# Modeling and Experimental Performances of a Domestic Thermal Water Heater in Nigeria

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**Abstract**

This paper presents the mathematical model and experimental performance of a thermal domestic electrical water heater in Nigeria. The domestic water heater consists of a 50litres single tank, control valves and water reservoir. The heating time, first-hour rating and recovery efficiency were calculated.The mathematical model developed has been validated by experiment. It was observed that the transient heat transfer to water for the tank was 1225.6W. The domestic heater produced 155litres of water in 6hours 16minutes. The recovery efficiency of the tank was found to have increased from 63% to 87%.

Keywords: Water Heater, Heat Transfer, Water Tank.

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## 1. Introduction

Domestic water heating represents a significant use of energy in many countries and is one of the most common ways of generating hot water heater during the winter. Domestic electric water heater is a simple system that is made up of an insulated tank, heating element and associated piping [1].The earliest families enjoyed warm baths in rare, but valued natural hot springs. As civilization advanced, so did development of the technology for hot water.The Romans were the first to heat water artificially for bathing, using a furnace below a cistern. After the Romans, no progress was made in heating water until industrial Revolution. In factories in which steam was available, it was injected "live" into water to be heated. In wealthy houses, a boiler forming part of the cooking range heated water. In England, Municipal bodies were first allowed by Act of parliament to build public hot water baths and washhouses in 1846.

In the 1870s, Englishman, Maugham invented the first instant water heater. Little was known about Maugham's invention; however, his invention influenced the designs of Edwin Ruud. Edwin Ruud, a Norwegian Mechanical engineer was the inventor of the automatic storage water heater in 1889.Ruud immigrated to Pittsburgh where he pioneered the early development of both residential and commercial water heaters. He founded the Ruud Manufacturing Company[2].

At first, people heated water in vessels. The affluent uses gas-fired, coil type, tankless water heaters to heat the water in their Victorian homes. These water heater pipes formed in a coil, which acted as a heat exchanger. After a hot water fixture was opened, the gas burner would then be lit, heating water as it flowed through coil.Unfortunately, this type of water heater had no water temperature control. Instead, the ultimate output temperature of the hot water depends upon the heating capacity of the burner size and the volume of the water sent-through the coil.

Many studies were performed on water heaters in order to analyze their performances under different conditions. Studies was undertaken to determine the structure of domestic hot water consumption using data acquired from seven New Zealand households. The study shows that there is a high proportion of low volume hot water draw-offs depending on the household. In addition, draws tend to be separated by relatively short intervals of not more than 10-20 minutes. It is also estimated that thermal losses in the piping exceed 20% of the heat energy supplied by the hot water heater and the losses can be reduced by 5-7% when pipes are insulated [3,4].

## 2. Modeling of the Domestic Water Heater

### *2.1 Heat Transfer*

The three modes (conduction, convection and radiation) of heat transfer are experienced in the domestic water heater.

The rate of conduction heat transfer is express by the Fourier's law rate equation and is expressed for one dimensional analysis as:

$$Q_x = -\frac{K_A dT}{Dx} \quad (1)$$

The rate of convective heat transfer is given as:

$$Q_c = h_A(T_w - T_a) \quad (2)$$

The Stefan – Boltzmannlaw gives the minima flux that can be emitted by radiation from a body surface as

$$q_r = \sigma T_w{}^4 \quad (3)$$

The total energy in the system is given as

$$E_{in} + E_g - E_{out} = E_{st} \quad (4)$$

Where, $E_{in}$ is the rate at which thermal and mechanical energy enters the heater tank, $E_{out}$ is the rate at which thermal and mechanical energy leaves the heater tank, $E_g$ is the rate of thermal energy entering the water heater tank due to conversion from other energy sources, $E_{st}$ is the changes in the amount of the internal thermal energy stored by the material in the control volume of water heater.
For a three dimensional control volume [5-7].

