# Analysis of Power flowand Torque of Asynchronous Induction Motor Equivalent circuits

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## ABSTRACT

Asynchronous motors are used for domestic and industrial purposes. Due to the wide range of application areas of this kind of motors, there is a need for careful analysis of their circuits' equivalent. The analysis is lacking in most of the research associated with this kind of system. In this paper, the steady state per phase equivalent circuit ofasynchronous induction motors, the variations of induction motor parameters from standstill to running conditions and the power flow diagram areinvestigated for the purpose of studying the performance characteristics.The advantages of wound rotor (slip ring) induction motor over a squirrel cage rotor induction motor are also investigated.It was observed that squirrel cage rotor induction motor (IM) has low starting torque and high starting current while the wound rotor IM has high starting torque and low starting current.The system behavior is observed using simulation approachand the results were analyzed by MATLAB.

**Keywords:** Equivalent circuit per phase, Asynchronous induction motor, Standstill conditions, Running conditions, Motor parameters, Squirrel cage induction motor, Wound rotor induction motor.

## 1.0 INTRODUCTION

Asynchronous motor converts electrical power into mechanical power. Three phase power supply is given to the stator winding and rotating magnetic field is created inside the stator. The rotating magnetic field induces current in the rotor conductors. The induced current interacts with the rotating magnetic field which causes the rotor to rotate at a speed less than the synchronous speed of the rotating magnetic field.The speed of the rotor must be less than the synchronous speed otherwise there will be no induced current in the rotor and the rotating magnetic field will not move relative to rotor conductors. The difference between the speed of the rotor and that of the rotating magnetic field is called the slip. Squirrel cage rotor and wound rotor are the two types of rotorsof asynchronous induction motor (IM). The rotor of squirrel cage is made of solid copper bars or aluminum strips which span the length of the rotor and shorted by end ring. The rotor bars are skewed to reduce noise and harmonics. The wound or slip ring rotor type replaces the bars of the squirrel cage rotor with insulated winding that is similar to that of the stator. The winding is wye-connected with the terminals brought out to three slip rings on the shaft. Graphite brushes are connected to the slip rings in order to provide external access to the rotor winding that is connected to rheostatic controller for the purpose of inserting additional resistance in each rotor phase in order toimprove the starting torque [1].

The input data of a squirrel cage rotor induction motor are simulatedand the motor when replaced by wound rotor with variable resistances is investigated, simulated and the results analyzed. The equivalent circuit per phase of three phaseinduction motor shows the electrical parametersof the stator and the rotor core. The stator core is the stationary part of the motor and it consists of a series connection of the stator reactance per phase and the stator resistance per phase and both are connected parallel to the parallel connection of the resistance representing core losses and the magnetizing reactance per phase. The magnetizing component and iron loss are voltage dependant and not on load dependant. The space between the stator and the rotor is the air gap where energy conversion takes place. The second part is the rotor which is the rotating part of the motor.The rotor consists ofa series connection of both rotor resistance per phase and the rotor reactance per phase as the motor remains stationary.Under running conditions, each of these parameters such as rotor induced electromotive force,rotor frequency and rotor reactance per phase are multiplied by the slip of the motor [2,3].

Induction motor at rest is like a short-circuited transformer. When an induction motor is connected to a full voltage supply, it draws high current known as Locked Rotor Current (LRC) and the torque produced at that moment is known as the Locked Rotor Torque (LRT). The locked rotor current and the locked rotor torque are functions of the terminal voltage to the motor and the motor design. The torque and the current alter as the motor accelerates under constant supply voltage.The power flow diagram provides the stages of power loss from the input to the shaft of the motor. The advantages of wound rotor (slip ring) induction motor over a squirrel cage rotor induction motor are discussed by virtue of the magnitude of the starting torque and the starting current developed by each of the motors are investigated.

The remainder of this paper is organized as follows: Section 2 describes the theory of Slip of an induction motor. In Section 3, we analyze the equivalent circuit per phase of an induction motor. Section 4 gave a detailed analysis of power flow in an induction motor. Analysis of Torque of an induction motor is given in Section 5. In Section 6, simulation results and the results analysis is given. We give concluding remarks in Section 7.

## 2.0 SLIP

The slip of an induction motor (IM) is the ratio of the difference between the rotor speed and the synchronous speed which is expressed as $s = \frac{N_s - N_r}{N_s}$ where Ns and Nr are the synchronous speed and rotor or shaft speed respectively.The rotor normally slips number of revolutions behind the magnetic rotating field of the stator. The larger the load on the shaft of the motor, the larger the rotor slips behind the synchronous speed of the rotating magnetic field. If the load is doubled, the slip becomes doubled. A motor on no load has very small slip of which the rotor speed is nearly equal to the synchronous speed. The slip of induction motors vary depending on the designs. A well designed induction motor should have a slip between 2% to 5% at full load. Any increase in inrush current will reduce slip and leads to the increase in efficiency.When the rotor is at its standstill, the frequency of the rotor current equals the frequency of the supply ($f_1$) and the slip, s=1. The motor starts to rotate at a frequency which depends upon the relative speed or slip-speed. The frequency of the rotor current $f_2=sf_1$ [4,5].

## 3.0 ANALYSIS OF EQUIVALENT CIRCUIT PER PHASE OF AN IM

The equivalent circuit of an IM is shown in Fig. 1. The first part of the circuit on the left side is the stator and the second part on the right side is the rotor circuit on running conditions.$I_1$ and $V_1$ are the current drawn by the stator and the supply voltage respectively. R1 and X1 are stator resistance and stator reactance per phase respectively. In [6], at a standstill, $R_2$ and $X_2$ are known as rotor resistance per phase and rotor reactance per phase respectively. As the rotor begins to operate on running conditions, $f_{run}=sf$, $E_{run}=Er=sE_2$ and $X_{run}=sX_2$.The IM on no load draws current from the supply producesa flux in the air gap in order to supply the line losses. The exciting circuit in the stator circuit consists of active current (Ic) which supplies no load losses and the magnetizing current ($I_m$) which sustains flux in the core and the air gap [7]. The elements of the exciting circuit consist of no load resistance $R_0 = \frac{V1}{Ic}$ and no load reactance $X_0 = \frac{V1}{\mathrm{Im}}$. The no load current is $I_0 = \sqrt{Ic^2 + \mathrm{Im}^2}$ .

