# PREDICTING THE CALCULATED MAXIMUM SUM OF TWO COMPONENTS THROUGH SHOCK MODEL

P. Pandiyan[1*], A.Loganathan[2]

[1]Department of Statistics, Annamalai University, Annamalainagar, Chidambaram, Tamilnadu, India.

[2]Department of Statistics, Manonmaniam Sundranar University, Tirunelveli, Tamil Nadu, India.

## ABSTRACT

The human immunodeficiency virus (HIV) mutates rapidly during the course of an individual's infection. At any given time the virus population in an infected patient does not consist of a single uniform sequence but a distribution of different variants, generated by mutation and selection. The maximum of the two components is estimated using three parameter generalized Pareto distribution, in this paper by finding the expected time and variance. The analytical results are numerically illustrated by assuming the distribution for the practical use of the model.

*Keyword*: Components; HIV; Threshold; Three parameter generalized Pareto distribution.

## INTRODUCTION

Mathematical modelling can be applied to evaluating multiple aspects involved in exposing a population to the AIDS virus and predicting the spread of AIDS within the population. These mathematical models can be readily converted into a computer project that evaluates effects of the disease and potential spread of the disease. The threshold beyond which the human immune system cannot withstand is represented as sum of two random variables.The three parameter generalized Pareto distribution, which is a special case of both exponential and Wakeby distribution, has good potential for the analysis of flood peaks because of its inherent properties. Point after which radical changes are likely to occur is called threshold level. The generalized Pareto distribution has been widely used to model rare events in several fields. The generalized Pareto distribution, introduced by Pickands (1975), is a limit distribution for the excess over a (large) threshold for data coming from generalized extreme value distributions, as well as a generalization of the Pareto distribution.One can see for more detail in Esary et al., (1973), V.S.Bhuvana and P.Pandiyan, (2012), discussed about the expected time to cross threshold level period.

Sexual risk and needle sharing are the source of HIV infection. The threshold of any individual is a random variable. If the total damage crosses a threshold level Y which itself is a random variable, the seroconversion occurs and a person is recognized as an infected. The inter-arrival times between successive contacts, the sequence of damage and the threshold are mutually independent.

**NOTATION**

$X_i$ : a continuous random variable denoting the amount of loss caused to the system on the i$^{th}$ occasion of policy announcement (Shock), 1, 2….k and $X_i's$ are i.i.d

$Y_1, Y_2$ : continuous random variable denoting the threshold levels for the two components which follows three parameter generalized Pareto distribution.

$U_i$ : a random variable denoting the inter-arrival times between contact with c.d.f. $F_i(.)$, $i = 1,2,3 \dots k$.

$g(.)$ : the probability density function of $X_i$; $g^*(.)$ : Laplace transform of $g(.)$

$g_k(.)$ : the k- fold convolution of $g(.)$i.e., p.d.f. of $\sum_{j=1}^{k} X_i$

$f(.)$ : p.d.f. of random variable denoting between successive eventwith the corresponding c.d.f. $F(.)$.

$F_k(.)$ : k-fold convolution of $F(.)$; $S(.)$ : Survival function.

$V_k(t)$ : Probability of exactly k policy announcements; $L(t)$: $1 - S(t)$.

## MODEL DESCRIPTION

Any component exposed to shocks which cause damage to the component is likely to fail when the total cumulated damage exceed a level called threshold. In general, assuming that the threshold Y, follows three-parameter generalized Pareto distributiondiscussed by Pickands (1975).

$$\bar{H}(x) = 2e^{\left(\frac{d-x}{b}\right)} - e^{\left(\frac{2d-2x}{b}\right)} \tag{1}$$

There may be no practical way to inspect an individual item to determine its threshold Y. Three-parameter generalized Pareto distribution with parameter $b$ and $d$, can be proved that

$$P(X_i < Y) = \int_0^\infty g^*(x)\bar{H}(x)dx \tag{2}$$

Now the threshold Y is such that it has two components namely $\mathrm{Y}_1$ and $\mathrm{Y}_2$. Transfer of component from $\mathrm{Y}_1$ to $\mathrm{Y}_2$ is also possible. We have the breakdown of the system at $\mathrm{Y} = \max(\mathrm{Y}_1, \mathrm{Y}_2)$.