$$E_{in} - E_{out} = \left[\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)\right]dxdydz \quad (5)$$

$$E_g = qdxdydz \quad (6)$$

$$E_{st} = PCp\frac{\partial T}{\partial t}dxdydz \quad (7)$$

Equations (3), (4) and (5) becomes

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + q = PCp\frac{\partial T}{\partial t} \quad (8)$$

If the thermal conductivity is a constant, the heat equation from equation (8) becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{K} = \frac{1}{\alpha_P}\frac{\partial T}{\partial t}$$
$$where\ \alpha_p = K / PCp \quad (9)$$

If the water heater equation contains no heat sources, equation (9) becomes the Fourier equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{K} = \frac{1}{\alpha_P}\frac{\partial T}{\partial t} \quad (10)$$

If steady state is considered, that is, if the temperature distribution does not vary with time, the Poisson's equation (10) results to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{K} = 0 \quad (11)$$

***2.2 Overall coefficients of heat transfer through water tank.***

The value of the coefficient will depend on the mechanism by which heat is transferred, on the fluid dynamics of both the heated and the cooled fluids, on the properties of the materials through which the heat must pass, and on the geometry of the fluids paths. In solid like the mild steel sheet used in the construction of this water heater tank, heat is normally transferred by conduction and have a high thermal conductivity while the fiberglass used as the insulating material have a low thermal conductivity. Water in the tank also transmits heat readily by conduction, though circulating currents are frequently set up and the resulting convective transfer may be considerably greater than the transfer by conduction.

If the heat is being transmitted through a number of media in series, the overall heat transfer coefficient may be broken down into individual coefficients. The Figure1 below illustrates the individual coefficients of water heater tank.

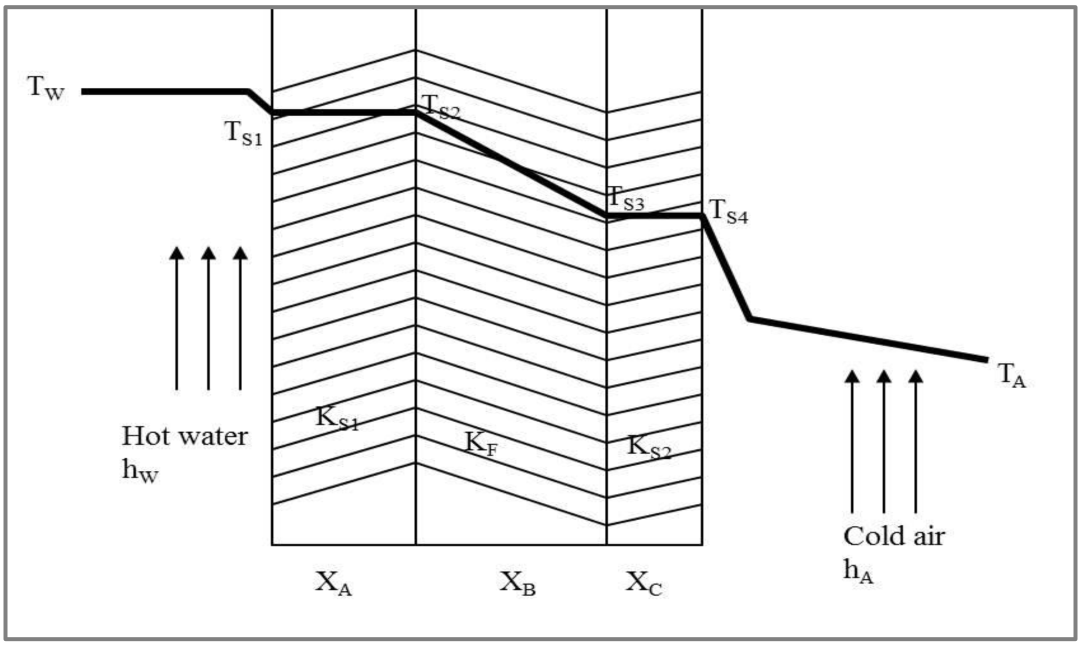


Figure1. Heat Transfer through a water heater tank

Consider the series composite wall of Figure1. The one – dimensional heat transfer rate for the water heater tank may be determined from separate consideration of each element in the network. That is,

