Weight Minimization of Functionally Graded Structures

Using ICA and ANN

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Abstract:

The main goal of the structural optimization is to minimize the weight of structures while satisfying all design requirements imposed. In this study, optimization of four-parameter volume fraction of functionally graded (FG) beams with objective of minimizing the density to achieve a specified fundamental frequency obtained by means of the generalized differential quadrature (GDQ) method is presented. The primary optimization variables are the four parameters of the volume fraction of ceramic. Since the search space is large, the optimization processes becomes so complicated and too much time consuming. Thus a novel meta-heuristic called Imperialist Competitive Algorithm (ICA) which is a socio-politically motivated global search strategy is applied to find the optimal solution. Applying the proposed algorithm to some of benchmark cost functions, shows its ability in dealing with different types of optimization problems. The performance of ICA is evaluated in comparison with other nature inspired technique Genetic Algorithm (GA). Comparison shows the success of combination of ANN and ICA for design of material profile of beam. Finally the optimized material profile for the optimization problem is presented.

Keywords: Optimization, imperialist competitive algorithm, artificial neural network, functionally graded beam

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1. Introduction

A new class of materials known as (FGMs) has attracted much attention as advanced structural materials in many structural members. Functionally graded materials FGMs are composite materials that are microscopically inhomogeneous, and the mechanical properties vary continuously in one (or more) direction(s). Recently, Tornabene [1] has used four-parameter power law distribution to study the dynamic behavior of moderately thick functionally graded conical and cylindrical shells and annular plates. Static and free vibration analyses of continuously graded fiber-reinforced cylindrical panels using generalized power-law distribution are presented by Sobhani Aragh and Yas [2]. One of the advantages of using four-parameter power law distribution is the ability of controlling the materials volume fraction of FG structures for considered applications. In another words, these kinds of distribution can be optimized for various purposes.

Beams and columns supported along their length are very common in structural configurations. Beams are often found to be resting on earth in various engineering applications. These include railway lines, geotechnical areas, highway pavement, building structures, etc. This motivated many researchers to analyze the behavior of beam structures on elastic foundations [3-7].

Optimization is the task of finding one or more solutions which correspond to minimizing (or maximizing) one or more specified objectives and which satisfy all constraints (if any) [8]. Optimization is implemented for various objective functions in mechanical problems, such as buckling loads [9], weight (either as a constraint or as an objective to be minimized) [10, 11], stiffness [12], fundamental frequencies [9], deflection [10], etc. When the search space becomes large, enumeration is soon no longer feasible simply because it would take far too much time. In this it's needed to use a specific technique to find the optimal solution. In the present work, Imperialist Competitive Algorithm (ICA) is implemented that has recently been

introduced by Atashpaz-Gargary and Lucas (2007) for dealing with different applications, such as designing PID controller [13], characterizing materials properties [14], error rate beam forming [15], designing vehicle fuzzy controller [16], etc.

M. Abouhamze et al. [9] optimized stacking sequence of laminated cylindrical panels with respect to the first natural frequency and critical buckling load. They used genetic algorithm and neural network for optimization. The concept of neural networks has been introduced to different branches of engineering, analytical procedure of structural design, structural optimization problems and functionally graded materials [9, 17-20]. As a simple modeling technique, in this work, ANN is employed to reproduce the fundamental frequency parameter and the density of FG beam in order to reduce the time of the optimization process.

Many, or even most, real engineering problems may contain a number of constrains which any feasible solution (including all optimal solutions) must satisfy; this kind of problems are stated as constrained optimization problems. In the present work, penalty method is applied for handling the constraints in Imperialist competitive algorithm.

The aim of this study is to present useful results on FG beams on elastic foundations and then optimize generalized power-law distribution for minimizing the density with constraint on the first natural frequency of FG beam using ICA, penalty method and neural network. The Frequency parameter of beam is obtained by using numerical technique termed the generalized differential quadrature (GDQ) method based on the DQ technique [21].

