

TOTAL VARIATION OF CONVEX FUZZY NUMBER MAPPINGS

K.P .Deepa

Assistant Professor

Department of Mathematics, Maria College of Engineering & Technology

Affiliated to Anna University of Thirunelveli, TamilNadu, India

Dr.S.Chenthur Pandian

Principal

Dr.Mahalingam College of Engineering and Technology

Affiliated to Anna University of Coimbatore, TamilNadu, India

ABSTRACT

In this paper we discuss the concept of total variation for the convex fuzzy number mapping. Also here we frame some results based on the convexity and continuity of total variation of convex fuzzy number mappings and forward the concept of differentiability on convex fuzzy number mappings.

Keywords: Fuzzynumbers, fuzzynumber space, Convexfuzzynumber mappings, bounded - variation, total variation, continuity, differentiability.

Corresponding Author: K.P .Deepa

1.INTRODUCTION

Fuzzy sets were introduced simultaneously by Lotfi.A.Zadeh and Dieter Klaua in 1965 as an extension of the classical notion of a set. Many contributions in fuzzy set theory are dispersed over a broad range providing utility of knowledge. The concept of fuzzy number mappings was first introduced by S.L.Chang and Lotfi.A.Zadeh in 1972.

A fuzzy number is an ordinary number whose precise value is somewhat uncertain. Fuzzy numbers are used in Statistics, Computer Programming, Engineering and Experimental Science. Fuzzy number represents a real number interval whose boundary is fuzzy. Fuzzy numbers are fuzzy subsets of the set of real numbers satisfying some additional conditions. Fuzzy numbers are used in Statistics, Computer Programming, Engineering and Experimental Science. Fuzzy number represents a real number interval whose boundary is fuzzy[4]. Arithmetic operations on fuzzy numbers have also been developed and are based mainly on the extension principle [11] or on interval arithmetic. When operating with fuzzy numbers, result of the calculations depend on the shape of the membership functions of these numbers. Less regular membership functions lead to more complicated calculations. Moreover, fuzzy numbers with simpler shape of membership functions often have more intuitive and more natural interpretation. Trapezoidal or triangular fuzzy numbers are most common fuzzy numbers. But usually the fuzzy numbers which are used in practical

applications are trapezoidal. Also in 1992, Nanda and Kar discussed the concept of convex fuzzy mappings in a vector space over the field \mathbb{R} [8].

The aim of the present paper is to give the concept of total variation of convex fuzzy number mappings and its properties on convex fuzzy number space. For that we are going to frame some results connecting total variation to bounded variation, convexity, continuity and differentiability on Fuzzy number mappings.

2.BACKGROUND

A Fuzzy set \mathbf{A} in \mathbb{R} (real line) is defined to be a set of ordered pairs

$\mathbf{A} = \{x, f_A(x) / x \in \mathbb{R}\}$, where $f_A(x)$ is called membership function for the Fuzzy set \mathbf{A} .

A Fuzzy set is called **Normal** if there is at least one point $x \in \mathbb{R}$ with $f_A(x) = 1$

A Fuzzy set is **Convex** if for any $x, y \in \mathbb{R}$ and any $\lambda \in [0, 1]$,

$$f_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{f_A(x_1), f_A(x_2)\}$$

A convex fuzzy set (Fig.1) is described by a membership function whose membership values are strictly monotonically increasing or whose membership values are strictly monotonically decreasing (Or whose membership values are strictly monotonically increasing then decreasing or whose membership values are strictly monotonically decreasing then increasing).

A fuzzy set is strictly convex if the sets $A_\alpha = \{x / f_A(x) \geq \alpha\}$ for all $\alpha \in (0, 1]$ are strictly convex

A convex fuzzy set with maximum membership value 1 is called convex normal fuzzy set (Fig.2). Also a normal fuzzy set which is not convex is a non convex normal fuzzy set (Fig.3)

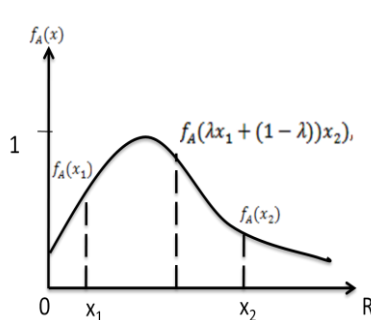


Fig.(1)