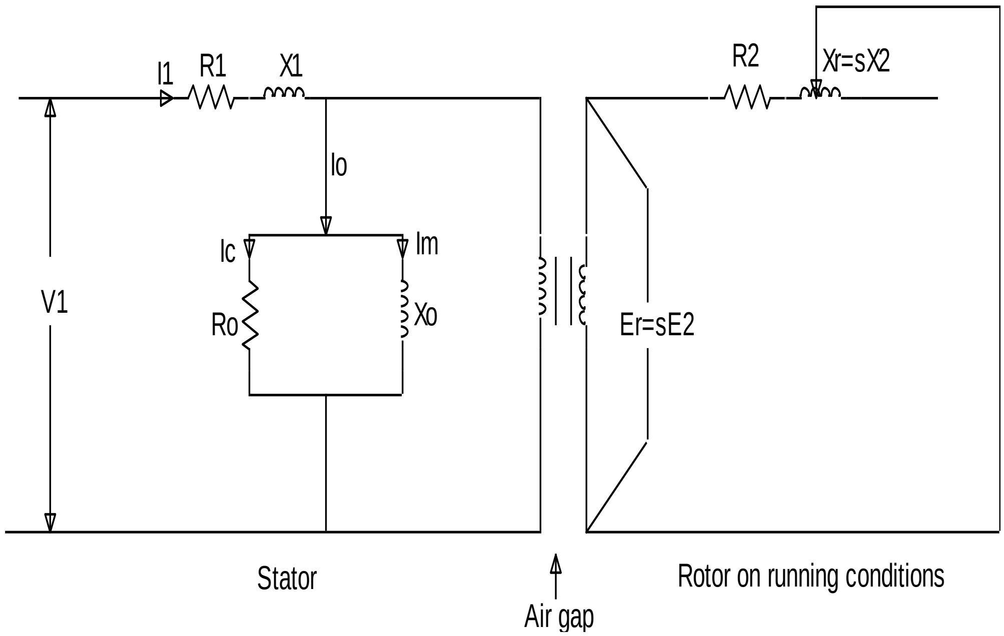


Fig.1: Equivalent circuit of an Induction Motor

Fig. 1 shows the equivalent circuit of an induction motor on running conditions. At standstill, slip will be absent in the circuit.

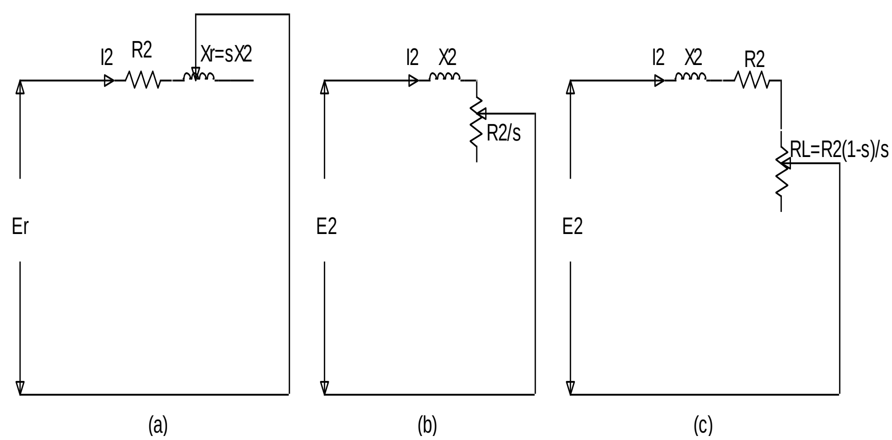


Fig.2: Rotor impedance equivalent circuits

Fig. 2(a) represents the rotor circuit impedance:

$$Z_{run} = \sqrt{R_2^{\ 2} + (sX_2)^2} \tag{1}$$

The rotor current as: $I_{2run} = \frac{E_r}{Zr} = \frac{sE_2}{\sqrt{R_2{}^2 + (sX_2)^2}}$ (2)

The rotor current can also be represented in Fig. 2(b) as:

$$I_{2run} = \frac{E_r}{Zr} = \frac{sE_2}{\sqrt{\frac{s^2}{s^s}(R_2{}^2 + (sX_2)^2)}} = \frac{E_2}{\sqrt{(\frac{R_2}{s})^2 + X_2{}^2}} \quad (3)$$

The value of $\frac{R_2}{s}$ can be further expressed as:

$$\frac{R_2}{s} = \frac{R_2}{s} + R_2 - R_2 = R_2 + \frac{R_2}{s} - R_2$$
$$= R_2 + R_2(\frac{1-s}{s}) \quad (4)$$

The rotor circuit impedance can be further expressed as:

$$Z_{run} = \sqrt{X_2{}^2 + (R_2 + R_2(\frac{1-s}{s}))^2} \quad (5)$$

The rotor circuit impedance represented in Fig. 2(b) can be further transformed into circuit impedance in Fig. 2c

## 3.1 Induction motor without iron losses

The iron losses in a circuit can be neglected so that we obtain the circuit in Fig. 3.

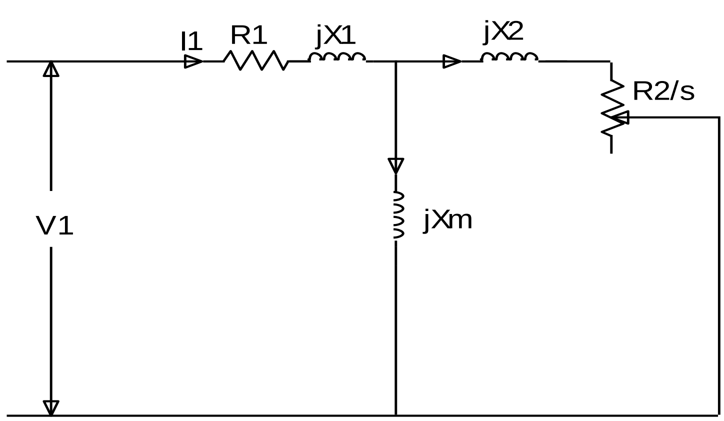


Fig.3: Equivalent circuit of an induction motor without iron losses

The phase current $I_1$ is given by:

$$I_1 = \frac{V1}{Z_{ph}} \tag{6}$$

Zph is the phase impedance and it is given by:

$$Z_{ph} = R1 + jX_1 + \frac{jX_m(\frac{R_2}{s} + jX_2)}{\frac{R_2}{s} + j(X_2 + X_m)} \tag{7}$$

The rotor current without core loss resistance is given by:

$$I_2 = \frac{jX_m}{\frac{R_2}{s} + j(X_2 + X_m)}.I_1 \tag{8}$$

## 3.2Equivalent circuit referred to stator side

All the parameters of the rotor are transferred to the stator side. The transformation ratio is given as: $k = \frac{E_2}{E_1} = \frac{N_2}{N_1}$ (9) $E_2^{'} = \frac{E_2}{k}$ (10)

The reflected component of rotor current on the stator side is

$$I_{2run}' = kI_{2run} = \frac{ksE_2}{\sqrt{R_2^{\ 2} + (sX_2)^2}} \tag{11}$$

$X_2^{'} = \frac{X_2^{\ 2}}{k^2}$ is the reflected rotor reactance (12) $R_2^{'} = \frac{R_2^{\ 2}}{k^2}$ is thereflected rotor resistance (13)and

$$R_L^{'} = \frac{R_L}{k^2} = \frac{R_2}{k^2}(\frac{1-s}{s})$$

$R_L^{'} = R_2'(\frac{1-s}{s})$ is the reflected mechanical load on stator (14)

The equivalent circuit referred to the stator is shown in Fig. 4.