2. Problem description

Consider a FG beam resting on two-parameter elastic foundation as shown in Fig.1 where k(x), $k_I(x)$ are Winkler foundation modulus and second parameter foundation modulus respectively.

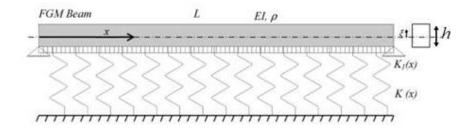


Fig.1 FG beam supported on variable two-parameter elastic foundation

The Young's modulus E_{fgm} , Poisson's ratio v_{fgm} and mass density ρ_{fgm} of the functionally graded beam can be expressed as a linear combination:

$$E_{fgm} = (E_c - E_m)V_c + E_m$$

$$\rho_{fgm} = (\rho_c - \rho_m)V_c + \rho_m$$

$$\upsilon_{fgm} = (\upsilon_c - \upsilon_m)V_c + \upsilon_m$$
(1)

Where ρ_m , E_m , ν_m , V_m and ρ_c , E_c , ν_c , V_c represent mass density, Young's modulus, poisson's ratio and volume fraction of the metal and ceramic constituent materials, respectively. In the present work, V_c is considered as follow [1];

$$V_c = (1 - a \left(\frac{1}{2} + \eta\right) + b \left(\frac{1}{2} + \eta\right)^c)^p \tag{2}$$

where volume fraction index p $(0 \le p \le \infty)$ and the parameters a,b,c dictate the material variation profile through the FG beam thickness. It should be noticed that the values of parameters a,b and c must be chosen so that $0 \le V_c \le 1$.

For FG beam resting on two-parameter elastic foundation in the absence of body force, the governing equation can be expressed as:

$$-D_{fgm}\frac{\partial^4 w}{\partial x^4} - \frac{\partial}{\partial x}(k_1(x)\frac{\partial w}{\partial x}) - k(x)w - \rho A\frac{\partial^2 w}{\partial t^2} = 0 \qquad 0 < x < L$$
(3)

Where

$$D_{fgm} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{11} z^2 dz$$
 , $Q_{11} = \frac{E_{fgm}}{1 - v_{fgm}^2}$

To obtain the natural frequency, Eq. (2) is formulated as an eigenvalue problem by using the following periodic function $w(x,t) = W(x)e^{-i\alpha x}$, where W(x) is the mode shape of the transverse motion of the beam.

3. GDQ solution of governing equation

The GDQ approach was developed by Shu and coworkers [22, 23] that approximates the spatial derivative of a function of given grid point as a weighted linear sum of all the functional value at all grid point in the whole domain. In GDQ method, the nth order partial derivative of a continuous function f(x,z) with respect to x at a given point x_i can be approximated as a linear sum of weighting values at all of the discrete point in the domain of x, i.e.

$$\frac{\partial f^{n(x_i,z)}}{\partial x^n} = \sum_{k=1}^{N} c_{ik}^n f(x_{ik}, z), \qquad (i=1,2,...N, n=1,2,...N-1)$$
(4)

Where N is the number of sampling points, and c_{ij}^n is the x_i dependent weight coefficients.

GDQ approach approximates the spatial derivative of a function of given grid point as a weighted linear sum of all the functional value at all grid point in the whole domain. The computation of weighting coefficient by GDQ is based on an analysis of a high order polynomial approximation and the analysis of a linear vector space. The weighting coefficients of the first-order derivative are calculated by a simple algebraic formulation, and the weighting coefficient of the second-and higher-order derivatives are given by a recurrence relationship. The nth order of a continuous function f(x, z) with respect to x at a given point x_i can be approximated as a linear sum of weighting values at all of the discrete point in the domain of x, i.e.

$$\frac{\partial f^{n(x_i,z)}}{\partial x^n} = \sum_{k=1}^{N} c_{ik}^n f(x_{ik}, z), \qquad (i=1,2,...N, n=1,2,...N-1)$$
(5)

Where N is the number of sampling points, and c_{ij}^n is the x_i dependent weight coefficients.

