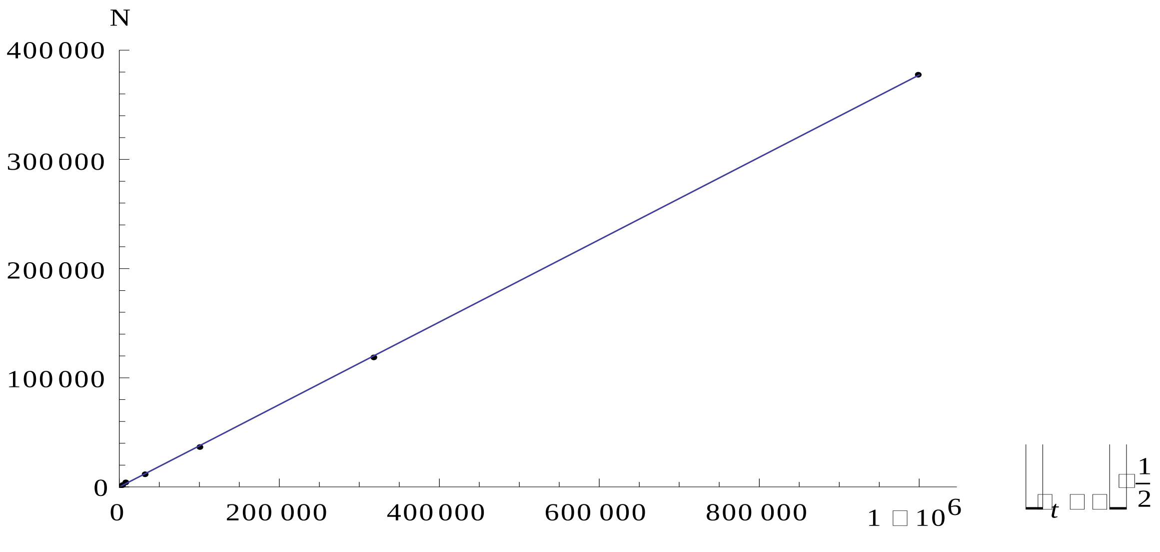


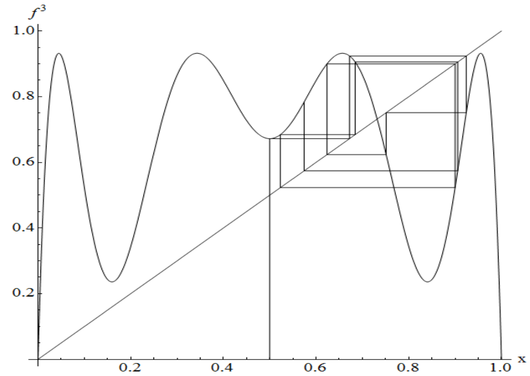
Figure 9

The slope of the above points when fitted with a straight line [figure 9] by least square method is found to be 0.377413with a mean deviation of0.000382827

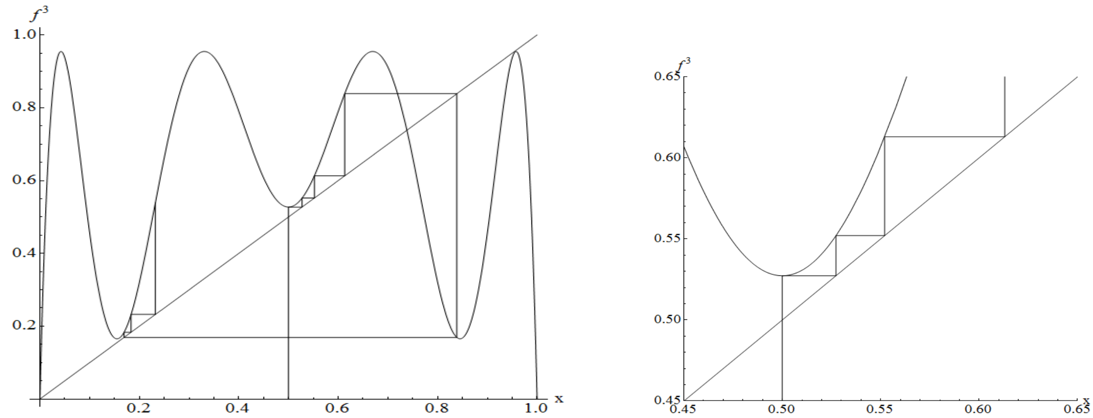
From table 3 it is observed that as μ gets close to μ_t the number of iterations increase rapidly. The last column lists the product$N(\mu_a)\sqrt{\mu_t - \mu_a}$ and reveals that it converges to approximately