$$q_x = \frac{T_W - T_{S1}}{{}^{1}\!/_{h_w}A} = \frac{T_{S1} - T_{S2}}{{}^{X_A}\!/_{K_{S1}}A} = \frac{T_{S2} - T_{S3}}{{}^{X_B}\!/_{K_F}A} = \frac{T_{S3} - T_{S4}}{{}^{X_C}\!/_{K_{S2}}A} = \frac{T_{S4} - T_A}{{}^{1}\!/_{h_A}A} \tag{12}$$

Because the conduction and convection resistances are in series and may be summed,

$$Rt = \frac{1}{h_W A} + \frac{X_A}{K_{S1}A} + \frac{X_B}{K_F A} + \frac{1}{h_A A} \tag{13}$$

$$q_x = \frac{T_W - T_A}{\sum Rt} \tag{14}$$

With composite systems it is often convenient to work with an overall heat transfer coefficient, U, which is defined by an expression analogous to Newton's law of cooling. That is,

$$q_x = UA\Delta T \tag{15}$$

$$U = \frac{1}{RtA} = \frac{1}{\left[\left(\frac{1}{h_w}\right) + \left(\frac{X_A}{K_{S1}}\right) + \left(\frac{X_B}{K_F}\right) + \left(\frac{1}{h_A}\right)\right]} \tag{16}$$

***2.3 Thermal Analysis***

The thermal analysis is based on the following assumptions; that constant rates of electricity supply, the incoming cold water temperature and the surrounding air temperature ($T_A$) are the same, convective heat transfer coefficients on the air and water sides are independent of temperature, time, location and are evaluated at their mean temperature., uniform water temperature and properties, during start – up losses is equal to zero.

$$E_{in} + E_g - E_{out} = \frac{dE_{st}}{dt} \quad (17)$$

$$\frac{dE_{st}}{dt} = PVCp\frac{dT_W}{dt} \quad (18)$$

$$E_{in} = PQCp(T_A - T_W) \quad (19)$$

$$E_g = P \quad (20)$$

$$E_{out} = Q_{Loss} \quad (21)$$

Where p represents the electric power in watts supplied to the water heater. $Q_{Loss}$ are the heat loss from the water heater tank to the surroundings. The various $Q_{Loss}$'s may be given as follows:

$$Q_S = U_S A_S(T_W - T_A) \quad (22)$$

$$Q_T = U_T A_T(T_W - T_A) \quad (23)$$

$$Q_B = U_B A_B(T_W - T_A) \quad (24)$$

Where the $U_s$, $U_T$, and $U_B$ represents the overall heat transfer coefficients of the sides, top and bottom.

$$U_S == \frac{1}{\left[\left(\frac{1}{h_{wS}}\right) + \left(\frac{X_A}{K_{S1}}\right) + \left(\frac{X_B}{K_F}\right) + \left(\frac{X_C}{K_{S2}}\right) + \left(\frac{1}{h_{AS}}\right)\right]} \quad (25)$$

$$U_T == \frac{1}{\left[\left(\frac{X_A}{K_{S1}}\right) + \left(\frac{X_B}{K_F}\right) + \left(\frac{X_C}{K_{S2}}\right) + \left(\frac{1}{h_{AT}}\right)\right]} \quad (26)$$

$$U_B == \frac{1}{\left[\left(\frac{X_A}{K_{S1}}\right) + \left(\frac{X_B}{K_F}\right) + \left(\frac{X_C}{K_{S2}}\right) + \left(\frac{1}{h_{AB}}\right)\right]} \quad (27)$$