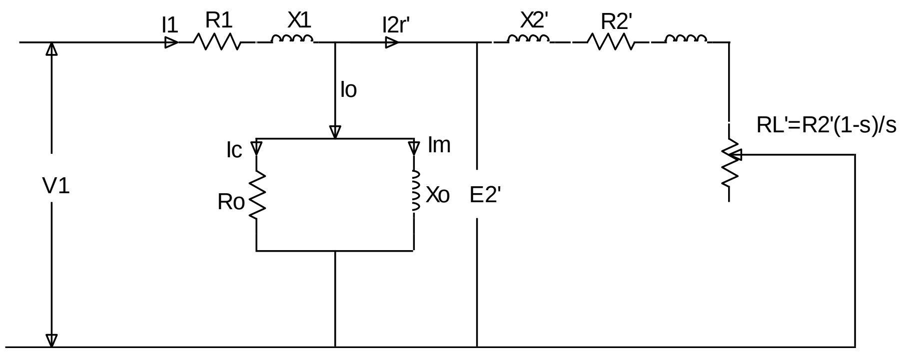


Fig.4: Equivalent circuit referred to stator

The value of Io is so small that it can be neglected, and the circuit in Fig. 4can be modified by shifting the exciting circuit across the supply. The circuit now obtained is the approximate equivalent circuit which is shown in Fig. 5.

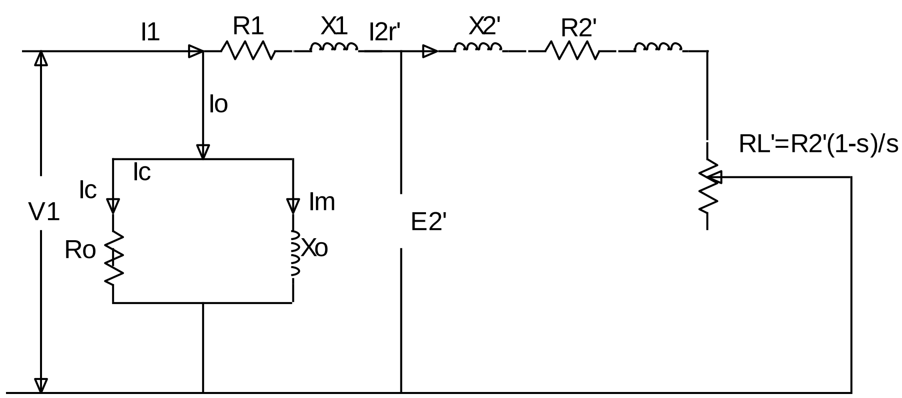


Fig.5: Approximate equivalent circuit

## 4.0 POWER FLOW IN ANIM

The power flow diagram in [8] is transformed intoFig. 6 shows the power losses from the input power to the output. The input power to a three phase induction motor is given by:

$$P_{input} = \sqrt{3} V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta \quad (15)$$

The losses in IM are the stator copper loss $P_{scl} = I_1{}^2 R_1$, rotor copper loss $P_{rcl} = I_{2r}{}^2 R_2$, core loss or iron loss, eddy current loss and hysteresis losses in the laminations. The core loss, friction, windage and stray loss are rotational losses. These losses are subtracted from the input power to obtain the output power.The rotor input power is $P_2 = \dfrac{I_{2r}{}^2 R_2}{s}$

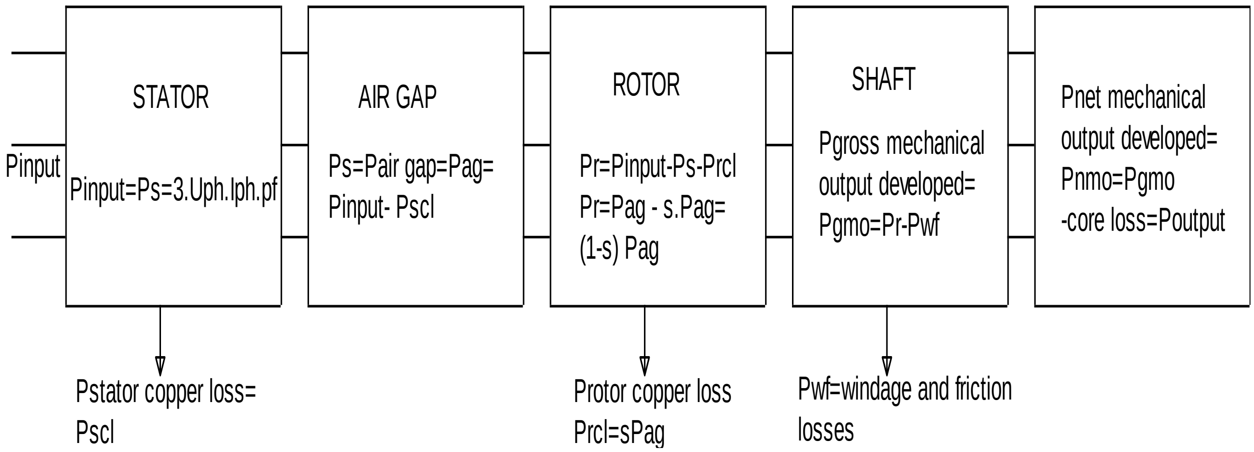


Fig.6: Power flow diagram

The air gap power transferred to the rotor is given by:

$$P_{ag} = 3I_2{}^2 . \frac{R_2}{s} . \tag{16}$$

The power converted into mechanical system is given by:

$$\begin{aligned} P_{conv} &= P_{ag} - P_{rcl} \\ &= 3I_2 . \frac{R_2}{s} - 3I_2{}^2 R_2 \\ &= 3I_2{}^2 R_2 (\frac{1-s}{s}) = P_{ag}(1-s) \end{aligned} \tag{17}$$

The output power can be obtained by subtracting the rotational losses from the power converted to mechanical energy which is expressed as $P_{out} = P_{conv} - P_{rotational}$..The mechanical power developed $P_{mech} = P_2 - P_{rcl}$ .The input power of the rotor can be expressed in term of torque and angular velocity (rad/s). $P_2 = T \times \varpi_2 = T \times \frac{2\pi N_s}{60}$ and the mechanical power $P_{mech} = T \times \frac{2\pi N_r}{60}$ . The rotor copper loss $P_{rcl} = T \times \frac{2\pi(N_s - N_r)}{60}$ and $\frac{P_{rcl}}{P_2} = \frac{N_s - N_r}{N_r} = s$. Since $P_{rcl} = sP_2$, $P_2 - P_{crl} = P_{mech}$ and $P_2 - sP_2 = P_2(1-s) = P_{mech}$ in [9,10], then $\frac{P_{crl}}{P_{mech}} = \frac{s}{1-s}$ $and \frac{P_2}{P_c} = \frac{1}{s}$.