0.377413… as the parameter μ approaches to μ_t and which is in agreement to the theoretical value already found in (4), up to three decimal places.

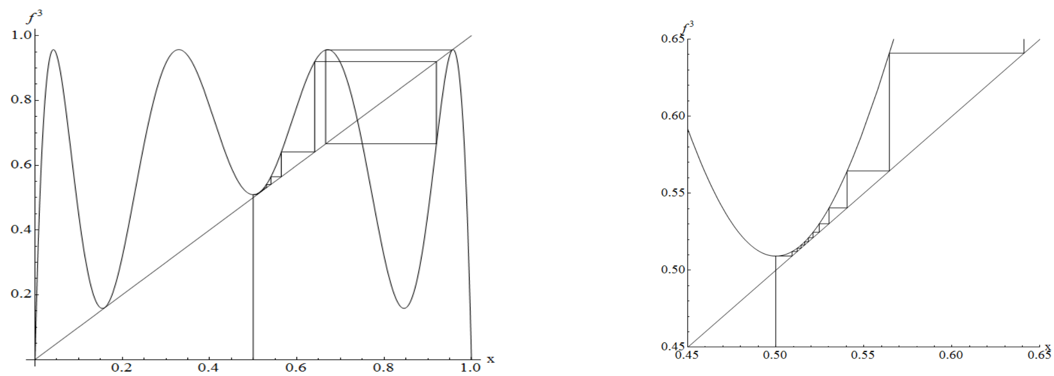
In figure 10 from the cobweb diagram plot it is quite evident that for the parameter value $\mu_a = \mu_t - \frac{1}{10^a}$ it shows chaotic behaviour for $a = 1$whereas it shows intermittent behaviour for $a = 2$ onwards as the iterates passes through narrow channels in the neighbourhood of the point *0.514355* ...



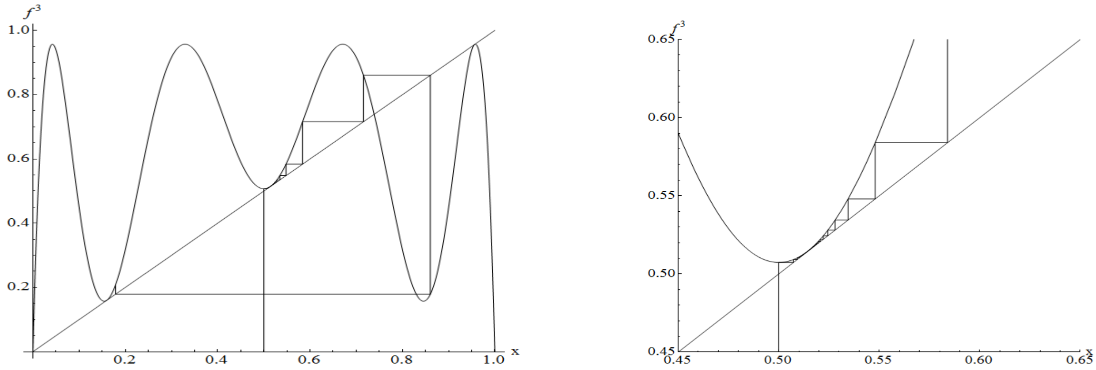
$a = 1, \mu = 3.72842712474619$
(i)



$a = 2, \mu = 3.81842712474619$
(ii)