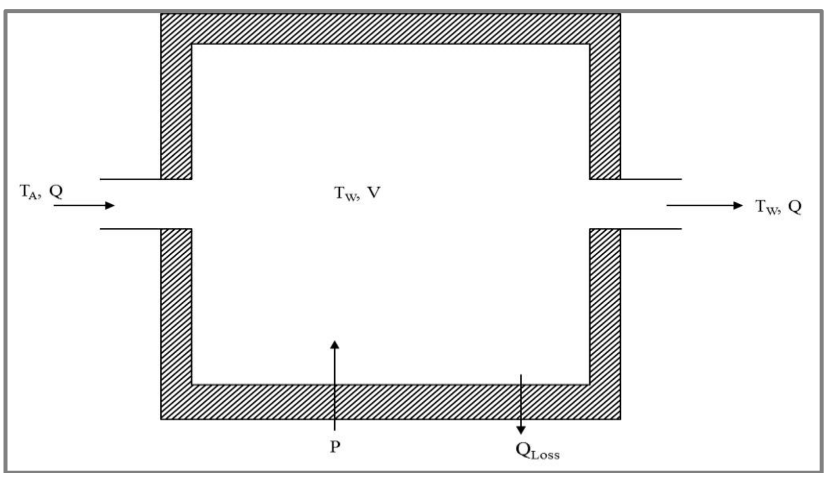


Figure 2. Schematic representation of a single tank water heater

The transient energy balance for single – tank water heater is

$$\rho \mathrm{VCp}\frac{\mathrm{dT_W}}{\mathrm{dt}} = p + \rho QCp(T_A - T_W) - Q_{Loss} \quad (28)$$

At heating when no hot water flows out, the value of Q = 0.
Therefore, equation 28 becomes,

$$\rho VCp\frac{dT}{dt} = \rho - Q_{Loss} \quad (29)$$

$$Q_{Loss} = Q_S + Q_T + Q_B \tag{30}$$

$$Q_{Loss} = (T_W - T_A)(U_S A_S + U_T A_T + U_B A_B) \tag{31}$$

Substituting 31 into 29

$$\rho \mathrm{VCp}\frac{\mathrm{dT_W}}{\mathrm{dt}} = p + T_A(U_S A_S + U_T A_T + U_B A_B) - T_W(U_S A_S + U_T A_T + U_B A_B) \tag{32}$$

$$\frac{\mathrm{dT_W}}{\mathrm{dt}} = \frac{p + T_A(U_S A_S + U_T A_T + U_B A_B)}{\rho \mathrm{VCp}} - \frac{T_W(U_S A_S + U_T A_T + U_B A_B)}{\rho \mathrm{VCp}} \tag{33}$$

$$Let\ N = \frac{p + T_A(U_S A_S + U_T A_T + U_B A_B)}{\rho VCp} \tag{34}$$

$$\beta = \frac{U_S A_S + U_T A_T + U_B A_B}{\rho VCp} \tag{35}$$

$$\frac{dT_W}{dt} = N - T_W \beta \tag{36}$$

$$\frac{dT_W}{dt} + T_W \beta = N \tag{37}$$

Equation 37 is called linear differential equation, integrating factor (IF) = $\ell^{\int \beta t} = \ell^{\beta t}$

$$\frac{d}{dt}\left(\ell^{\beta t} T_W\right) = \int N \ell^{\beta t} \tag{38}$$

$$T_W \ell^{\beta t} = \frac{N\ell^{\beta t}}{\beta} + C \tag{39}$$

$$T_W = \frac{N}{\beta} + C\ell^{-\beta t} \tag{40}$$

Introducing initial condition

At t = 0; $T_W = T_A$ (41)

Substituting equation (41) into (40)

$$C = T_A - \frac{N}{\beta} \tag{42}$$

Substituting 42 into 40

Therefore

$$T_W = \frac{N}{\beta} + \left(T_A - \frac{N}{\beta}\right)\ell^{-\beta t} \tag{43}$$

Therefore, the theoretical temperature at a given time, t, for the single – tank water heater is $T_W$.