Convex fuzzy sets

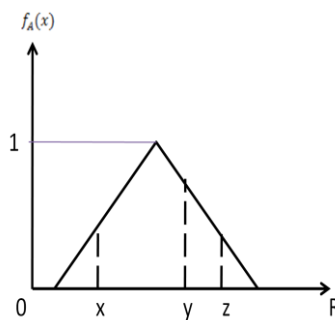


Fig.(2)

Convex normal fuzzy sets

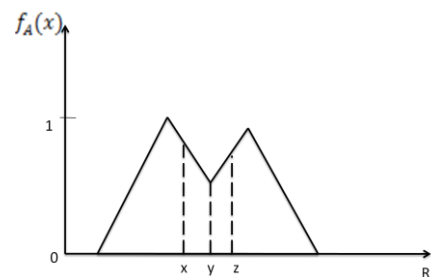


Fig.(3)

Non convex normal fuzzy sets

A fuzzy set \mathbf{A} is bounded if and only if the set $A_\alpha = \{x / f_A(x) \geq \alpha\}$ are bounded for all $\alpha > 0$ i.e., for every $\alpha > 0$, there exists finite $R(\alpha)$ such that $\|x\| \leq R(\alpha)$ for all $x \in A_\alpha$

If \mathbf{A} is a convex single point normal fuzzy set defined on the real line then \mathbf{A} is often termed as a fuzzy number. A fuzzy number should be normalized and convex, condition for normalised implies that maximum membership value is 1. Generally a fuzzy number represents a real number interval whose boundary is fuzzy and the fuzzy interval is represented by two end points.

By the concept of ordered fuzzy numbers, a fuzzy number \mathbf{A} can be identified with an ordered pair of continuous real functions defined on the interval $[0,1]$

ie, $\mathbf{A} = (f, g)$ with $f, g : [0,1] \rightarrow \mathbb{R}$ are continuous functions where f and g , the up and down parts of the fuzzy number \mathbf{A} respectively.

The continuity of both parts implies that their images are bounded intervals.

3. PRELIMINARIES

A Fuzzy number μ is defined as $\mu : \mathbb{R} \rightarrow [0,1]$ which is normal, fuzzy convex, upper semi-continuous with bounded support.

Now $\mathcal{E} = \{ \mu \mid \mu : \mathbb{R} \rightarrow [0,1] \}$ is called a fuzzy number space

Each $r \in \mathbb{R}$ can be considered as a Fuzzy number and is defined as

$$\check{r}(k) = \begin{cases} 1, & \text{if } k = r \\ 0, & \text{if } k \neq r \end{cases} \quad \text{for any } k \in \mathbb{R}$$

Obviously, a Fuzzy set $\mu : \mathbb{R} \rightarrow [0,1]$ is a fuzzy number if and only if $[\mu]^r$ is a closed and bounded interval for each $r \in [0,1]$ and $[\mu]^1 \neq \emptyset$, null set

Also, $[\mu]^r = [\mu^-(r), \mu^+(r)]$, $r \in [0,1]$, where $\mu^-(r)$ denotes the left-hand side end point and $\mu^+(r)$ denotes the right-hand side end point of $[\mu]^r$

Addition and scalar multiplication of fuzzy numbers as follows

Let $\mu, \lambda \in \mathcal{E}$ and $k \in \mathbb{R}$, $r \in [0,1]$

$$\begin{aligned} [\mu + \lambda]^r &= [\mu]^r + [\lambda]^r \\ [k\mu]^r &= k[\mu]^r \\ [\mu \lambda]^r &= [\mu]^r [\lambda]^r \end{aligned}$$

A mapping $\mathbf{F} : \mathcal{E} \rightarrow \mathcal{E}$ is said to be a **Fuzzy Number Mapping**

Also $[F(\mu)]^r = [F(\mu^-(r)), F(\mu^+(r))]$, $\mu \in \mathcal{E}$ and $r \in [0,1]$

4. CONCEPTS OF FUZZY NUMBER MAPPINGS (continuity-bounded variation)

Let $F : \mathcal{E} \rightarrow \mathcal{E}$ be a Fuzzy Number Mapping

If F is bounded on all bounded subsets of \mathcal{E} then F is bounded

If $\mu \rightarrow \mu_c \Rightarrow F(\mu) \rightarrow F(\mu_c)$ then F is continuous

ie, $|F(\mu) - F(\mu_c)| < \epsilon_0$ for $|\mu - \mu_c| < \delta_0$; $\epsilon_0, \delta_0 > 0$