$\mathrm{P}[\max(\mathrm{Y}_1, \mathrm{Y}_2)] = \mathrm{P}[(\mathrm{Y}_1 < y) \cap (\mathrm{Y}_2 < y)] = \mathrm{P}[\mathrm{Y}_1 < y]\mathrm{P}[\mathrm{Y}_2 < y]$

Now that, $\mathrm{Y}_1$ and $\mathrm{Y}_2$ follow three parameter generalized Pareto distribution with parameter$b, d$.

$$P\left(\sum_{i=1}^{k} X_i < Y\right) = 2\int_0^\infty g^*(x)e^{-\left(\frac{x-d}{b}\right)} - \int_0^\infty g^*(x)e^{\left(\frac{2x-2d}{b}\right)}dx$$

$$= 2\left[g^*\left(\frac{1-d}{b}\right)\right]^k - \left[g^*\left[2\left(\frac{1-d}{b}\right)\right]\right]^k \tag{3}$$

Survival analysis is a class of statistical methods for studying the occurrence and timing of events.The survival function $\mathrm{S(t)}$ which is the probability that an individual survives for a time t

$\mathrm{S(t)} = \mathrm{P}(\mathrm{T} > t) =$ Probability that the total damage survives beyond $t$

$$= \sum_{k=0}^{\infty} \mathrm{P}\{there\ are\ exactly\ k\ epochs\ in\ (0,t] * P\ (the\ total\ cumulative\ \ (0,t]\}$$

$$S(t) = P(T > t) \ = \sum_{k=0}^{\infty} V_k(t) P\big(X_i < max(Y_1, Y_2)\big)$$

It may happen that successive shocks become increasingly effective in causing damage, even though they are independent. This means that $V_k(t)$, the distribution function of the k$^{th}$ damage is decreasing in k=1, 2... foreach t.It is also known from renewal process that

$$\mathrm{P(exactly\ k\ policy\ decesions\ in\ (0,t]) = F_k(t) - F_{K+1}(t)} \qquad \text{with} \qquad \mathrm{F_0(t) = 1}$$

$$= \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[g^*\left(\frac{1-d}{b}\right)\right]^k + \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[g^*\left(\frac{1-d}{b}\right)\right]^k - \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[g^*\, 2\left(\frac{1-d}{b}\right) + \left(\frac{1-d}{b}\right)\right]^k$$

$L(t) = 1 - S(t)$, TakinglaplacetransformofL(t), Weget

$$L(t) = 1 - \left\{2\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[g^*\left(\frac{1-d}{b}\right)\right]^k - \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[g^*\left(\frac{2-2d}{b}\right)\right]^k\right\}$$

On simplification we get,

$$L(t) = 2\left[1 - g^*\left(\frac{1-d}{b}\right)\right]\sum_{k=1}^{\infty} F_k(t)\left[g^*\left(\frac{1-d}{b}\right)\right]^{k-1} - \left[1 - g^*\left(\frac{2-2d}{b}\right)\right]\sum_{k=1}^{\infty} F_k(t)\left[g^*\left(\frac{2-2d}{b}\right)\right]^{k-1}$$

$$l^*(t) = \frac{2\left[1 - g^*\left(\frac{1-d}{b}\right)\right] f^*(s)}{\left[1 - g^*\left(\frac{1-d}{b}\right) f^*(s)\right]} - \frac{\left[1 - g^*\left(\frac{2-2d}{b}\right)\right] f^*(s)}{\left[1 - g^*\left(\frac{2-2d}{b}\right) f^*(s)\right]} \tag{4}$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $\mathrm{f^*(s)} = \left(\frac{\mathrm{c}}{\mathrm{c+s}}\right)$, substituting in the above equation (4) we get

$$= \frac{2\left[1 - g^*\left(\frac{1-d}{b}\right)\right]\left(\frac{c}{c+s}\right)}{\left[1 - g^*\left(\frac{1-d}{b}\right)\left(\frac{c}{c+s}\right)\right]} - \frac{\left[1 - g^*\left(\frac{2-2d}{b}\right)\right]\left(\frac{c}{c+s}\right)}{\left[1 - g^*\left(\frac{2-2d}{b}\right)\left(\frac{c}{c+s}\right)\right]} \tag{5}$$

$$E(T) = -\frac{d}{ds} l^*(s) given s = 0, \qquad E(T^2) = -\frac{d^2}{ds^2} given s = 0$$