$a = 3, \mu = 3.8274271247461904$
(iii)



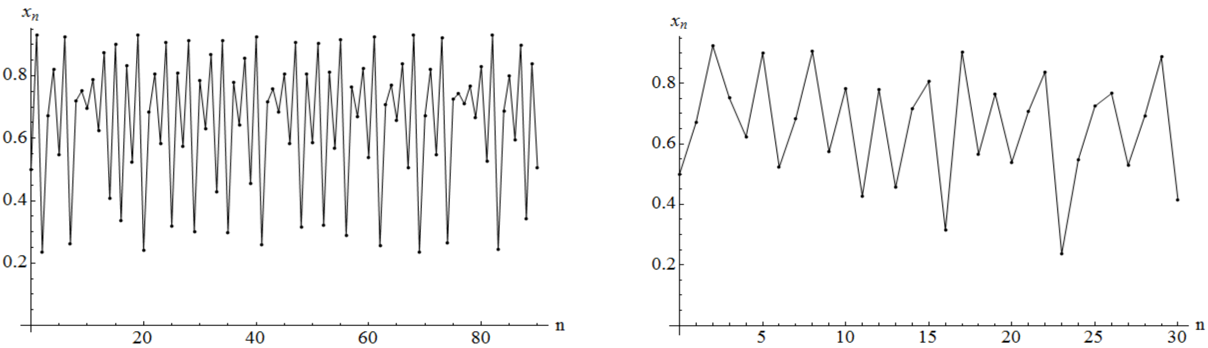
$a = 4, \mu = 3.82832712474619$
(iv)

Figure 10.[(i),(ii),(iii),(iv)]

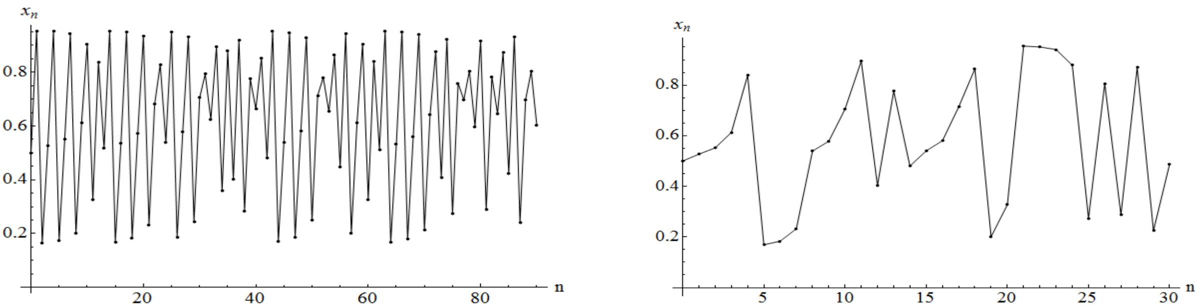
In table 4 we have compared the observed value of $N(\mu_a)$ which was used in table (3) and its corresponding theoretical value given by relation (2) considering $x_{in} = 0.504$ ...and $x_{out} = 0.525$ ... . It further verifies that the phenomena of intermittency occurs in the region $|\mu_t - \mu| \leq \frac{1}{10^2}$, which was reflected in the cobweb diagrams of figure 10.

Table 4:

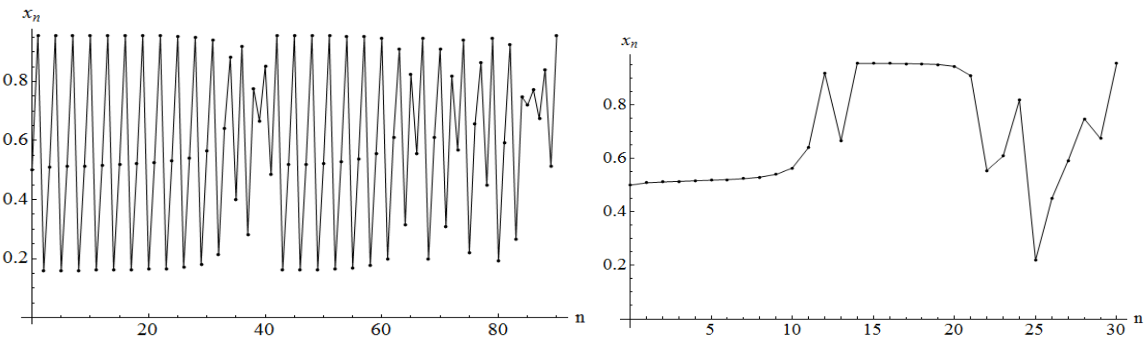
a	$\mu_a = \mu_t - 10^{-a}$	$1/\sqrt{\mu_t - \mu}$	Iteration no. for x_{in}	$x_{in} = 0.504$ ...	Iteration No. for x_{out}	$x_{out} = 0.525$ ...	$N(\mu_k)$ Observed value	$N(\mu_k)$ Theoretical value
1	3.728427124746…	$\sqrt{10^1}$					0	0
2	3.818427124746…	$\sqrt{10^2}$					0	0
3	3.827427124746…	$\sqrt{10^3}$	1	0.509194…	7	0.524674…	7	7
4	3.828327124746…	$\sqrt{10^4}$	1	0.507348…	32	0.524432…	32	32
5	3.828417124746…	$\sqrt{10^5}$	1	0.507163…	113	0.523317…	113	113
6	3.828426124746…	$\sqrt{10^6}$	1	0.507144…	371	0.523272…	371	371
7	3.828427024746…	$\sqrt{10^7}$	1	0.507142…	1187	0.523225…	1187	1187
8	3.828427114746…	$\sqrt{10^8}$	1	0.507142…	3768	0.524372…	3768	3768
9	3.828427123746…	$\sqrt{10^9}$	1	0.507142…	11928	0.523656…	11928	11928
10	3.828427124646…	$\sqrt{10^{10}}$	1	0.507142…	37733	0.523337…	37733	37733
11	3.828427124736…	$\sqrt{10^{11}}$	1	0.507142…	119337	0.523305…	119337	119337
12	3.828427124745…	$\sqrt{10^{12}}$	1	0.507142…	377407	0.524453…	377407	377388
13	3.828427124746…	$\sqrt{10^{13}}$	1	0.507142…	1195148	0.524176…	1195148	1193420
μ_t	$1+\sqrt{8}$	0						∞



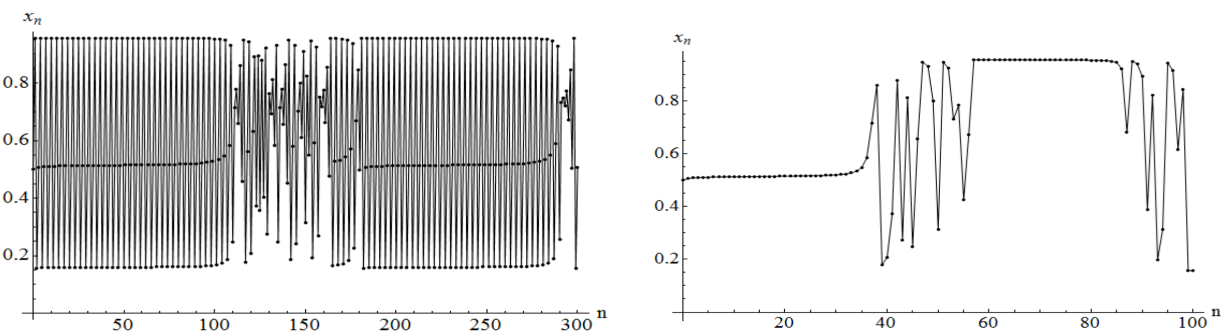
$a = 1, \mu = 3.72842712474619$
(i)