If $\mu \rightarrow \mu_c \Rightarrow F(\mu) \xrightarrow{r} F(\mu_c)$ F is the w-cut-continuous

ie, $|F(\mu)(r) - F(\mu_c)(r)| < \epsilon_0$ for $|\mu - \mu_c| < \delta_0$; $\epsilon_0, \delta_0 > 0$

If $\mu \xrightarrow{r} \mu_c \Rightarrow F(\mu) \xrightarrow{r} F(\mu_c)$ then F is the cut-continuous

ie, $|F(\mu)(r) - F(\mu_c)(r)| < \epsilon_0$ for $|\mu(r) - \mu_c(r)| < \delta_0$; $\epsilon_0, \delta_0 > 0$

Also Continuity \Rightarrow the w-cut continuity

The cut-continuity \Rightarrow the w-cut continuity

Let $F : \mathcal{E} \rightarrow \mathcal{E}$ be a Fuzzy Number Mapping, a Convex (or Concave) Fuzzy Number Mapping is defined as follows:

$$F(t\mu + (1-t)\lambda) \leq tF(\mu) + (1-t)F(\lambda)$$

(or $F(t\mu + (1-t)\lambda) \geq tF(\mu) + (1-t)F(\lambda)$), where $\mu, \lambda \in \mathcal{E}$, $t \in [0,1]$.

Let $F: \mathcal{E} \rightarrow \mathcal{E}$ be a convex Fuzzy number mapping and $\mu \in \mathcal{E}$ be a Fuzzy number.

Then foreach $r \in [0,1]$, $[\mu]^r$ is a closed and bounded interval so that it can be partitioned.

Let $\Psi = \{\mathbb{Q}_0(r), \mathbb{Q}_1(r), \dots, \mathbb{Q}_n(r)\}$ be a partition of $[\mu]^r$ such that $\mathbb{Q}_n(r) = \mathbb{Q}^+(r)$, where $\mathbb{Q}_0(r) < \mathbb{Q}_1(r) < \mathbb{Q}_n(r)$, $i = 1, 2, \dots, n-1$, $\mathbb{Q}_0(r) = \mathbb{Q}^-(r)$

Define a metric d on $[F(\mu)]^r$ as

$$d([F(\mu)]^r) = \sup\{|F(\mathbb{Q}_j(r)) - F(\mathbb{Q}_i(r))| : r \in [0,1]\} \text{ for any } i, j = 0, 1, 2, \dots, n, i < j$$

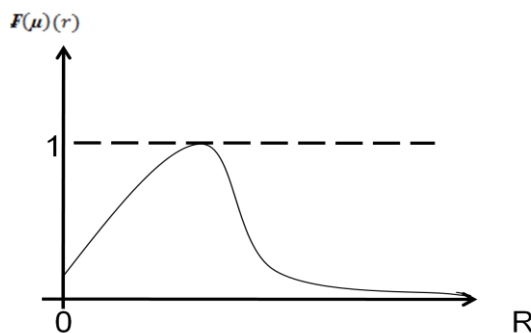
If there exists a real number $0 < \eta \leq 1$ such that $d([F(\mu)]^r) < \eta$ then F is of **bounded variation** on $[F(\mu)]^r$

Let $F: \mathcal{E} \rightarrow \mathcal{E}$ be a convex Fuzzy number mapping and $\mu \in \mathcal{E}$ be a fuzzy number.

If F is of bounded variation on $[F(\mu)]^r$ then F is of bounded on $[F(\mu)]^r$, $r \in [0,1]$.

If F is the cut-continuous on $[\mu]^r$ then F is of bounded variation on $[F(\mu)]^r$, $r \in [0,1]$

If F is the w-cut continuous on $[\mu]^r$ then F is of bounded variation on $[F(\mu)]^r$, $r \in [0,1]$



5.MAIN RESULTS

Definition5.1 : Let $F: \mathcal{E} \rightarrow \mathcal{E}$ be a convex Fuzzy number mapping and $\mu \in \mathcal{E}$ be a fuzzy number. Let $\Psi = \{\mathbb{Q}_0(r), \mathbb{Q}_1(r), \dots, \mathbb{Q}_n(r)\}$ be a partition of $[\mu]^r$ such that $\mathbb{Q}_0(r) = \mathbb{Q}^-(r)$ and $\mathbb{Q}_n(r) = \mathbb{Q}^+(r)$, where $\mathbb{Q}_0(r) < \mathbb{Q}_1(r) < \mathbb{Q}_n(r)$, $i = 1, 2, \dots, n-1$

Define

$$V([F(\mu)]^r) = \sup\{\sum |F(\mathbb{Q}_j(r)) - F(\mathbb{Q}_i(r))| : r \in [0,1]\} \text{ for } i, j = 0, 1, 2, \dots, n \text{ and } i < j$$

Then V is the **Total variation** of F on $[F(\mu)]^r$, $r \in [0,1]$, for every $\mu \in \mathcal{E}$

Obviously $0 \leq V \leq 1$.