In order to determine the weighting coefficient c_{ij}^n , the Lagrange interpolation basic function are used as test function, and explicit formulation for computing these weighting coefficient can be obtained:

$$c_{i,j}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_j)M^{(1)}(x_j)} \qquad i, j = 1, 2, ..., N, \qquad i \neq j$$
(6)

Where:

$$M^{(1)}(x_i) = \prod_{j=1, i \neq j}^{N} (x_i - x_j)$$
(7)

For the first-order derivative; i.e. n=1 and for higher-order derivative, one can use the following relations iteratively:

$$c_{i,j}^{n} = n \left(c_{i,i}^{(n-1)} c_{i,j}^{(1)} - \frac{c_{i,j}^{(n-1)}}{(x_i - x_j)} \right) \qquad i, j = 1, 2, ..., N, \quad i \neq j, \quad n = 2, 3, ..., N - 1$$
(8)

$$c_{i,i}^{(n)} = -\sum_{j=1, i \neq j}^{N} c_{i,j}^{(n)} \qquad i = 1, 2, ..., N, \qquad n = 1, 2, ..., N - 1$$
(9)

The Chebyshev-Gauss-Labatto quadrature points are used, that is

$$x_{i} = \frac{1}{2} \left(1 - \cos \left(\frac{i - 1}{n - 1} \pi \right) \right) \qquad i = 1, 2, ..., N$$
 (10)

4. Neural network modeling

ANN modeling is an equation-free, data-driven modeling technique that tries to emulate the learning process in the human brain by using many examples [24]. ANN can be defined as a massive parallel-distributed information processing system that has a natural propensity for recognizing and modeling complicated input-output systems. The basic element of an NN is

the artificial neuron as shown in Fig. 2 which consists of three main components namely as weights, bias, and an activation function.

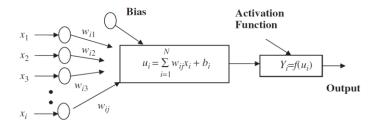


Fig. 2 Basic elements of an artificial neuron

NNs are commonly classified by their network topology (i.e. feedback, feed forward) and learning or training algorithms (i.e. supervised, unsupervised). There is no well-defined rule or procedure to have optimal network architecture. In this study, the feedforward multi-layer perceptron (MLP) network has been applied. MLP networks are one of the most popular and successful neural network architectures which are suited to a wide range of applications such as prediction and process modeling.

5. Imperialist Competitive Algorithm

Imperialist Competitive Algorithm is a novel global search heuristic for optimization that uses imperialism an imperialistic competition process as a source of inspiration. Like other evolutionary algorithms, ICA starts with an initial population called countries that are divided in two types: imperialists (in optimization terminology, countries with least cost) and colonies (the remained countries). In ICA, the more powerful imperialist, have the more colonies. Based on the power of countries (the counterpart of fitness value in Genetic Algorithm which is inversely proportional to its cost), all of them are divided among the mentioned imperialists. Starting the competition, imperialists attempt to achieve more

colonies and colonies in each of them start to move toward their relevant imperialist which this movement is shown in Fig.3. In this movement θ , x are random numbers with uniform distribution as illustrated in formula (11) and d is the distance between colony and the imperialist. In formula (11), β and γ are parameters that modify the area that colonies randomly search around the imperialist. Weak empires will lose their power and will be eliminated from the competition (in another word, they will be collapsed) and the powerful ones will be improved and remain. At last, only one imperialist will remain that in this stage, colonies have the same position and power as the imperialist.

$$x \sim U(0, \beta \times d), \quad \theta \sim U(-\gamma, \gamma)$$
 (11)

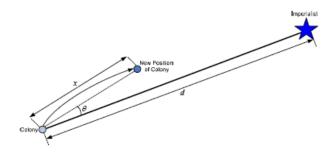


Fig. 3 Motion of colonies toward their relevant imperialist

In imperialistic competition which is shown in Fig.4, all empires try to take possession of colonies of other empires and control them. Based on the total power of empires which is observed in formula (12), the more powerful an empire, the more likely it will possess the colonies. Imperialistic competition can be modeled as Fig.4.

