Slightly gα-continuous; Somewhat gα-continuous and Somewhat gα-open functions

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Abstract: In this paper we discuss new type of continuous functions called slightly $g\alpha$–continuous; somewhat $g\alpha$-continuous and somewhat gα-open functions; its properties and interrelation with other such functions are studied.

Keywords: slightly continuous functions; slightly semi-continuous functions; slightly pre-continuous; slightly β–continuous functions; slightly γ–continuous functions and slightly ν–continuous functions; somewhat continuous functions; somewhat semi-continuous functions; somewhat pre-continuous; somewhat β–continuous functions; somewhat γ–continuous functions and somewhat ν–continuous functions; somewhat open functions; somewhat semi-open functions; somewhat pre-open; somewhat β–open functions; somewhat γ–open functions and somewhat ν–open functions

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1.Introduction

In 1995 T.M.Nour introduced slightly semi-continuous functions. After him T.Noiri and G.I.Chae further studied slightly semi-continuous functions in 2000. T.Noiri individually studied about slightly β–continuous functionsin 2001. C.W.Baker introduced slightly precontinuous functions in 2002. Erdal Ekici and M. Caldas studied slightly γ–continuous functions in 2004. Arse Nagli Uresin and others studied slightly δ–continuous functions in 2007. Recently S. Balasubramanian and P.A.S. Vyjayanthi studied slightly ν–continuous functions in 2011.

b-open sets are introduced by Andrijevic in 1996. K.R.Gentry introduced somewhat continuous functions in the year 1971. V.K.Sharma and the present authors of this paper defined and studied basic properties of *v*-open sets and *v*-continuous functions in the year 2006 and 2010 respectively. T.Noiri and N.Rajesh introduced somewhat b-continuous functions in the year 2011. Inspired with these developments we introduce in this paper slightly $g\alpha$–continuous, somewhat $g\alpha$-continuous functions and somewhat $g\alpha$-open functions and study its basic properties and interrelation with other type of such functions. Throughout the paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2.Preliminaries

Definition 2.1: A⊂ X is called
(i) closed if its complement is open.
(ii) rα-open[*v*-open] if ∃ U∈ αO(X)[RO(X)] such that U⊂ A⊂ αcl(U)[U⊂ A⊂cl(U)].
(iii)semi-θ-open if it is the union of semi-regular sets and its complement is semi-θ-closed.
(iv) Regular closed[α-closed; pre-closed; β–closed] if A = cl{A^o}[resp:(cl(A^o))^o⊆A; cl(A^o)⊆A; cl((cl{A}))^o⊆A].
(v) Semi closed[*v*-closed] if its complement if semi open[*v*-open].
(vi) g-closed[rg-closed] if cl A⊆ U whenever A⊆ U and U is open in X.
(vii) sg-closed[gs-closed] if s(cl A) ⊆ U whenever A⊆ U and U is semi-open{open} in X.
(viii) αg-closed[gα-closed; rgα-closed] if α(cl A) ⊆ U whenever A⊆ U and U is{α-open; rα-open}open in X.
(x) *v*g-closed if *v*cl(A) ⊆ U whenever A⊆ U and U is *v*-open in X.
(xi) b-open if A⊂(*cl*{A})^o∩ *cl*{A^o}.