Definition5.2 : Let $F: \mathcal{E} \rightarrow \mathcal{E}$ be a convex Fuzzy number mapping and $\mu \in \mathcal{E}$ be a fuzzy number. Let $\Psi = \{\mathbb{Q}_0(r), \mathbb{Q}_1(r), \dots, \mathbb{Q}_n(r)\}$ be a partition of $[\mu]^r$ such that $\mathbb{Q}_0(r) = \mathbb{Q}^-(r)$ and $\mathbb{Q}_n(r) = \mathbb{Q}^+(r)$, where $\mathbb{Q}_0(r) < \mathbb{Q}_1(r) < \mathbb{Q}_n(r)$, $i = 1, 2, \dots, n-1$.

For $\mu, \lambda \in \mathcal{E}$, $t \in [0,1]$

$$V([tF(\mu) + (1-t)F(\lambda)]^r) \leq tV([F(\mu)]^r) + (1-t)V([F(\lambda)]^r)$$

Then V is said to be **Convex** on \mathcal{E} .

Definition5.3 : Let $F: \mathcal{E} \rightarrow \mathcal{E}$ be a convex Fuzzy number mapping and $\mu \in \mathcal{E}$ be a fuzzy number. Let $\Psi = \{\mathbb{Q}_0(r), \mathbb{Q}_1(r), \dots, \mathbb{Q}_n(r)\}$ be a partition of $[\mu]^r$ such that $\mathbb{Q}_0(r) = \mathbb{Q}^-(r)$ and $\mathbb{Q}_n(r) = \mathbb{Q}^+(r)$, where $\mathbb{Q}_0(r) < \mathbb{Q}_1(r) < \mathbb{Q}_n(r)$, $i = 1, 2, \dots, n-1$.

For $t \in [0,1]$, we can find $0 < \epsilon, \delta \leq 1$ such that

$$V([F(\mathbb{Q})]^r) = \left\{ \sum |F(\mathbb{Q}_j(r)) - F(\mathbb{Q}_i(r))| : r \in [0,1] \text{ for } i, j = 0, 1, 2, \dots, n \text{ and } i < j \right\} \\ < \epsilon, \text{ for every } |\mathbb{Q}_j(r) - \mathbb{Q}_i(r)| < \delta$$

Then V is said to be **Continuous** on any closed interval $[F(\mathbb{Q})]^r, \mathbb{Q} \in \mathbb{E}, r \in [0,1]$

Definition 5.4 : Let $F: \mathbb{E} \rightarrow \mathbb{E}$ be a convex Fuzzy number mapping and $\mu \in \mathbb{E}$ be a fuzzy number. Let $\Psi = \{\mathbb{Q}_0(r), \mathbb{Q}_1(r), \dots, \mathbb{Q}_n(r)\}$ be a partition of $[\mathbb{Q}]^r$ such that $\mathbb{Q}_0(r) = \mathbb{Q}^-(r)$ and $\mathbb{Q}_n(r) = \mathbb{Q}^+(r)$, where $\mathbb{Q}_0(r) < \mathbb{Q}_i(r) < \mathbb{Q}_n(r), i = 1, 2, \dots, n-1$.

$$\text{Now } F'(\mathbb{Q})(r) = \lim_{\mathbb{Q}(r) \rightarrow \mathbb{Q}_c(r)} \frac{F(\mathbb{Q})(r) - F(\mathbb{Q}_c)(r)}{\mathbb{Q}(r) - \mathbb{Q}_c(r)}, \text{ where } r \in [0,1], \mathbb{Q}_c(r) \in \Psi$$

Then $F'(\mathbb{Q})(r)$ is the derivative of F and if such an F' existing on every closed interval of \mathbb{E} then F is **Differentiable** on \mathbb{E}

Theorem 5.1 : The total variation of a convex Fuzzy number mapping is convex.

Proof

Let $F: \mathbb{E} \rightarrow \mathbb{E}$ be a convex Fuzzy number mapping and $\mu \in \mathbb{E}$ be a fuzzy number.