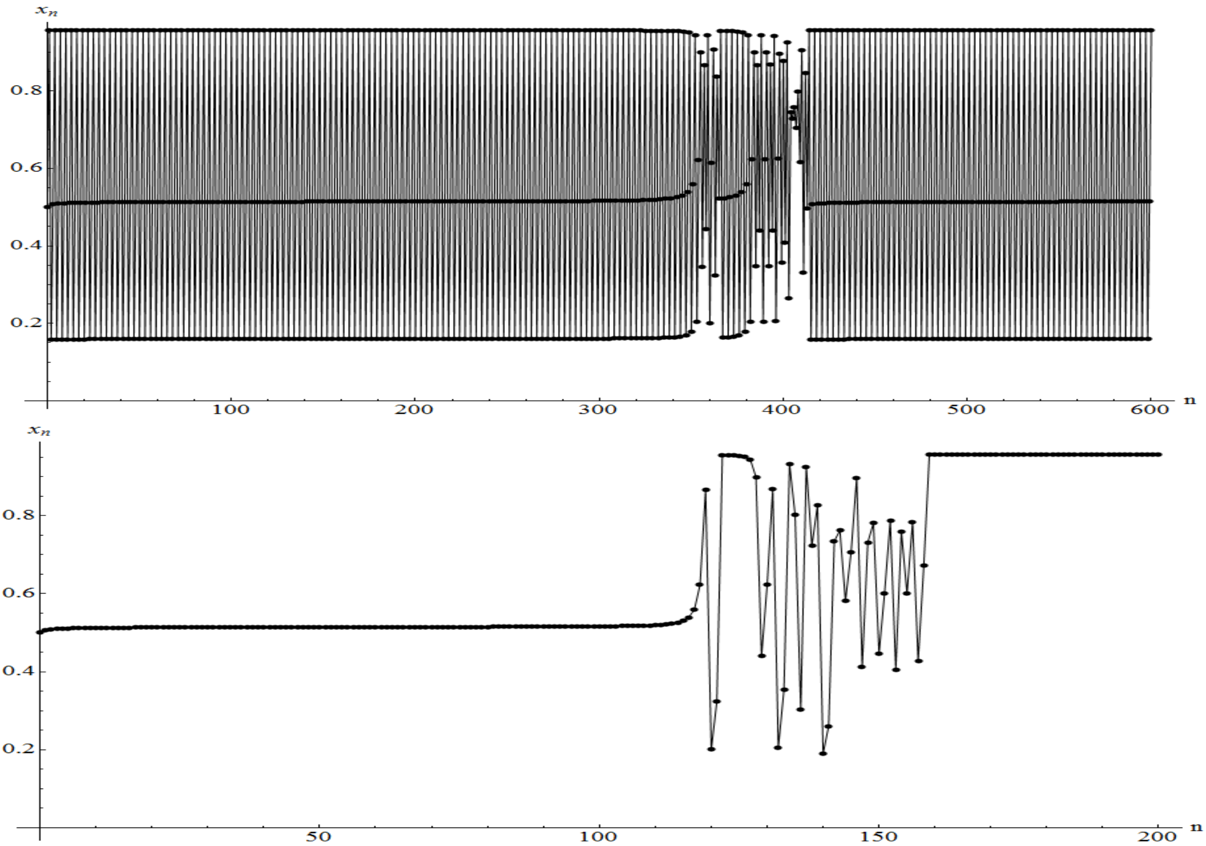
$a = 2, \mu = 3.81842712474619$
(ii)



$a = 3, \mu = 3.8274271247461904$
(iii)



$a = 4, \mu = 3.82832712474619$
(iv)



$a = 5, \quad \mu = 3.8284261247 \ldots$

Figure 11 $[(i),(ii),(iii),(iv),(v)]$

In figure 11 $[(i),(ii),(iii),(iv),(v)]$we have plotted the time series plot for f and f^3 in pair for different parameter values within $\mu_a = \mu_t - \frac{1}{10^a}$and μ_t. The above figures clearly reflect that the length of the laminar region where periodic nature is present gets elongated as it approaches the parameter value μ_t after which that dynamics become totally regular till it reaches the end of the periodic window through an infinite series of period doubling bifurcation.