Definition 2.2: A function *f*:X→ Y is said to be

(i) continuous[resp**:** nearly-continuous; rα-continuous; *v*-continuous; α-continuous; semi-continuous; β−continuous; pre-continuous] if inverse image of each open set is open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β -open; preopen].
(ii) nearly-irresolute[resp**:** rα-irresolute; *v*-irresolute; α-irresolute; irresolute; β−irresolute; pre-irresolute] if inverse image of each regular-open[resp**:** rα-open; *v*-open; α-open; semi-open; β-open; preopen] set is regular-open[resp**:** rα-open; *v*-open; α-open; semi-open; β-open; preopen].
(iii) almost continuous[resp**:** almost nearly-continuous; almost rα-continuous; almost *v*-continuous; almost α-continuous; almost semi-continuous; almost β−continuous; almost pre-continuous] if for each x in X and each open set (V, f(x)), ∃ an open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen] set (U, x) such that $f(U) \subset (cl(V))^{o}$.
(iv) weakly continuous[resp**:** weakly nearly-continuous; weakly rα-continuous; weakly *v*-continuous; weakly α-continuous; weakly semi-continuous; weakly β−continuous; weakly pre-continuous] if for each x in X and each open set (V, f(x)), ∃ an open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen] set (U, x) such that $f(U) \subset cl(V)$.
(v) slightly continuous[resp**:** slightly semi-continuous; slightly pre-continuous; slightly β−continuous; slightly γ−continuous; slightly α-continuous; slightly r-continuous; slightly *v*-continuous] at x in X if for each clopen subset V in Y containing f(x), ∃ U∈ τ (X)[∃ U∈ SO(X); ∃ U∈ PO(X); ∃ U∈ βO(X); ∃ U∈ γ O(X); ∃ U∈ α O(X); ∃ U∈ RO(X); ∃ U∈ *v* O(X)] containing x such that $f(U) \subseteq V$.
(vi) slightly continuous[resp**:** slightly semi-continuous; slightly pre-continuous; slightly β−continuous; slightly γ−continuous; slightly α-continuous; slightly r-continuous; slightly *v*-continuous] if it is slightly-continuous[resp**:**slightly semi-continuous; slightly pre-continuous; slightly β−continuous; slightly γ−continuous; slightly α-continuous; slightly r-continuous; slightly *v*-continuous] at each x in X.
(vii) almost strongly θ-semi-continuous[resp**:** strongly θ-semi-continuous] if for each x in X and for each V∈ σ(Y, f(x)), ∃ U∈ SO(X, x) such that $f(scl(U)) \subset scl(V)$[resp**:** $f(scl(U)) \subset V$].
(viii) somewhat continuous[resp: somewhat b-continuous; somewhat *v*-continuous] if for U∈σ and $f^{-1}(U) \neq \varphi$, there exists a non empty open[resp: non empty b-open; non empty *v*-open] set V in X such that V≠ φ and $V \subset f^{-1}(U)$.
(ix) somewhat-open[resp: somewhat b-open; somewhat *v*-open] provided that if U∈τ and U≠ φ, then there exists a non empty b-open set[resp: non empty b-open; non empty *v*-open] V in Y such that V ≠ φ and V⊂ f(U).
(x) somewhat *v*-irresolute if for U∈ *v*O(σ) and $f^{-1}(U) \neq \varphi$, there exists a non-empty *v*-open set V in X such that $V \subset f^{-1}(U)$.

Note 1: From the above Definitions we have the following interrelations among the closed sets.

↑ → → rα-closed → *v*-closed → → ↓
Regular closed → π-closed → closed → α-closed → semi closed → β-closed
↓
gα-closed

Definition 2.3: X is said to be a
(i) compact[resp**:** nearly-compact; rα-compact; α-compact; mildly-compact] space if every open[resp**:** regular-open; rα-open; α-open; clopen] cover has a finite subcover.
(ii) Lindeloff[resp**:** nearly-Lindeloff; rα-Lindeloff; α-Lindeloff; mildly-Lindeloff] space if every open[resp**:** regular-open; rα-open; α-open; clopen] cover has a countable subcover.
(iii) Extremally disconnected[briefly e.d] if the closure of each open set is open.

Definition 2.4: X is said to be a

(i) T_0[resp: r-T_0; rα-T_0; α-T_0; Ultra T_0] space if for each $x \neq y \in X$ $\exists$ $U \in \tau$ (X)[resp: rO(X); rαO(X); αO(X); CO(X)] containing either x or y.

(ii) T_1[resp: r-T_1; rα-T_1; α-T_1; Ultra T_1] space if for each $x \neq y \in X$ $\exists$ U, $V \in \tau$ (X)[resp: rO(X); rαO(X); αO(X); CO(X)] such that $x \in U - V$ and $y \in V - U$.

(iii) T_2[resp: r-T_2; rα-T_2; α-T_2; Ultra T_2] space if for each $x \neq y \in X$ $\exists$ U, $V \in \tau$ (X)[resp: rO(X); rαO(X); αO(X); CO(X)] such that $x \in U$; $y \in V$ and $U \cap V = \phi$.

(iv) C_0[resp: r-C_0; rα-C_0; α-C_0; Ultra C_0] space if for each $x \neq y \in X$ $\exists$ $U \in \tau$ (X)[resp: rO(X); rαO(X); αO(X); CO(X)]whose closure contains either x or y

(v) C_1[resp: r-C_1; rα-C_1; α-C_1; Ultra C_1] space if for each $x \neq y \in X$ $\exists$ U, $V \in \tau$ (X)[resp: rO(X); rαO(X); αO(X); CO(X)]whose closure contains x and y.

(vi) C_2[resp: r-C_2; rα-C_2; α-C_2; Ultra C_2] space if for each $x \neq y \in X$ $\exists$ disjoint U, $V \in \tau$ (X)[resp: rO(X); rαO(X); αO(X); CO(X)]whose closure contains x and y.

(vii) D_0[resp: r-D_0; rα-D_0; α-D_0; Ultra D_0] space if for each $x \neq y \in X$ $\exists$ $U \in D(X)$[resp: rD(X); rαD(X); αD(X); COD(X)] containing either x or y.