The first–hour rating, which a measure of the energy consummations of the water heaters is given as:

$$F_{hr} = \sum_{i=1} Vi \tag{44}$$

Where: n is the number of draws that are completed during the first–hour rating experiment, $V_i$ is the volume of water removed during the draw of the first–hour rating experiment.

The recovery efficiency of each draw during the first – hour rating of the experiment is computed as:

$$\eta_r = \frac{\rho V_n Cp(T_W - T_A)}{Et} \tag{45}$$

Where $V_n$ is the volume of the nth draw during the first – hour rating.

## 3. Materials and Experimental Method

### 3.1 Materials

The material used in the construction of the these electric water heaters tanks were carefully selected on the basis of their properties such as strength of the materials, temperature, effect of heat, thermal conductivity and expansion coefficient.The materials include; galvanized sheet, steel sheet, filer glass, cylinder pipe and heating elements with its accessories (thermostats).

### 3.2 Experimental setup

The setup of the experiment is shown in Figure 3 below and consists of a bucket for cold water, siphons, inlet and outlet control valves, power source, thermostatsand a 50 litres water heater tank. The buckets are calibrated to collect a measured draw of hot water.

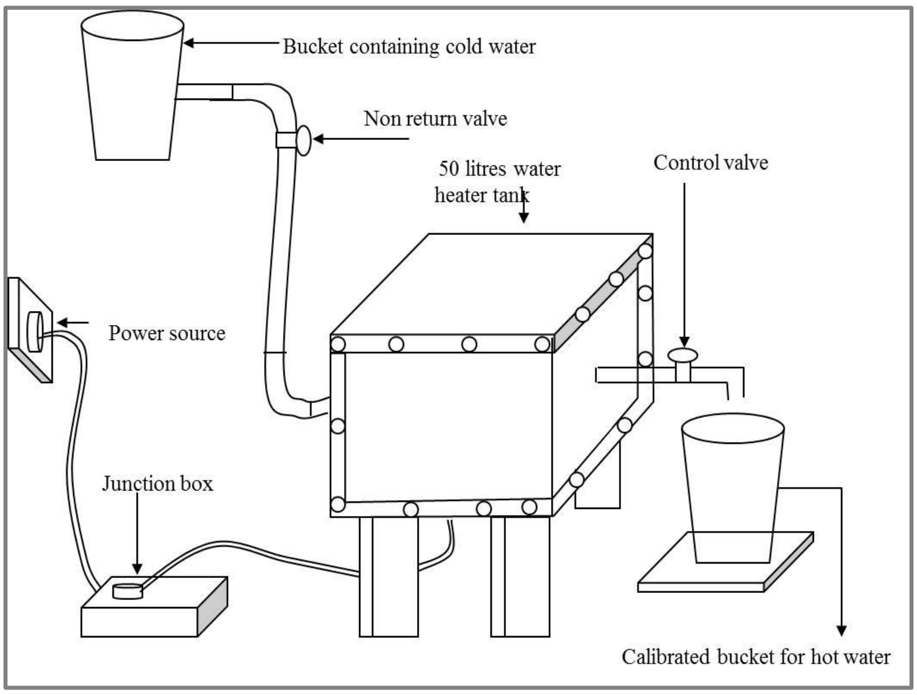


Fig 3. Experimental set up of a single tank water heater

Table 1.Specifications

| Description | Tank | Jacket |
|---|---|---|
| Length, L (m) | 0.46 | 0.51 |
| Breadth, B (m) | 0.24 | 0.26 |
| Height, H (m) | 0.47 | 0.52 |
| Area, A ($m^2$) | 0.11 | 0.13 |
| Volume, V ($m^2$) | 0.05 | 0.07 |

### 3.3 Experimental procedure

The cold water tap was opened, 45 litres of cold water at ambient temperature was poured into the tank. The gap between the 50 litres of tank and 45 litres of water was allowed to give the required volume of water during the maximum design temperature without exceeding the maximum allowable operating pressure and to maintain the required minimum pressure when the system is cold. The experiment was carried out in two phases. The first was to determine the heating time and the second for the first hour rating.