From which variance $V(T) = E(T^2) - [E(T)]^2$ can be obtained

$$E(T) = \frac{2}{c\left[1 - g^*\left(\frac{1-d}{b}\right)\right]} - \frac{1}{c\left[1 - g^*\left(\frac{2-2d}{b}\right)\right]} on simpilification$$

$$E(T^2) = \frac{4}{c^2\left[1 - g^*\left(\frac{1-d}{b}\right)\right]^2} - \frac{2}{c^2\left[1 - g^*\left(\frac{2-2d}{b}\right)\right]^2} on simpilification$$

$$g^*(.) \sim Mittag Leffler Distribution \frac{1}{1+\lambda^\alpha}$$

$$g^*(.) \sim exp\ (\mu), g^*(\lambda) \sim exp\left(\frac{\mu}{\mu+\lambda}\right) and\, g^*(\lambda\theta) \sim exp\left(\frac{\mu}{\mu+\lambda\theta}\right)$$

$$E(T) = \frac{2}{c\left[1 - \left(\frac{\mu}{\mu+\frac{1}{b}}\right) - \left(\frac{\mu}{\mu+\frac{d}{b}}\right)\right]} - \frac{1}{c\left[1 - \left(\frac{\mu}{\mu+\frac{2}{b}}\right) - \left(\frac{\mu}{\mu+\frac{2d}{b}}\right)\right]}$$

$$E(T) = \frac{2[b^2\mu^2 + db\mu + b\mu + d]}{c[b^2\mu^2 + 2b\mu + d]} - \frac{[b^2\mu^2 + 2db\mu + 2b\mu + 4d]}{c[b^2\mu^2 + 4b\mu + 4d]} \qquad (6)$$

$$E(T^2) = \frac{4[b^2\mu^2 + db\mu + b\mu + d]^2}{c^2[b^2\mu^2 + 2b\mu + d]^2} - \frac{2[b^2\mu^2 + 2db\mu + 2b\mu + 4d]^2}{c^2[b^2\mu^2 + 4b\mu + 4d]^2} (7)$$

**RESULTS**

$$E(T) = \frac{2[b^2\mu^2 + db\mu + b\mu + d]}{c[b^2\mu^2 + 2b\mu + d]} - \frac{[b^2\mu^2 + 2db\mu + 2b\mu + 4d]}{c[b^2\mu^2 + 4b\mu + 4d]} \qquad (8)$$

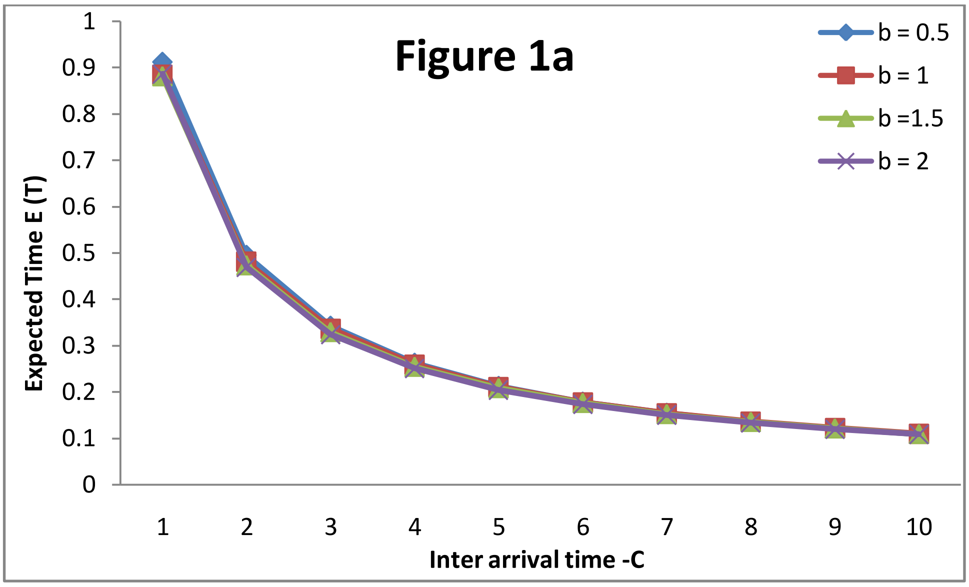
$$V(T) = \frac{4[b^2\mu^2 + db\mu + b\mu + d][b^2\mu^2 + 2db\mu + 2b\mu + 4d]}{c^2[b^2\mu^2 + 2b\mu + d][b^2\mu^2 + 4b\mu + 4d]}$$