(viii) D_1[resp: r-D_1; rα-D_1; α-D_1; Ultra D_1] space if for each $x \neq y \in X$ $\exists$ U, $V \in D(X)$[resp: rD(X); rαD(X); αD(X); COD(X)]such that $x \in U-V$ and $y \in V-U$.

(ix) D_2[resp: r-D_2; rα-D_2; α-D_2; Ultra D_2] space if for each $x \neq y \in X$ $\exists$ U, $V \in D(X)$[resp: rD(X); rαD(X); αD(X); CD(X)] such that $x \in U$; $y \in V$ and $U \cap V = \phi$.

(x) R_0[resp: r-R_0; rα-R_0; α-R_0] space if for each x in X $\exists$ $U \in \tau(X)$[resp: RO(X); rαO(X); αO(X)]$cl\{x\} \subseteq U$[resp: $rcl\{x\} \subseteq U$; $\alpha cl\{x\} \subseteq U$] whenever $x \in U \in \tau(X)$[resp: $x \in U \in RO(X)$; $x \in U \in \alpha O(X)$]

(xi) R_1[resp: r-R_1; rα-R_1; α-R_1] space if for $x,y \in X$ such that $cl\{x\} \neq cl\{y\}$[resp: such that $rcl\{x\} \neq rcl\{y\}$; such that $r\alpha cl\{x\} \neq r\alpha cl\{y\}$] $\exists$ disjoint U; $V \in \tau(X)$ such that $cl\{x\} \subseteq U$[resp: RO(X) such that $rcl\{x\} \subseteq U$;RαO(X) such that $r\alpha cl\{x\} \subseteq U$] and $cl\{y\} \subseteq V$ [resp: RO(X) such that $rcl\{y\} \subseteq V$;RαO(X) such that $r\alpha cl\{y\} \subseteq V$]

Lemma 2.1:

(i) Let A and B be subsets of a space X, if $A \in g\alpha O(X)$ and $B \in RO(X)$, then $A \cap B \in g\alpha O(B)$.

(ii) Let $A \subset B \subset X$, if $A \in g\alpha O(B)$ and $B \in RO(X)$, then $A \in g\alpha O(X)$.

3.Slightly gα-continuous functions:

Definition 3.1: A function f**:**$X \rightarrow Y$ is said to be

(i) slightly gα-continuous function at x in X if for each clopen subset V in Y containing f(x), $\exists$ $U \in g\alpha O(X)$ containing x such that $f(U) \subseteq V$.

(ii) slightly gα-continuous function if it is slightly gα-continuous at each x in X.

Note 2: Here after we call slightly gα-continuous function as sl.gα.c function shortly.

Example 3.1: X = Y = {a, b, c}; τ = {ϕ, {a}, X} and σ = {ϕ, {a}, {b, c}, Y}. Let f**:**$X \rightarrow Y$ defined as f(a) = b; f(b) = c and f(c) = a, then f is sl.gα.c.

Example 3.2: X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {a}, {b, c}, Y}. Let f**:**$X \rightarrow Y$ defined as follows:

(i) f(a) = b; f(b) = c and f(c) = a, then f is not sl.gα.c.

(ii) f(a) = b; f(b) = a and f(c) = c, then f is not sl.gα.c.

Theorem 3.1: The following are equivalent:
(i) f**:**X→ Y is sl.gα.c.
(ii) f^{-1}(V) is gα-open for every clopen set V in Y.
(iii) f^{-1}(V) is gα-closed for every clopen set V in Y.
(iv) f(gαcl(A)) ⊆ gαcl(f(A)).

Corollary 3.1: The following are equivalent.
(i) f**:**X→ Y is sl.gα.c.
(ii) For each x in X and each clopen subset V∈ (Y, f(x)) ∃ U∈ gαO(X, x) such that f(U) ⊆ V.

Theorem 3.2: Let ∑ = {U_i**:**i∈ I} be any cover of X by regular open sets in X. A function f is sl.gα.c. iff $f_{/Ui}$**:** is sl.gα.c., for each i∈ I.

Proof: Let i∈ I be an arbitrarily fixed index and U_i∈ RO(X). Let x∈ U_i and V∈ CO(Y, f_{Ui}(x)) Since f is sl.gα.c, ∃ U∈gαO(X, x) such that f(U) ⊂ V. Since U_i∈ RO(X), by Lemma 2.1 x∈ U∩ U_i∈gαO(U_i) and ($f_{/Ui}$)U∩ U_i = f(U∩ U_i)⊂ f(U) ⊂ V. Hence $f_{/Ui}$ is sl.gα.c.
Conversely Let x in X and V∈ CO(Y, f(x)), ∃ i∈ I such that x∈ U_i. Since $f_{/Ui}$ is sl.gα.c, ∃ U∈ gαO(U_i, x) such that $f_{/Ui}$(U) ⊂ V. By Lemma 2.1, U∈gαO(X) and f(U) ⊂ V. Hence f is sl.gα.c.