$$T.C._n = Cost(imperialist_n) + \zeta mean\{Cost(colonoes of impire_n)\}$$
 (12)

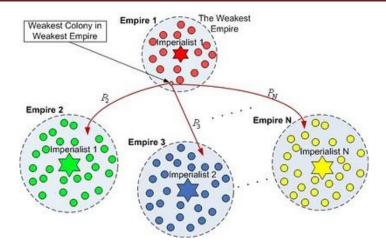


Fig. 4 Imperialistic competition

The main steps in the ICA are summarized as follows:

- Select some random points on the function and initialize the empires.
- Move the colonies toward their relevant imperialist (Assimilating).
- If there is a colony in an empire which has lower cost than that of imperialist, exchange the
- positions of that colony and the imperialist.
- Compute the total cost of all empires (related to the power of both imperialist and its colonies).
- Pick the weakest colony (colonies) from the weakest empire and give it (them) to the empire that has the most likelihood to possess it (imperialistic competition).
- Eliminate the powerless empires.
- If there is just one empire, stop, if not go to 2.

Also, the flowchart of the ICA is shown in Fig.5. More details about the proposed algorithm are found in [25].

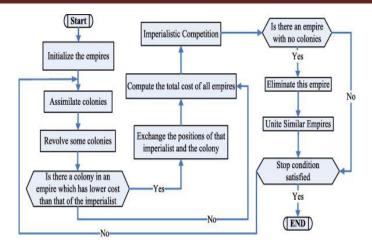


Fig. 5 Flowchart of the Imperialistic competitive algorithm

6. Penalty method

Many, or even most, real engineering problems are stated as constrained optimization problems. In general, a constrained numerical optimization problem with q inequality and m-q equality constraints is defined as:

Minimize
$$f(X)$$
 $X = (x_1, ..., x_n)^t \in F \subseteq S \subseteq R^n$
Subject to $g_i(X) \le 0$ $i = 1, ..., q$
 $h_j(X) = 0$ $j = q + 1, ..., m$ (13)

Where $X = (x_1,...,x_n)^t$ is a feasible solution of the problem that satisfies all constrains, F is the feasible region and S is the whole search space [26].

In this method the constrained problem in (13) is transformed into an unconstrained one as follows:

$$Minimize \quad \{f(X) + c P(X)\} \tag{14}$$

Where c is a positive constant and P is a function on \mathbb{R}^n satisfying:

- (i) P(X) is continuous
- (ii) $P(X) \ge 0$ for all $X \in \mathbb{R}^n$, and
- (iii) P(X) = 0 if and only if $X \in S$

7. Result and discussion

7.1. Free vibration analysis

In this study, ceramic and metal are particle mixed to form the functionally graded material.

The relevant material properties for the constituent materials are as fallow:

$$E_c = 380 \,\text{Gp}a$$
 , $\rho_c = 3800 \,kg/m^3$, $E_m = 70 \,\text{Gp}a$, $\rho_c = 2707 \,kg/m^3$

The influence of the parameters a, b, c, p on the fundamental frequency parameter $(\Omega = \omega L^2 \sqrt{\frac{\rho_m A}{E_m I}})$ and density of a FG beam for clamped boundary conditions is shown in

Fig.6-8. In these figures it is assumed that $k_1 = 1$, $k = 1000 (1 - 0.4 x^2)$

The new and interesting result is that by using the four-parameter volume fraction, we can control the profile of ceramic volume fraction. In other words, there are different values of density for a constant frequency and on the other hand, for a constant density, we can have different values for density.