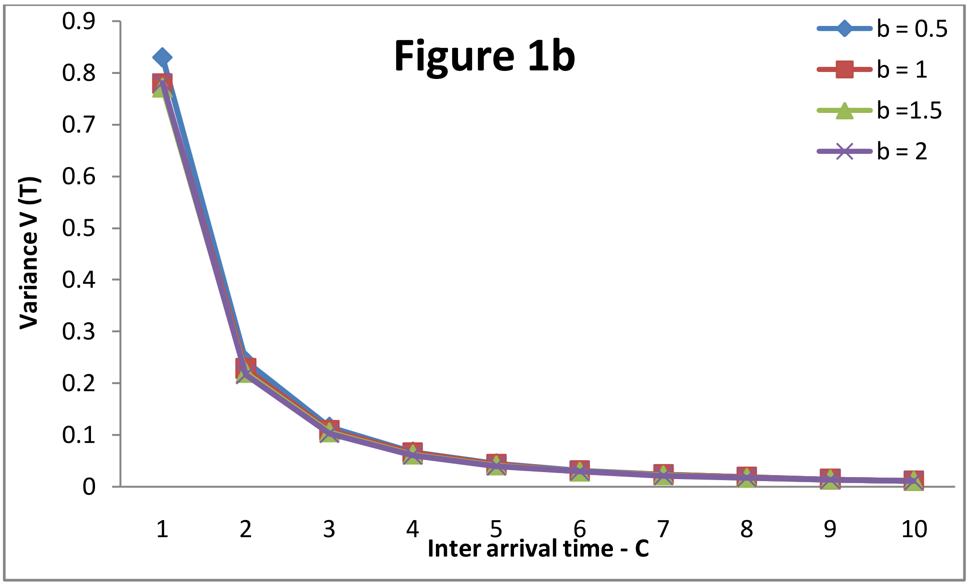
$$- \frac{3[b^2\mu^2 + 2db\mu + 2b\mu + 4d]^2}{c^2[b^2\mu^2 + 4b\mu + 4d]^2} \qquad (9)$$

Issue 2 volume 4, August 2012
ISSN 2249-9954

## NUMERICAL ILLUSTRATION

On the basis of the numerical illustration from the equation 8 and 9 the following conclusions regarding expected time and variance consequent to the changes in the different parameters can be observed in Figures 1 to 3 that follow.