## 5.0 TORQUE OF AN IM

The Torque of an IM is directly proportional to the product of flux per pole, the rotor current and the power factor of the rotor. The Torque is expressed by:

$T = k\phi I_2 \cos\varphi_2$

Where $I_2$ is the rotor current at standstill, $\varphi_2$ is the angle between the rotor electro-magnetomotive force (e.m.f) and rotor current $I_2$ and k is a constant. The rotor e.m.f at standstill is directly proportional to flux per pole so that the torque $T = kE_2 I_2 \cos\varphi_2$. The impedance per phase at standstill is $Z_2 = \sqrt{R_2^{\,2} + X_2^{\,2}}$ , where $R_2$ is the rotor resistance per phase.

The rotor current per phase at standstill is given by $I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^{\,2} + X_2^{\,2}}}$. (18)

The power factor is given by:

$$\cos\varphi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^{\,2} + X_2^{\,2}}}. \tag{19}$$

The standstill or starting torque is expressed as:

$$\begin{aligned} T_{st} &= kE_2 I_2 \cos\varphi_2 \\ &= kE_2 I_2 \frac{E_2}{\sqrt{R_2^{\,2} + X_2^{\,2}}} \cdot \frac{R_2}{\sqrt{R_2^{\,2} + X_2^{\,2}}} = \frac{kE_2^{\,2} I_2 R_2}{R_2^{\,2} + X_2^{\,2}} \end{aligned} \tag{20}$$

As long as the supply voltage is constant, $E_2$ and $I_2$ remain constant and the starting torque can now be expressed as:

$$T_{st} = k_2 \frac{R_2}{R_2^{\,2} + X_2^{\,2}} \tag{21}$$

The value of the resistance per phase of squirrel cage rotor IM is small when compared to its reactance per phase. At starting as in [11], the frequency of the rotor current is equal to that of the supply frequency. Hence, the starting current is large (5 to 7 times the full-load current) in magnitude and lags by a large angle behind $E_2$ and this result to low starting torque which is 1.5 of the full-load torque. Squirrel cage rotor IM is required where low starting torque is required.

## 5.1 Maximum starting torque

The maximum starting torque can be obtained by differentiating the starting torque as shown in equation 21with respect to rotor resistance per phase and equate it to zero.

$$\frac{d}{dR_2}(T_{st}) = \frac{k_2 \frac{d}{dR_2}(R_2{}^2 + X_2{}^2) - (R_2{}^2 + X_2{}^2)\frac{d}{dR_2}(k_2 R_2)}{(R_2{}^2 + X_2{}^2)^2}$$

$$0 = \frac{k_2(X_2{}^2 - R_2{}^2)}{(R_2{}^2 + X_2{}^2)^2} \tag{22}$$

$$R_2 = X_2$$

The maximum starting torque is obtained when the rotor resistance per phase is equal to the rotor reactance per phase at standstill. The expression for maximum torque as $R_2$=$X_2$ is given by:

$$T_{st} = k_2 \frac{R_2}{R_2{}^2 + R_2{}^2} = k_2 \frac{1}{2R_2} = k_2 \frac{1}{2X_2} \tag{23}$$

## 5.2 Torque under running conditions

As the motor begins to run, running torque is given by:

$$T_{run} = \frac{k(sE_2\phi)R_2}{R_2{}^2 + (s^2 X_2{}^2)} = \frac{k(sE_2{}^2)R_2}{R_2{}^2 + (sX_2)^2} \tag{24}$$

where $k = \frac{3}{2\pi n_s}$ *and* $n_s$ *is in rps*

## 5.3 Maximum torque under running conditions

Equation (24)can be differentiated with respect to slip in order to obtain a maximum slip and equating the resultant expression to zero.

$$T_{\max(run)} = \frac{d}{ds}\left(\frac{k(sE_2{}^2 R_2)}{R_2{}^2 + (sX_2)^2}\right.$$

$$= \frac{R_2{}^2 + (sX_2)\frac{d}{ds}(ksE_2{}^2 R_2) - ksE_2{}^2 R_2 \frac{d}{ds}(R_2{}^2 + s^2 X_2{}^2)}{(R_2{}^2 + s^2 X_2{}^2)^2} \tag{25}$$

$$0 = R_2{}^2 - s^2 X_2{}^2,\ s = s_{\max} = \frac{R_2}{X_2}$$

The maximum torque at the maximum slip ($s_{max)}$ is obtained by substituting the value of $s_{max}$ in equation 24

$$T_{\max\ (run)} = \frac{k(\frac{R_2}{X_2}E_2\phi)R_2}{R_2^{\ 2} + ((\frac{R_2}{X_2})^2 X_2^{\ 2})} = \frac{k(\frac{R_2}{X_2}E_2^{\ 2})R_2}{R_2^{\ 2} + (\frac{R_2}{X_2}.X_2)^2} \tag{26}$$

$$T_{\max\ (run)} = \frac{kE_2\phi}{2X_2} = \frac{kE_2^{\ 2}}{2X_2} \tag{27}$$

The maximum torque which is the breakdown torque depends on rotor reactance per phase at a standstill and independent of rotor resistance per phase, but the speed or slip at which it occurs is determined by the rotor resistance per phase. In slip ring motor or wound rotor of an IM, rotor circuit resistances are varied in order to obtain any desired speed or slip. The wound rotor ac motor is generally started with secondary resistance in the rotor circuit and as the speed of the motor increases, the resistance is sequentially reduced in order to bring the motor to full speed and torque. The motor develops substantial torque as the locked rotor current is now limited [12].

The mechanical torque developed can be found from the expression:

$$T = \frac{P_{ag}}{\varpi_{synch}} \tag{28}$$

where $\varpi_s = \frac{120f}{p} \times \frac{2\pi}{60} = \frac{4\pi f}{P}$ is the synchronous speed in radian per second.

## 6.0 SIMULATION RESULTS

### 6.1Motor input parameters

A three phase squirrel cage rotor IM of resistance r1 ohm with input parameters in Table 1 is investigated and simulated by MATLAB and the results of the characteristics are shown in Fig.7, 8,9 and10. The squirrel cage rotor is replaced by the wound rotor with additional resistances r2, r3 and r4 and the results investigated and simulated as shown in Fig.11, 12,13 and 14.