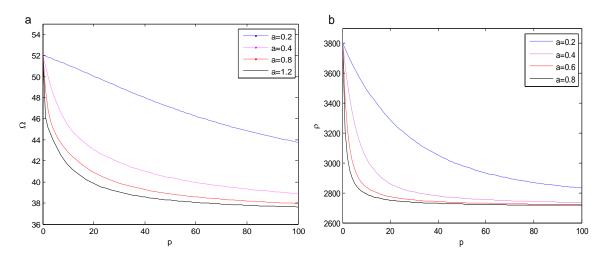


Fig. 6 Variation of a: the first natural frequency parameter b: density vs. the power-law exponent p for various values of the parameter a for clamped boundary conditions $(0.2 \le a \le 1.2, b = 0.2, c = 2)$

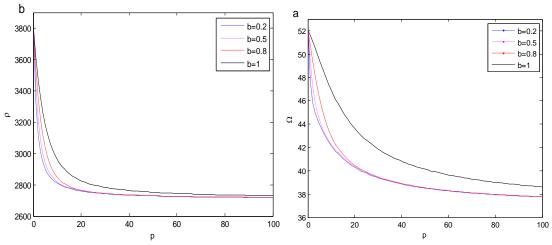


Fig. 7 Variation of a: the first natural frequency parameter b: density vs. the power-law exponent p for various values of the parameter b for clamped boundary conditions $(a=1, 0 \le b \le 1, c=2)$

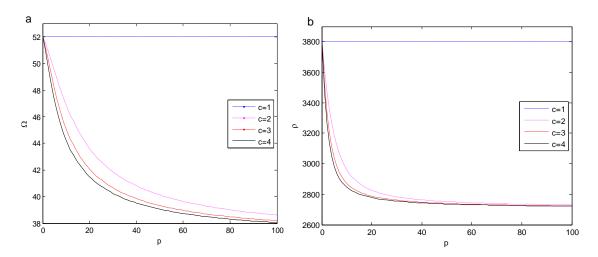


Fig. 8 Variation of a: the first natural frequency parameter b: density vs. the power-law exponent p for various values of the parameter a for clamped boundary conditions $(a = 1, b = 1, 1 \le c \le 4)$

7.2. Optimization procedure

The objective of optimization in this paper is to find the best values of the parameters a, b, c, p in the four-parameter power-law distribution so that to minimize density with constraint on the fundamental frequency parameter of beam. In other words, we optimize the material profile of the FG beam for minimum density when the value of firs natural frequency parameter of FG beam shouldn't be less than 45. Also, there is an important point that

considered parameters must be obtained so that the ceramic volume fraction is between zero and one $(0 \le V_c \le 1)$. The boundary conditions of the FG beam are considered clamped and the parameters are considered in the following ranges:

$$-10 \le a \le 10$$
, $-10 \le b \le 10$, $0 \le c \le 15$, $0 \le p \le 15$, $k = 1000(1 - 0.4x^2)$, $k_1 = 1$

Therefore, the constrained optimization problem is defined as:

$$\begin{aligned} \textit{Minimize} & & f(a,b,c,p) = \rho \\ \textit{Subject to} & & \begin{cases} \Omega \geq 45 \\ 0 \leq V_c \leq 1 \\ -10 \leq a \leq 10 \end{aligned}, & -10 \leq b \leq 10 \text{ , } 0 \leq c \leq 15 \text{ , } 0 \leq p \leq 15 \end{aligned}$$

If GDQ method is applied for frequency parameters, the optimization process becomes so complicated and time consuming. For example even if the increment of the parameters (a,b,c,p) is assumed 0.1, the formed discrete space contains more than 900,000,000 design choices to be searched for an optimum point. Also, if it is assumed that the process of one search takes 0.1 second in average, the optimization process takes about 25000 hours. Thus in the present work, ANN and ICA are implemented for increasing the speed of optimization. For this purpose, 5216 training examples have been given to ANN as inputs. The MLP network has been used having three hidden layers. A program was developed in MATLAB which handles the trial and error process automatically. The program tries varying number of hidden layers neurons is tested from two up to fifteen for a constant epoch for 5 times for different back propagation training algorithms. 10, 8, 12 neurons for first, second and third hidden layer and also, Levenberg-Marquardt (LM) algorithm are chosen for the network because it performs better than other cases. The ability of trained network to reproduce the fundamental frequency parameter and density is shown in Fig.9 for 36 test points which selected far from the training point randomly. As observed, the neural network has been accurately designed and we can use ANN in the optimization process.