Let $\Psi = \{\mathbb{Q}_0(r), \mathbb{Q}_1(r), \dots, \mathbb{Q}_n(r)\}$ be an arbitrary partition of $[\mathbb{Q}]^r$ such that $\mathbb{Q}_0(r) = \mathbb{Q}^-(r), \mathbb{Q}_n(r) = \mathbb{Q}^+(r)$, where $\mathbb{Q}_0(r) < \mathbb{Q}_i(r) < \mathbb{Q}_n(r), i = 1, 2, \dots, n-1, r \in [0,1]$

Since F is convex, for $\mu, \lambda \in \mathbb{E}, t \in [0,1], F(t\mu + (1-t)\lambda) \leq tF(\mu) + (1-t)F(\lambda)$

Let V be the total variation of F on \mathbb{E}

$$\text{Then } V([F(\mathbb{Q})]^r) = \left\{ \sum |F(\mathbb{Q}_j(r)) - F(\mathbb{Q}_i(r))| : r \in [0,1] \right\} \text{ for } i, j = 0, 1, 2, \dots, n \text{ and } i < j \\ < \epsilon, \text{ for every } |\mathbb{Q}_j(r) - \mathbb{Q}_i(r)| < \delta, \mu \in \mathbb{E}, 0 < \epsilon, \delta \leq 1$$

Let $\Phi = \{\lambda_0(r), \lambda_1(r), \dots, \lambda_n(r)\}$ be an arbitrary partition of $[\lambda]^r$ such that $\lambda_0(r) = \lambda^-(r)$ and $\lambda_n(r) = \lambda^+(r)$, where $\lambda_0(r) < \lambda_i(r) < \lambda_n(r), i = 1, 2, \dots, n-1, r \in [0,1]$

Now for $\mathbb{Q}, \lambda \in \mathbb{E}, t \in [0,1], r \in [0,1]$

$$\begin{aligned} V([tF(\mathbb{Q}) + (1-t)F(\lambda)]^r) &= \sup \left\{ \sum |tF(\mathbb{Q}_j(r)) - (1-t)F(\lambda_j(r)) - (tF(\mathbb{Q}_i(r)) - (1-t)F(\lambda_i(r)))| \right\} \\ &\quad \text{for } i, j = 0, 1, 2, \dots, n \text{ and } i < j \\ &= \sup \left\{ \sum |tF(\mathbb{Q}_j(r)) - tF(\mathbb{Q}_i(r)) - ((1-t)F(\lambda_j(r)) - (1-t)F(\lambda_i(r)))| \right\} \\ &= \sup \left\{ \sum |t(F(\mathbb{Q}_j(r)) - F(\mathbb{Q}_i(r))) - (1-t)(F(\lambda_j(r)) - F(\lambda_i(r)))| \right\} \\ &\leq \sup \left\{ \sum |t| |F(\mathbb{Q}_j(r)) - F(\mathbb{Q}_i(r))| \right\} + \sup \left\{ \sum |1-t| |F(\lambda_j(r)) - F(\lambda_i(r))| \right\} \\ &\leq t \sup \left\{ \sum |F(\mathbb{Q}_j(r)) - F(\mathbb{Q}_i(r))| \right\} + (1-t) \sup \left\{ \sum |F(\lambda_j(r)) - F(\lambda_i(r))| \right\} \\ &\leq tV([F(\mathbb{Q})]^r) + (1-t)V([F(\lambda)]^r) \end{aligned}$$

Thus implies V is Convex on \mathbb{E} .

Theorem 5.2 : Let F be a convex fuzzy number mapping which is of bounded variation on the closed interval $[F(\mathbb{Q})]^r, r \in [0,1], \mathbb{Q} \in \mathbb{E}$ and V is the total variation of F . If F is continuous on any closed interval of \mathbb{E} then V is also continuous on that interval.

Proof

Let $\Psi = \{\mu_0(r), \mu_1(r), \dots, \mu_n(r)\}$ be an arbitrary partition of $[\mu]^r$ such that $\mu_0(r) = \mu^-(r)$ and $\mu_n(r) = \mu^+(r)$, where $\mu_0(r) < \mu_i(r) < \mu_n(r), i = 1, 2, \dots, n-1, r \in [0,1]$

Assume that F is continuous on $[F(\mathbb{Q})]^r$

Then for $\mu \in \mathbb{E}$, we can find $0 < \epsilon, \delta \leq 1$ such that $|F(\mathbb{Q})(r) - F(\mathbb{Q}_c)(r)| < \frac{\epsilon}{n}$,

for all $|\mathbb{Q}(r) - \mathbb{Q}_c(r)| < \delta$, where n is the natural number.