Table 1. Motor input parameters

| | | |
|---|---|---|
| 1 | R1-stator winding resistance per phase | 0.398 ohms |
| 2 | $R2^1$-rotor winding resistance per phase | 0.39 ohms |
| 3 | s-slip, $X1+X2^1$ | 0.038, 2.18 ohms |
| 5 | Nr-rotor speed | 962 (rpm) |
| 6 | P-number of poles | 6 |
| 7 | Vphase-phase voltage | 220 volts |
| 8 | f-frequency of the supply | 50Hz |
| 9 | m-number of phase | 3 |
| 10 | Type of stator winding connection | Star |

The Torque produced in [13] by induction motor is given by:

$$T = \frac{mpU_{ph}^{2}\frac{r2^{1}}{s}}{2\pi f\left((r1+\frac{r2^{1}}{s})^{2}+(x1+x2^{1})^{2}\right)} \quad (29).$$

and the rotorcurrent is given by:

$$I2^{1} = \frac{U_{ph}}{\sqrt{(r1+\frac{r2^{1}}{s})^{2}+(x1+x2^{1})^{2}}} \quad (30)$$

## 6.2Squirrel cage rotor Induction Motor

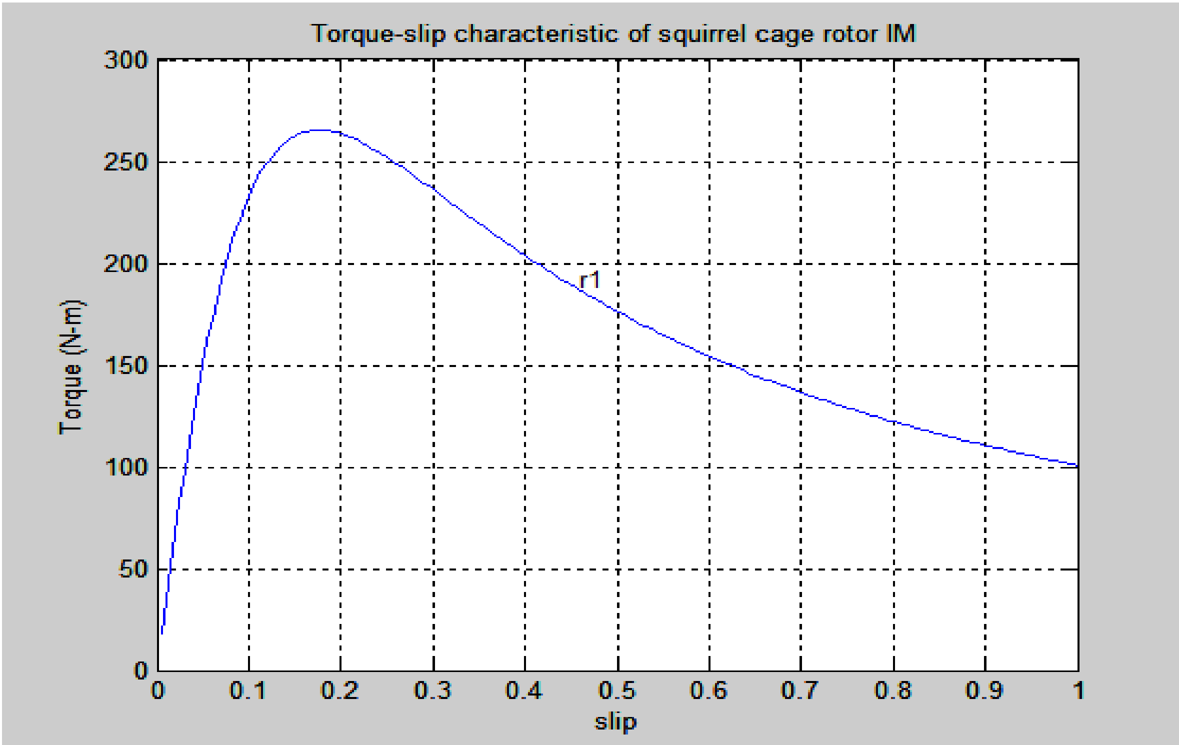


Fig. 7: Torque-slip characteristic

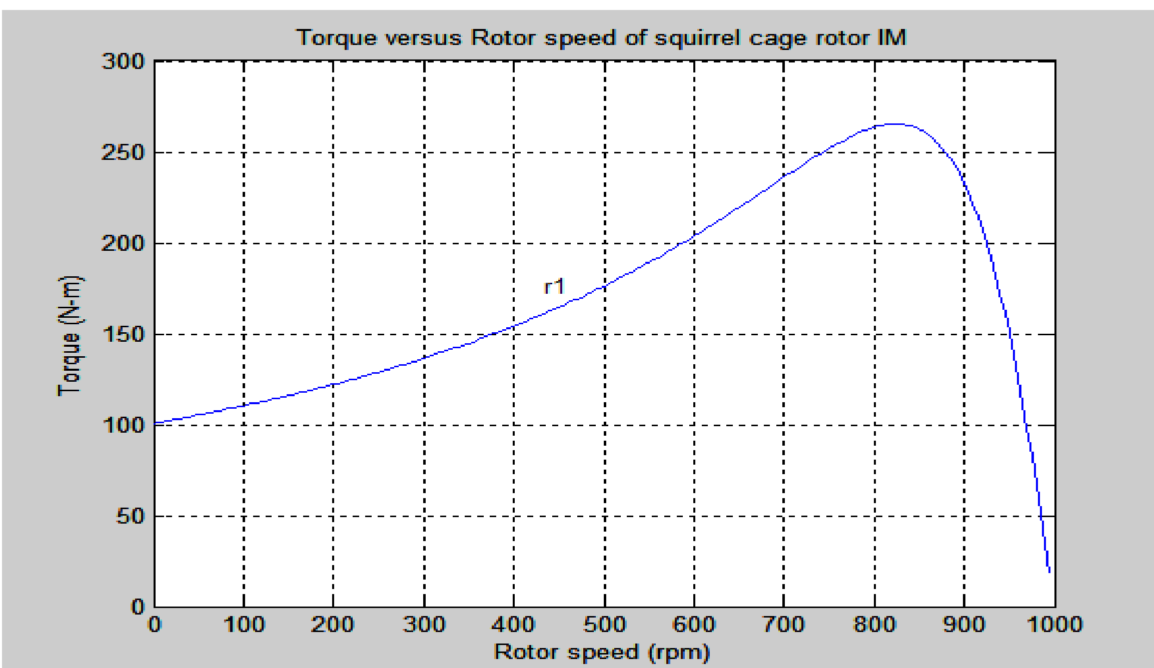


Fig. 8: Torque-rotor speed characteristic

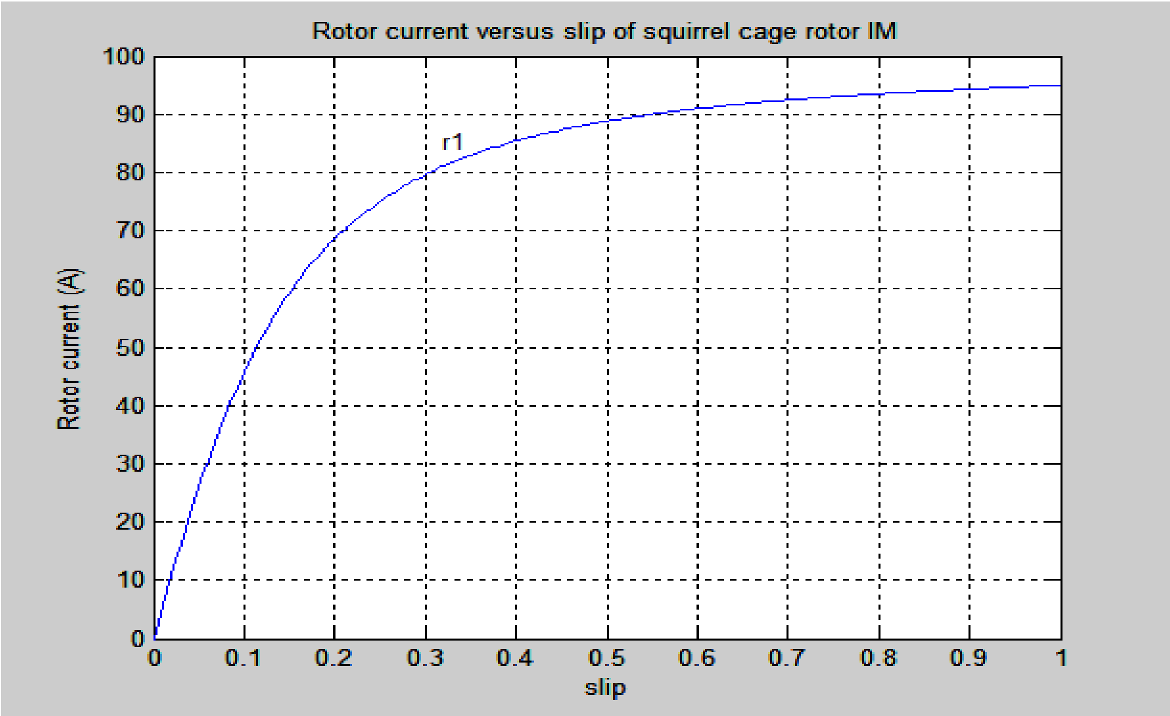