Average length of the laminar region near the period three window:

In table 3 and 4 we calculated the length of laminar region for different values of a with initial point 0.5 which is the critical point for the logistic map. During the phenomena of intermittency the iterates of the map pass through some narrow channel and after sometime (number of iterations) escapes from the channelbut gets re-injected to the narrow channel after some intermittent turbulent bursts. We cannot expect that every time it will get re-injected to the channel with the initial value $x = 0.5$ and hence we made tables (table 5 and table 6 for different values of a) where we listed the length of the laminar region for different values of x_{in} and x_{out} . In table 5 and table 6 we iterated the logistic map 10000 times and 15000 times respectively.

Table 5 [For $a = 4$ and $\mu_a = 3.8283271247461...$]

x_{in}	Iteration No. (x_{in})	x_{out}	Iteration No.(x_{out})	Length of the Laminar region
0.507348…	1	0.524432…	32	32
0.507454…	171	0.524546…	202	32
0.504166…	241	0.522659…	272	32
0.510434…	382	0.524718…	411	30
0.507699…	619	0.524833…	650	32
0.507362…	1086	0.524447…	1117	32
0.512152…	1274	0.524656…	1300	27
0.514763…	1316	0.523356…	1331	16
0.509576…	1628	0.522667…	1657	30
0.508159…	2034	0.522856…	2064	31
0.511149…	2131	0.524642…	2159	29
0.506685…	2250	0.52384…	2281	32
0.510663…	2506	0.522921…	2534	29
0.507566…	2697	0.524673…	2728	32
0.508287…	2738	0.522994…	2768	31
0.51337…	2868	0.522258…	2889	22
0.508612…	2900	0.523389…	2930	31

Average length of the laminar region

(32+32+32+30+32+32+27+16+30+31+29+32+29+32+31+22+31)=500/17=29.4

Table 6 [For $a = 5$ and $\mu_a = 3.8284171247461...$]

x_{in}	Iteration No. (x_{in}))	x_{out}	Iteration No. (x_{out})	Length of the Laminar region
0.507163…	1	0.523317…	113	113
0.510063…	289	0.523534…	399	111
0.511664…	407	0.524395…	514	108
0.507554…	543	0.523681…	655	113
0.508044…	777	0.524268…	889	113
0.508649…	1427	0.522847…	1538	112
0.513213…	2130	0.522601…	2225	96
0.51067…	2538	0.523351…	2647	110
0.512545…	2832	0.522841…	2934	103
0.51375…	3586	0.52355…	3670	85
0.513617…	3710	0.524424…	3798	89
0.507172…	3815	0.523324…	3927	113
0.506633…	4518	0.522927…	4630	113
0.507925…	4840	0.52411…	4952	113

The average length of the laminar region is

$$(113 + 111 + 108 + 113 + 113 + 112 + 96 + 110 + 103 + 85 + 89 + 113 + 113 + 113)/14$$

$$= 106.57$$

From tables 5 and 6 it is seen that the number of iterations in first laminar region is in good conformity with the average length of the laminar region. We further calculated it for $a = 6$ and got them to be 365 for the laminar phase when initial value is taken as 0.5 and the average length of laminar phase was got 363 respectively.

7. Conclusion:

The phenomenon of intermittency is quite common in systems where the transition from periodic to chaotic behaviour takes place through a saddle node bifurcation.In our present investigation, we have verified the power law which states that the number of iterations $[N(\mu)]$ inside the narrow channel is proportional to $(\mu_t - \mu)^{-\frac{1}{2}}$; in case of the logistic map. We found out the constant of proportionality in the above case for period three window and this was found approximately to be 0.377413... .

We think that there is need of research to find the value of this constant for other periodic windows (of period 5, 7 and so on)andto see if there is any relationship between those values.