**Heating time:** The initial temperature of water and the ambient temperature were measured to be 27.7°C. The main to which the heater and the thermostat were connected was switch on simultaneously with the stop watch. The increase in water temperature was measured at an interval of ten minutes for heating mode (power on) until the temperature of water reaches the required temperature (60°C).

**First hour rating:** At steady temperature, first- hour rating of water heater was experimented. Each draw was made when the thermostats switched-off the electrical input supplied to each of the heating element. Also each of the draws was 10 litres, 15 litres, 25 litres, 30 litres, 35 litres and 40 litres.After each successive draws, the same amount of draw was returned back into the tank through the bucket containing cold water. Steady heat flow equation was used to determine the final temperature as the cold water enters the tank. This was repeated until the last draw was collected. The total volume of each draw was added to determine the first hour rating.

## 4. Results

Table 2: Experimental Results for heating times

| Time (Minute) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|---|---|---|---|
| Temp. (°C) | 27.7 | 30.0 | 35.0 | 40.0 | 45.0 | 48.0 | 50.0 | 55.0 | 58.0 | 60.0 |

Table 3: Experimental Results for first hour rating

| Draw in volume (litre) | 10 | 15 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|
| Time taken to return back to 60°C (min) | 30 | 42 | 65 | 73 | 80 | 86 |

### 4.1 Theoretical Calculations

The following parameters were used for the analysis.
Inlet water temperature, $T_A$ = 27.7°C
Temperature around tanks (Ambient temperature), $T_A$ = 27.7°C
Specific heat of water at constant pressure, Cp = 4190J/Kg K
Power input to tank of 50 litres, P = 1200W
Thermal conductivity of fiber glass, $K_F$ = 0.0385W/MK
Emissivity of polished mild steel, E = 0.07
Thermal conductivity of mild steel, $K_s$ = 48.5W/MK
Required hot – water temperature, $T_W$ = 60°C.

***Theoretical temperatures for heating time***
The convective heat transfer coefficients were determined using existing correlations.

Using equations(25) to (27), the values of $U_S$, $U_T$ and $U_B$ are calculated as 0.777W/m$^2$K, 1.902W/m$^2$K and 0.902W/m$^2$K respectively.

Also from equations (22) to (24)and substituting the values obtained for $U_S$, $U_T$ and $U_{B,}$ the values of $Q_S$, $Q_T$and $Q_B$are calculated as 19.83W, 4.77W and 3.79W respectively.

Using equations 43, the theoretical temperatures ($T_W$) at a given interval of time is calculated and results shown in Table 4 below.

Table 4. Theoretical temperature ($T_W$) of the single tank

| Time (min) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|---|---|---|---|
| Temp. (°C) | 27.7 | 31.44 | 35.17 | 38.88 | 42.59 | 46.29 | 49.98 | 53.65 | 57.32 | 60.97 |

***First – hour rating***
The first –hour rating is calculated using equation (44) as 10+15+25+30+35+40 = 155litres. The time taken for the total volume to return back to 60°C after each draws is 376 mins (6 hours 16 mins). The time of individual draw is tabulated in Table 5 below.

Table 5. Experimental Results for single – tank for first hour rating

| | | | | | | |
|---|---|---|---|---|---|---|
| Draw in volume (litre) | 10 | 15 | 25 | 30 | 35 | 40 |
| Time taken to return back to 60°C (min) | 30 | 42 | 65 | 73 | 80 | 86 |

***Recovery efficiency***

Using equation (45), the recovery efficiency after each draws from the tank is tabulated in Table 6 below.The average recovery efficiency for water heater tank is 77%.