Fig.9: Rotor current-slip characteristic

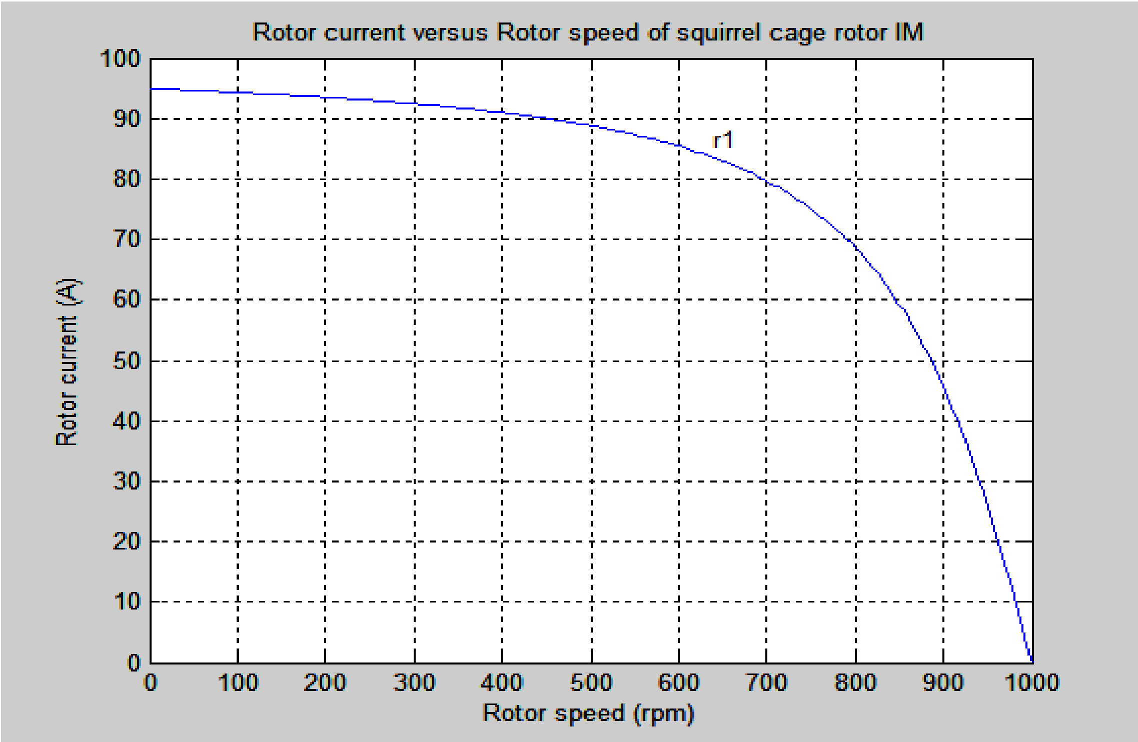


Fig.10: Rotor current-rotor speed characteristic

The simulation results in Fig.7, 8, 9 and 10 represent the torque/slip, torque/rotor speed, rotor current/slip and rotor current/rotor speed characteristics of squirrel cage rotor induction motor of constant rotor resistancer1.The starting current at fixed voltage drops slowly after it reaches 80% full speed as shown in Fig.10. The LRC ranges from 500% full load current to 1400% full load current, but the good motors fall between 550% to 750% full load current.The starting torque in Fig.7 and 8 is 100 N-m while the starting current in Fig.9 and 10 is 95A. The squirrel cage rotor induction motor has low starting torque and high starting current. The high starting current has heating effectsonmotor windings which results to more losses and consequently low efficiency. Torque and speed control is limited here. This type of motor is protected by starting the motor in start-delta connection.