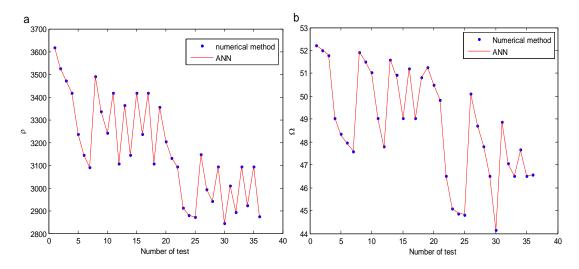


Fig. 9 Comparison of various predicted values of density and fundamental frequency parameter versus numerical data

Here, optimization is investigated for the FG beam. Table 1 shows the parameters of ICA used to find the optimal solution. Fig. 10 shows the convergence of the ICA to the optimal solution. As shown in this figure, ICA has reached to the optimal values of [a, b, c, p] in about 6 decades. The algorithm reached to optimal solution of [1.02, 1.02, 2, 15] which leads the 2838.8 for value of density.

Table.1 parameters of ICA approach

Parameters	Value
Number of initial countries	80
Number of initial imperialists	5
Number of decades	60
Revolution rate	0.3
eta	2
γ	0.5
ζ	0.06

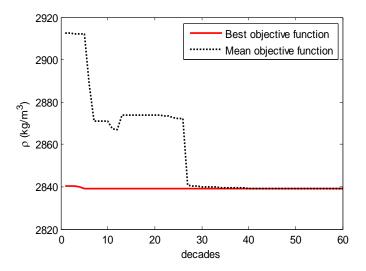


Fig.10 Convergence of the ICA to the optimal solution

Now, the performance of ICA is compared with Genetic Algorithm. Comparison shows that the results obtained by ICA are more optimized than genetic algorithm. This fact is shown in Table 2. It should be mentioned that the process of optimization in ICA took less than 50 seconds. It means CPU time is reduced by a considerable amount. Fig 11 shows the optimized material profile for the maximum frequency parameter.

Table.2 Comparison between the Imperialist competitive Algorithm and genetic algorithm

Algorithm _	Optimum parameters				$o(ka/m^3)$	0
	а	b	С	p	$-\rho(kg/m^3)$	22
ICA	1.02	1.02	2	15	2838.8	45.33
GA	1.78	1.60	3.93	4.04	2851.62	45.02

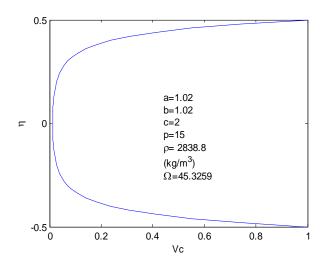


Fig. 11 Optimized material profile for the FG beam

Conclusion

Volume fraction optimization of four-parameter functionally graded beams with respect to the density was studied in this paper. Free vibration was analyzed by means of GDQ method. Imperialist Competitive Algorithm was performed to obtain the best material profile through the thickness to minimize the density so that the fundamental natural frequency constraint was satisfied. It was concluded that:

- By using four-parameter power law distribution, it is possible to control the materials volume fraction of FG structures for considered applications.
- By choosing suitable parameters in volume fraction relation we can obtain the best solution for our purposes.
- ICA can be applied for engineering optimization problems especially for those related to the FGM structures.
- Combination of NN and ICA reduces the CPU time by a considerable amount with losing negligible accuracy.
- The performance of ICA is better than other nature inspired technique Genetic Algorithm. In other word, the results obtained by ICA are more optimized than GA.

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