Table 6. The recovery efficiency

| | | | | | | |
|---|---|---|---|---|---|---|
| Draw in volume (litre) | 10 | 15 | 25 | 30 | 35 | 40 |
| Time taken to return back to 60°C (min) | 63 | 67 | 72 | 77 | 82 | 87 |

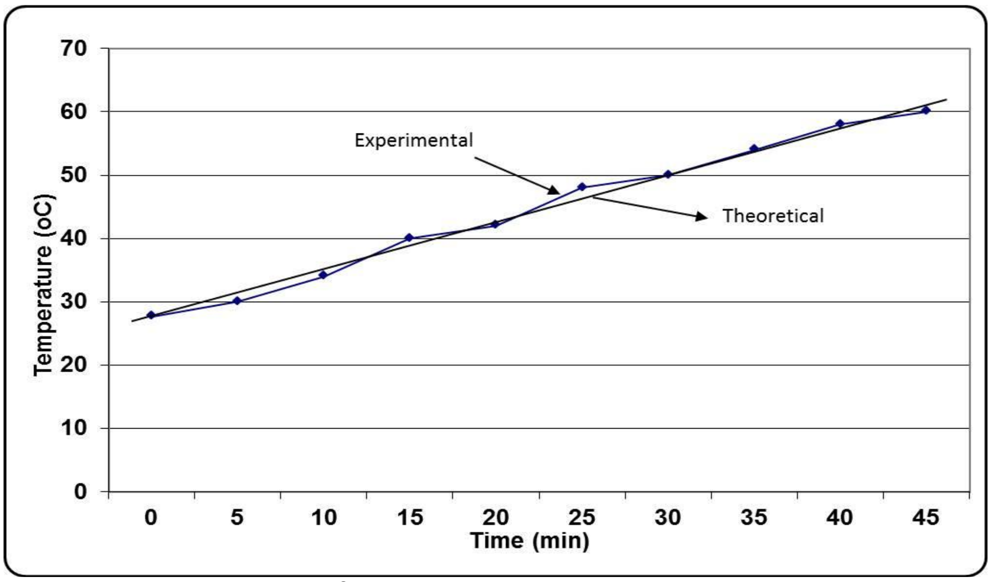


Fig 4. Graph of Temperature (°C) against time (min) for experimental and theoretical