### 6.3Wound rotor Induction Motor

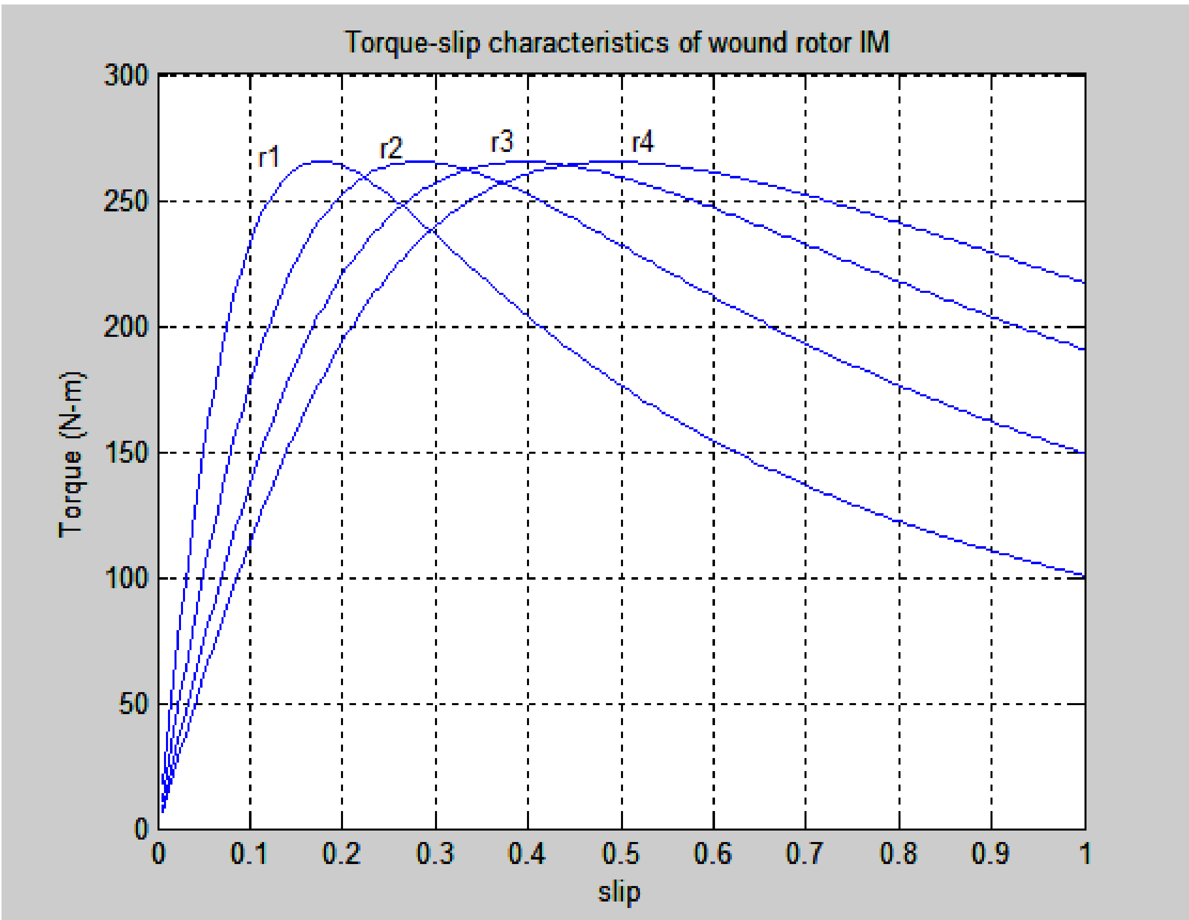


Fig. 11: Torque-slip characteristics

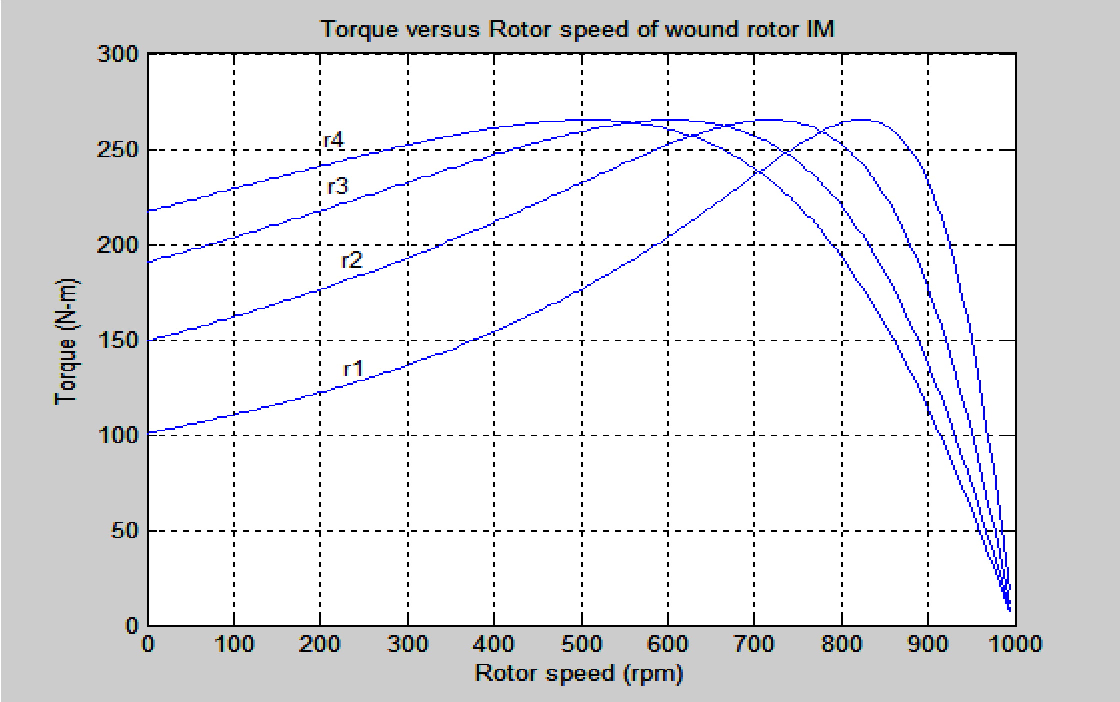


Fig. 12: Torque-rotor speed characteristics

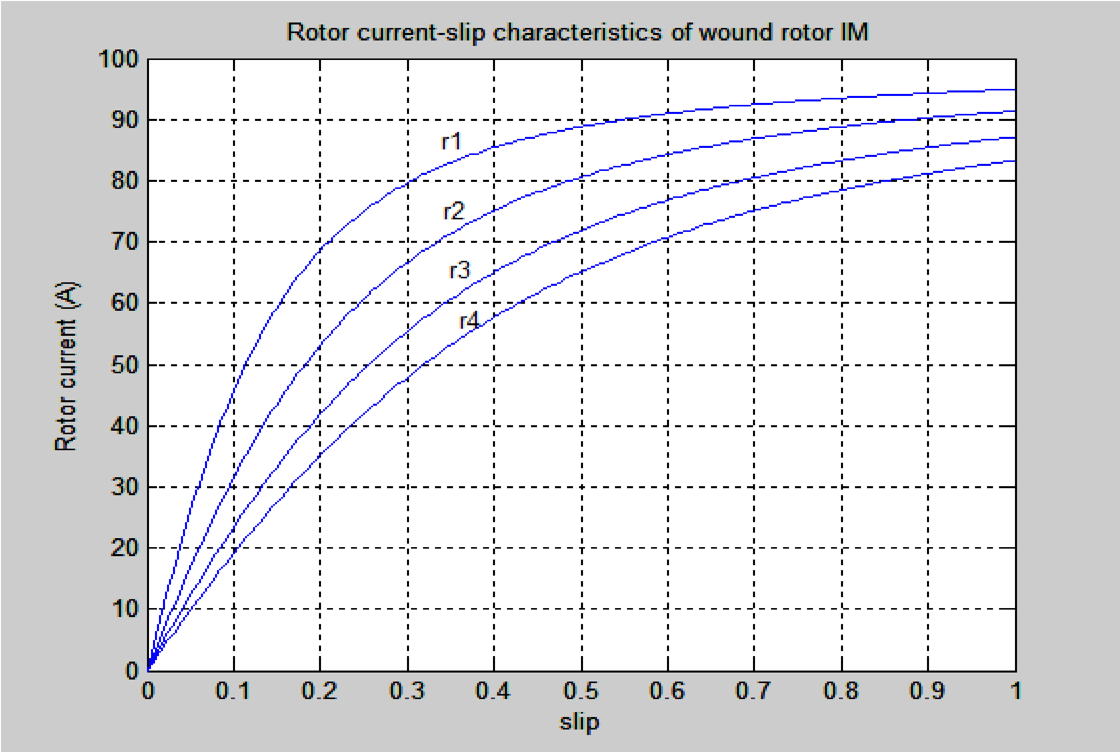


Fig. 13: Rotor current-slip characteristics

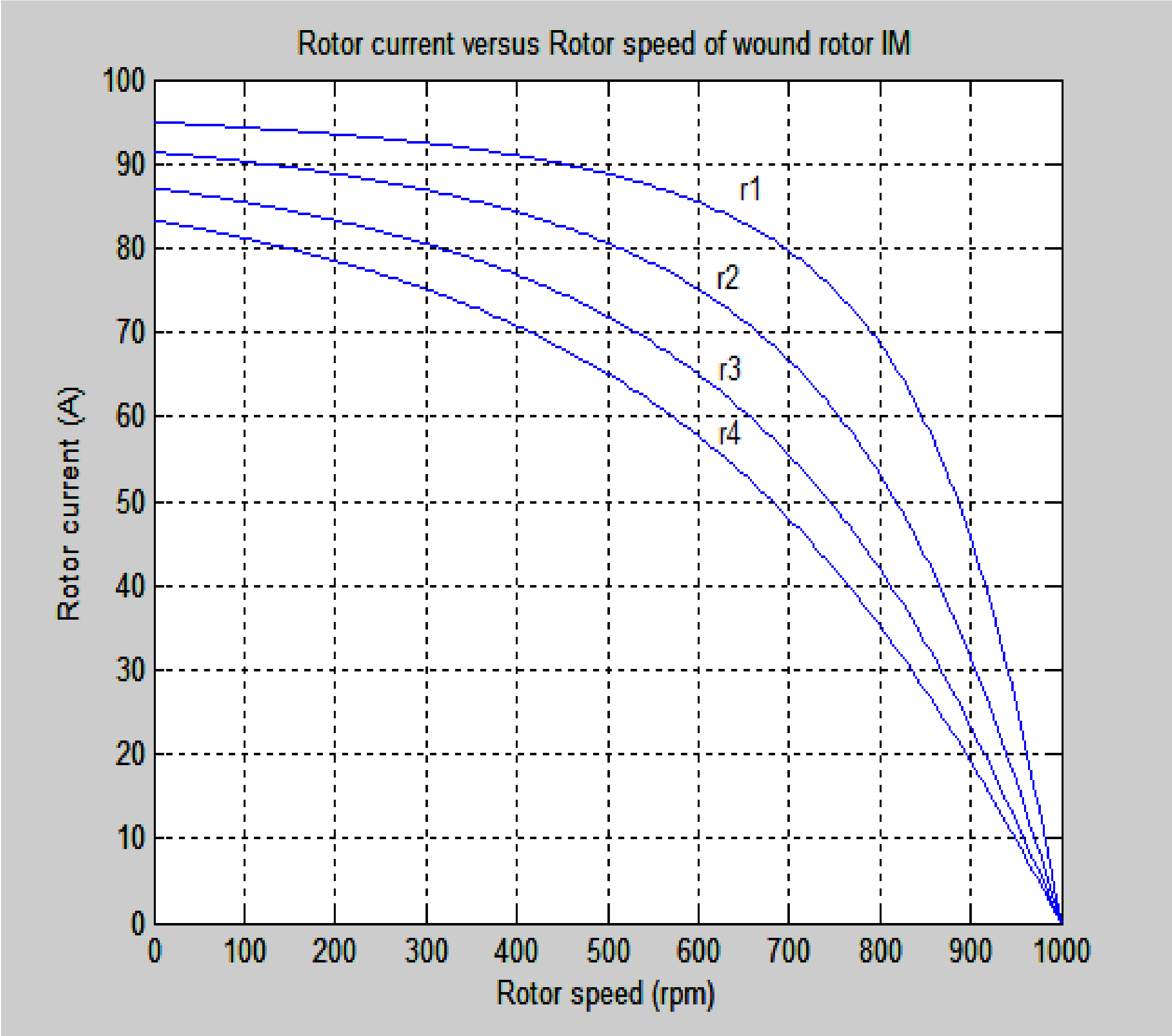


Fig.14: Rotor current-rotor speed characteristics

Due to accurate selection of resistance in slip rings (r1<r2<r3<r4) of wound motor, the starting torque of the motor can be controlled.Fig. 10 and 11 show the increase in the starting torque for resistances r2, r3, r4 compared to the resistance r1. The starting torque corresponding to r2, r3 and r4 are 150 N-m, 191 N-m, and 217.2 N-m respectively. As the resistance increases from r1 to r2, r3, and r4, the starting current reduces to from 95A to 91.37A, 87.13A and 83.3A respectively. The wound rotor has advantages over a squirrel cage rotor because of current, torque and speed control. High starting torque and low starting current can be obtained as a result the motor is applicable where high starting torque and low starting current is required.


Fig. 15: Torue-phase voltage of IM

The effects of supply voltage (phase) and rotor resistaceon torque of induction motors is shown in Fig. 15. The torque developed by squirrel cage induction motor is directly proportional to the supply voltage. As the resistance of the rotor circuit increases in wound rotor induction motor, the supply voltage and the torque produced decrease.The effects of the rotor induced voltage on torque is similar to that of the voltage supply of both the squirrel cage rotor and wound rotor induction motors.

## 6.4Results Analysis

Table 2. Output results of squirrel cage rotor IM

| Resistances (ohms) | Starting Torque (N-m) | Starting current (A) |
|---|---|---|
| r1 | 100 | 95 |

Table 3. Outputresults of wound rotor IM

| Resistances (ohms) | Starting Torque (N-m) | Starting current (A) |
|---|---|---|
| r1 | 100 | 95 |
| r2 | 150 | 91.37 |
| r3 | 191 | 87.13 |
| r4 | 217.2 | 83.3 |

The results in Table 2show that the squirrel cage rotorinduction motor has a low starting torque and a high starting current while the results in Table 3 shows that the torque of the wound or slip ring induction motor can be increased by addition of resistance in wound rotor circuit. In this case, wound rotor induction motor can be started with high starting torque and low starting current.

## 7.0 CONCLUSIONS

The equivalent circuit helps us to analyze the power flow and this lead us to the study of the steady state characteristics of asynchronous induction motor. The squirrel cage rotor IM has low starting torque and high starting current while the wound rotor IM has high starting torque and low starting current. The changing of impedance connected to the rotor circuit can alter the performance characteristics of wound rotor IM.

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