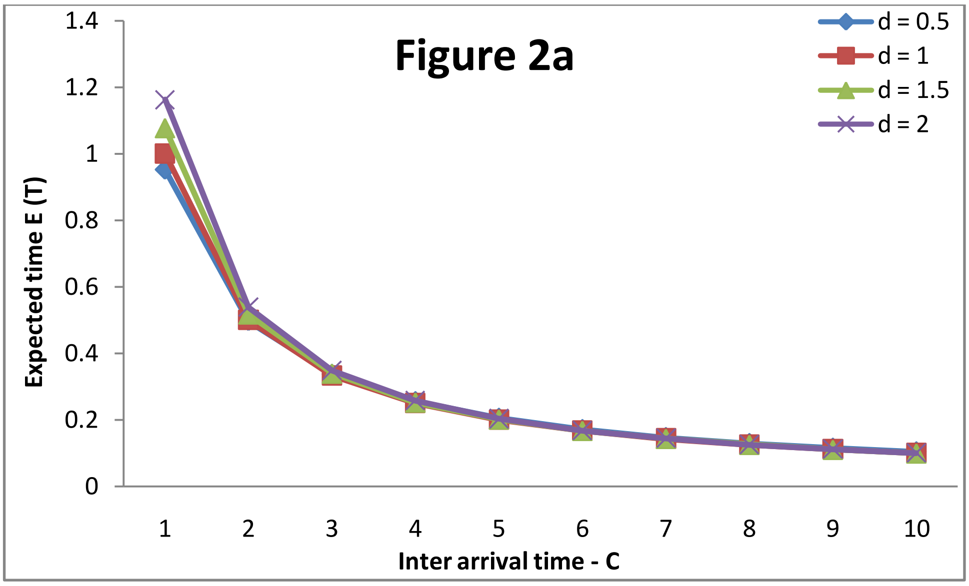


Figure 2a
Expected time E (T)
Inter arrival time - C
d = 0.5
d = 1
d = 1.5
d = 2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1
2
3
4
5
6
7
8
9
10

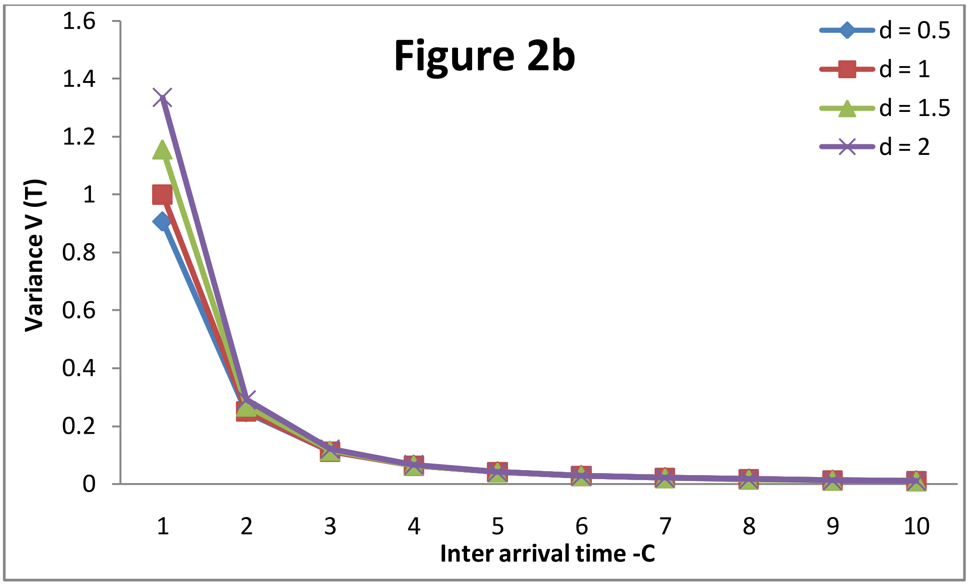
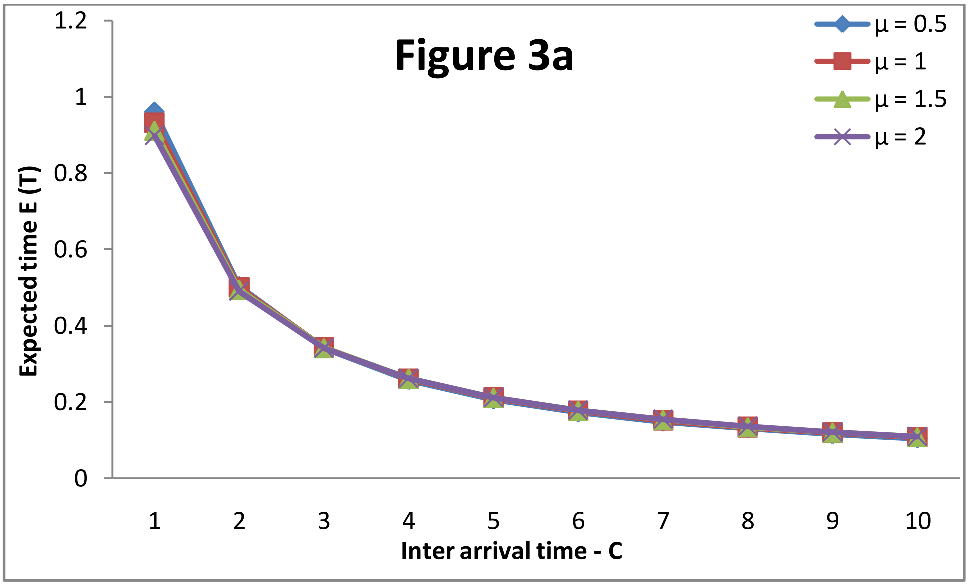
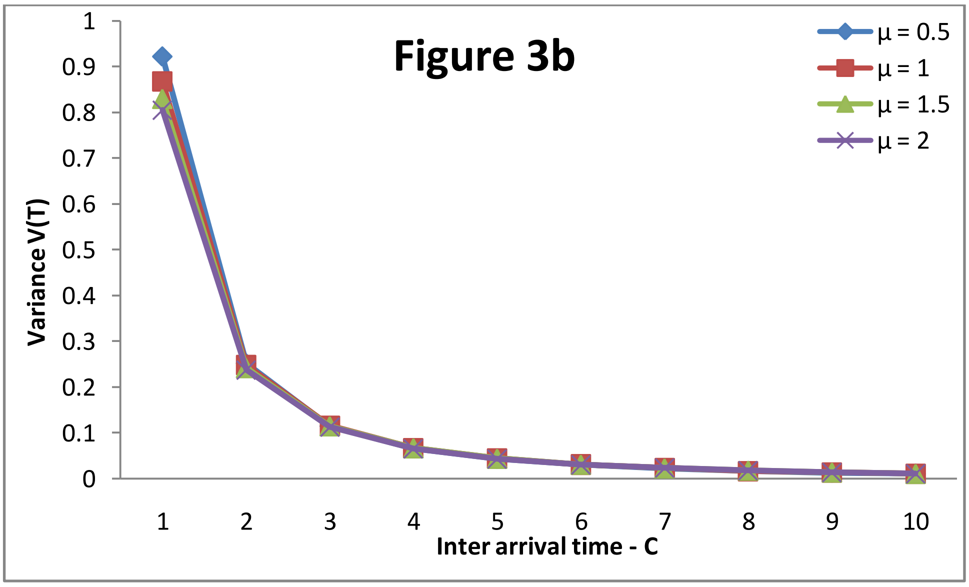