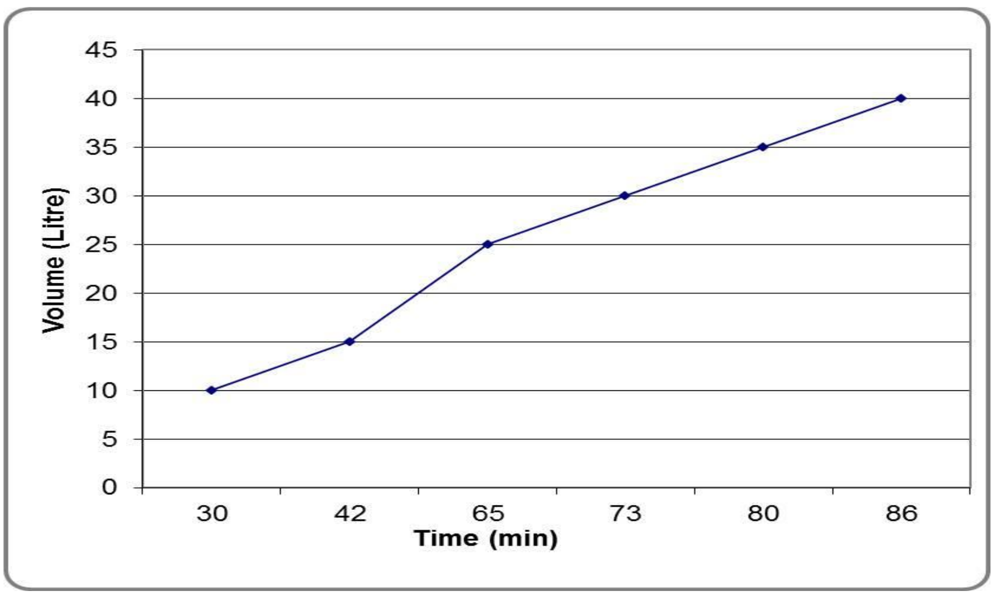


Fig 5. Graph of volume against time water heater tank

From the graph, the transient heat transfer to the water heaters can be estimated using equation (29) as 1225.6W. Where dT/dt is the slope of the graph and is given as 0.39°C/min

**5. Discussion of Results:**

The results obtained after the thermal performance of water heater showed that the aims and objectives of this paper were achieved. This can be seen from the results of experiments presented in terms of heating time with those obtained from the mathematical model Table 4 in which basic program was used in computing the values of temperatures. The theoretical and experimental results were in good agreement.The corresponding amount of heat lost from the tank was found to be 28.39W. The heat lost due to radiation from the water heater jacket was less because of the highly polished mild steel sheet with low emissive.

The graphs of the water heater shown in Figure 4 is said to be a thermal analysis curve for both theoretical and experimental values. The graphs are the change of temperature with time (dT/dt). Therefore, the transient heat transfer for heating the tank is 1225.6W. Also the graph of volume against time in Figure 5 shows the flow rate of the tanks. It was observed that an increase in time lead to a decrease in flow rate of the hot water. For instance higher increase in time shows little flow of hot water. Therefore, at 45 minutes the tank was able to make 0.465litres/minutes of hot water.

The first-hour rating of each draw was determined and the time taken for the addition of the same volume of cold water after each draw of hot water to return back to the required temperature ($60^0$) was also computed. The total volume used at the end of each draw was 155litres. It took the tank 6hours 5minutes to heat 155litres of water to the required temperature. The recovery efficiency of each heating of same addition of cold after each draw of hot water were determined. Table 6 shows the recovery efficiency for of the tank. The results show that as the number of draws increases also the efficiency increases. The value obtained for the tank is 87%.

**6.Conclusions**

A mathematical model for domestic water heating system was developed and validated by experiment. The experimental results were in good agreement with the model results. For a water heater carrying 155 litres of water, the time required to raise the temperature of the bulk of water from 27.7$^o$C to 60$^o$C is 365 minutes (6 hours 5minutes). The transient heat transfer was found to be 1225.6W for the single tank systems. The mathematical model developed is a useful tool to assess the performance of water heaters with a view to saving energy.

**References**

1. Kar, A. K. andAl-Dossary, K. M. (1995), Thermal Performances of water heaters in series, Applied Energy Journal, Vol. 52 (1).
2. Robert A. Meyers, Encyclopedia of Physical Science and Technology, 2$^{nd}$ Edition, McGraw-Hill, Vol. 16.
3. Carrington, C.G., Warrington, D. M. and Yak, Y.C., Structure of domestic hot water consumption. Energy Research, 9(1) (1985).
4. Philip Fairey, Dany S. Parker, (20 July, 2004), A Review of Hot Water Draw Profiles used in Performance Analysis of Residential Domestic Hot Water systems, Appendix E, (1-1-02 Edition).
5. A.Nasir and F. O. Akinbode (2002). Improved Efficiency of Electric Boiling Ring WaterHeating System Nigerian Journal of Technology Research. Vol. 1 (1).
6. Frank P. Incropera, David P. Dewitt, (1990), Introduction to Heat Transfer, 2$^{nd}$ Edition, John Wiley and Sons.
7. T. D. Eastop, A. Mcconkey, (1993), Applied Thermodynamics for Engineering Technologists, 5$^{th}$ Edition.