Now for $|\mathbb{Q}(r) - \mathbb{Q}_c(r)| < \delta$, for all $\mu \in \mathbb{E}$ and $i, j = 0, 1, 2, \dots, n, i < j$

$$\begin{aligned} & \sum |F(\mathbb{Q}_j)(r) - F(\mathbb{Q}_i)(r)| \\ &= |F(\mathbb{Q}_1)(r) - F(\mathbb{Q}_0)(r)| + |F(\mathbb{Q}_2)(r) - F(\mathbb{Q}_1)(r)| + \dots + |F(\mathbb{Q}_n)(r) - F(\mathbb{Q}_{n-1})(r)| \\ &\leq \frac{\epsilon}{n} + \frac{\epsilon}{n} + \dots + \frac{\epsilon}{n} = \epsilon \end{aligned}$$

$$\Rightarrow \sup \{ \sum |F(\mathbb{Q}_j)(r) - F(\mathbb{Q}_i)(r)| : r \in [0, 1] \} < \epsilon \text{ for } i, j = 0, 1, 2, \dots, n \text{ and } i < j$$

$$\text{and for every } |\mathbb{Q}_j(r) - \mathbb{Q}_i(r)| < \delta$$

$$\Rightarrow V([F(\mu)]^r) < \epsilon, \text{ for all } \mu \in \mathbb{E}$$

Hence V is continuous on $[F(\mu)]^r$ and the proof is completed.

Theorem 5.3: Let F be a convex fuzzy number defined on \mathbb{E} . If F is differentiable on \mathbb{E} , then F is continuous and is of bounded variation on \mathbb{E} . Moreover the total variation V of F is also continuous on \mathbb{E} .

Proof

Let $\Psi = \{\mu_0(r), \mu_1(r), \dots, \mu_n(r)\}$ be a partition of $[\mu]^r$ such that $\mu_0(r) = \mu^-(r)$ and $\mu_n(r) = \mu^+(r)$, where $\mu_0(r) < \mu_i(r) < \mu_n(r)$, $i = 1, 2, \dots, n-1, r \in [0, 1], \mathbb{Q} \in \mathbb{E}$

Suppose F is continuous on \mathbb{E} .

$$\text{Then for } \mu \in \mathbb{E}, r \in [0, 1] \quad F'(\mu)(r) = \lim_{\mu(r) \rightarrow \mu_j(r)} \frac{F(\mu)(r) - F(\mu_j)(r)}{\mu(r) - \mu_j(r)}$$

$$\Rightarrow |F(\mu)(r) - F(\mu_j)(r)| < \epsilon \text{ for } |\mu(r) - \mu_j(r)| < \delta, 0 < \epsilon, \delta \leq 1$$

$\Rightarrow F$ is continuous on \mathbb{E} .

Clearly continuity of $F \Rightarrow$ the cut continuity

Hence F is of bounded variation on any closed interval $[F(\mu)]^r$ so that F is of bounded variation on \mathbb{E} [12]

$$\begin{aligned} \text{Now } V([F(\mathbb{Q})]^r) &= \left\{ \sum |F(\mathbb{Q}_j)(r) - F(\mathbb{Q}_i)(r)| : r \in [0, 1] \right\} \text{ for } i, j = 0, 1, 2, \dots, n \text{ and } i < j \\ &< \epsilon, \text{ for every } |\mathbb{Q}_j(r) - \mathbb{Q}_i(r)| < \delta, \mu \in \mathbb{E} \text{ be the total variation of } F \end{aligned}$$

Then **Theorem 5.2** shows that V is continuous on \mathbb{E} .

CONCLUSION

The lack of strict monotonicity of the branches μ^- and μ^+ , the existency of constancy sub intervals imply that the inverse functions of μ^- and μ^+ do not exist in the classical sense. Thus to solve the above theorems we may assume that for both functions μ^- and μ^+ , there exists a finite (or at most countable) number of such constancy sub intervals and then the inverse functions exist in the generalized sense. From the above discussion we framed a definition of total variation of a fuzzy number mapping over fuzzy number space and explained some important results based on the convexity, continuity and differentiability for the total variation of convex fuzzy number mappings.

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