Figure 2b
Variance V (T)
Inter arrival time -C
d = 0.5
d = 1
d = 1.5
d = 2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1
2
3
4
5
6
7
8
9
10




## CONCLUSIONS

When $\mu, d$ is kept fixed with other parameters b the inter-arrival time $'c'$ which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time E (T) to cross the threshold is decreasing, for all cases of the parameter value $b = 0.5, 1, 1.5, 2$. When the value of the parameter b increases, the expected time is found increasing, this is observed in Figure 1a. The same case is found in Variance V (T) which is observed in Figure 1b.

When $\mu, b$ is kept fixed with other parameters $d$ the inter-arrival time $'c'$ increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value $d = 0.5,1,1.5,2$. When the value of the parameter d increases, the expected time is found increasing. This is indicated in Figure 2a. The same case is observed in the Variance V (T) which is observed in Figure 2b.

When $b, d$ is kept fixed with other parameters $\mu$ the inter-arrival time $'c'$ increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value $\mu = 0.5,1,1.5,2$. When the value of the parameter $\mu$ increases, the expected time is found increasing. This is indicated in Figure 3a. The same case is observed for the Variance V (T) which is observed in Figure 3b.

**REFERENCE**

1. Abd Elfattah, A. M., Elsherpieny, E. A. and Hussein, E. A. (2007) “Parameters estimation for new generalized Pareto distribution”. Submitted.
2. Esary, J.D., A.W. Marshall and F. Proschan. (1973), Shock models and wear processes. Ann. Probability, 1(4), 627-649.
3. Masoom, M. A. and Saralees Nadarajah, (2006) “A truncated Pareto distribution”, Computer Communications.
4. V.S. Bhuvana and P. Pandiyan (2012), Reduce the Spread of HIVthrough Estimating the Mean and Variance using Two Components,Advances in Applied Mathematical Biosciences, Volume 3, Number 1, pp. 25-30.
5. Pickands, J. (1975). Statistical inference using extreme order statistics. Ann. Statist. Vol 3, 119-131.
6. Singh, V.P. and H. Guo. 1995. Parameter estimation for 3-parameter generalized Pareto distribution by the principle of maximum entropy (POME). Hydrol. Sci. 40: 165-181.