References

1. Alligood, K. T. ; Sauer, T. D. ; Yorke, J. A. ; Chaos: An Introduction to Dynamical Systems ; Springer-Verlag New York, Inc, 1996.
2. Bechhoefer, J.; The birth of period three, revisited, Mathematics magazine, 69, 1996, pp 115-118
3. Berge, P.; Dubois, M.; Manneville, P. &Pomeau, Y. ;Intermittency in Rayleigh-Benard convection; J. Phys. Lett. 41, pp 341-354. 1980
4. Feudel, U.; Kuznetsov,S.; Pikovsky, A.; Strange Non Chaotic Attractors: Dynamics Between Order And Chaos in Quasiperiodically Forced Systems; World Scientific Publishing Co. Pte. Ltd. 2006.
5. Froyland, J.: Introduction to Chaos and Coherence. IOP Publishing Ltd., London, 1994.
6. Gordon, W. B. ; Period three trajectories of logistic map, Mathematics Magazine, Vol. 69, No. 2, April, 1996, pp. 118-120
7. Grebogi, C. ; Ott, E.; Romeiras, F. and Yorke, J. A. ; Critical exponents for crisis-induced intermittency, Physical Review A, Volume 36, Number 11, December 1, 1987, pp5365-5380.
8. Grebogi, C. ; Ott, E. and Yorke, J. A. ; Chaotic attractor in crisis, Physical Review Letters, Volume, 48, Number 22, 31 May 1982, pp 1507-1510
9. Hanssen, J. and Wilcox, W. ; Lyapunov Exponents for the Intermittent transition to chaos, International Journal of Bifurcation and Chaos, Vol. 9, No. 4, 1999, 657-670
10. Hilborn, R.C. ; Chaos and Nonlinear Dynamics. An introduction for Scientists and Engineers, Oxford University Press, 2006.
11. Hirsch, J. E. ; Huberman, B.A. ; Scalapino, D.J. ; Theory of intermittency, Physical Review A Volume 25, Number I, January 1982, pp 519-532
12. Jeffries, C. & Perez, J.;Observation of a PomeauManneville intermittent route to chaos in a nonlinearoscillator;Phys. Rev. A26, pp 2117-2122 ,1982
13. Klimaszewska K. and Zebrowski J. J.; Detection of type of intermittency using characteristic patterns in recurrence plots, Physical review E 80, 026214, 2009, pp 1-14
14. Manffra, E. F.; Caldas, I.L.; Viana, R. L.; Kalinowski H. J. ; Type-I Intermittency in a Semiconductor Laser under Injection Current Modulation.,Nonlinear Dynamics 27: 2002, pp 185-195.
15. Monnville P. and Pomeau, Y. : Intermittency and Lorentz Model; Physics. Lett. 75A, 1979, pp 1-2
16. Ott, E.; Chaos in Dynamical Systems; Cambridge University Press, New York 1993.
17. Peitgen, H.O.; Jürgens, H. and Saupe D.: Chaos and Fractals; New Frontiers of Science, Second Edition, Springer Verlag, New York, 2004.
18. Pomeau, Y. and Monnville P. : Intermittent Transition to Turbulence in Dissipative Dynamical Systems; Communication math. Phys. 74, 1980, pp 189-197.
19. Pomeau, Y.; Roux, J. C.; Rossi, A.; Bachelart, S. & Vidal, C.; Intermittent behavior in theBelousov-Zhabotinsky reaction," J. Phys. Lett. 41, 1981 Pp 271-273.
20. Saha, P. and Strogatz, S.H. : The Birth of Period Three, Mathematics Magazine, Vol. 68, No. 1, 1995, pp 42-47.

21. Schuster H.G. and Just W., Deterministic Chaos: An Introduction, Fourth, Revised and Enlarged Edition. WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim, 2005
22. Strogatz, S. H.; Nonlinear dynamics and Chaos, with applications to Physics, Biology, Chemistry and Engineering; Perseus Books Publishing, L.L.C. ,1994
23. Weh, W. J. & Kao, Y. H. ;IntermittencyinJosephson junctions; Appl. Phys. Lett. 42, 1983,pp 299-301.