Theorem 3.3:
(i) If f**:**X→ Y is gα-irresolute and g**:**Y→ Z is sl.gα.c.[slightly-continuous; α-continuous], then g • f is sl.gα.c.
(ii) If f**:**X→ Y is gα-continuous and g**:**Y→ Z is slightly-continuous, then g• f is sl.gα.c.
(iii) If f**:**X→ Y is α-continuous and g**:**Y→ Z is sl.gα.c. [slightly-continuous], then g• f is sl.gα.c.

Theorem 3.4: If f**:**X→ Y is gα-irresolute, gα-open and gαO(X) = τ and g**:**Y→ Z be any function, then g• f**:**X→ Z is sl.gα.c iff g**:**Y→ Z is sl.gα.c.

Proof: If part**:** Theorem 3.3(i)
Only if part**:** Let A be clopen subset of Z. Then $(g• f)^{-1}$(A) is a gα-open subset of X and hence open in X[by assumption]. Since f is gα-open $f(g• f)^{-1}$(A) is gα-open Y ⇒ g^{-1}(A) is gα-open in Y. Thus g**:**Y→ Z is sl.gα.c.

Corollary 3.2: If f**:** X→ Y is gα-irresolute, gα-open and bijective, g**:**Y→ Z is a function. Then g**:**Y→ Z is sl.gα.c. iff g• f is sl.gα.c.

Theorem 3.5: If g**:** X → X× Y, defined by g(x) = (x, f(x)) for all x in X be the graph function of f**:** X → Y. Then g**:**X→ X×Y is sl.gα.c iff f is sl.gα.c.

Proof: Let V∈ CO(Y), then X× V is clopen in X× Y. Since g**:**X→ Y is sl.gα.c., f^{-1}(V) = f^{-1}(X× V) ∈gαO(X). Thus f is sl.gα.c.

Conversely, let x in X and F be a clopen subset of X× Y containing g(x). Then F∩ ({x}× Y) is clopen in {x}× Y containing g(x). Also {x}× Y is homeomorphic to Y. Hence {y∈ Y**:**(x, y) ∈ F} is clopen subset of Y. Since f is sl.gα.c. ∪{f^{-1}(y)**:**(x, y) ∈ F} is gα-open in X. Further x∈ ∪{f^{-1}(y)**:**(x, y) ∈ F}⊆ g^{-1}(F). Hence g^{-1}(F) is gα-open. Thus g**:**X→ Y is sl.gα.c.

Theorem 3.6:
(i) If f**:** $X \rightarrow \Pi Y_\lambda$ is sl.gα.c, then $P_\lambda \bullet f$ **:** $X \rightarrow Y_\lambda$ is sl.gα.c for each $\lambda \in \Gamma$, where P_λ is the projection of ΠY_λ onto Y_λ.
(ii) f**:** $\Pi X_\lambda \rightarrow \Pi Y_\lambda$ is sl.gα.c, iff f_λ**:** $X_\lambda \rightarrow Y_\lambda$ is sl.gα.c for each $\lambda \in \Gamma$.

Remark 1:
- (i) Composition of two sl.gα.c functions is not in general sl.gα.c.
- (ii) Algebraic sum and product of sl.gα.c functions is not in general sl.gα.c.
- (iii) The pointwise limit of a sequence of sl.gα.c functions is not in general sl.gα.c.

However we can prove the following**:**

Theorem 3.7: The uniform limit of a sequence of sl.gα.c functions is sl.gα.c.

Note 3: Pasting Lemma is not true for sl.gα.c functions. However we have the following weaker versions.

Theorem 3.8: Let X and Y be topological spaces such that $X = A \cup B$ and let $f_{/A}$**:** $A \rightarrow Y$ and $g_{/B}$**:** $B \rightarrow Y$ are sl.r.c maps such that $f(x) = g(x)$ for all $x \in A \cap B$. Suppose A and B are r-open sets in X and RO(X) is closed under finite unions, then the combination α**:** $X \rightarrow Y$ is sl.gα.c continuous.

Theorem 3.9: Pasting Lemma Let X and Y be spaces such that $X = A \cup B$ and let $f_{/A}$**:** $A \rightarrow Y$ and $g_{/B}$**:** $B \rightarrow Y$ are sl.gα.c maps such that $f(x) = g(x)$ for all $x \in A \cap B$. Suppose A, B are r-open sets in X and gαO(X) is closed under finite unions, then the combination α**:** $X \rightarrow Y$ is sl.gα.c.

Proof: Let $F \in CO(Y)$, then $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$, where $f^{-1}(F) \in g\alpha O(A)$ and $g^{-1}(F) \in g\alpha O(B) \Rightarrow f^{-1}(F)$; $g^{-1}(F) \in g\alpha O(X) \Rightarrow f^{-1}(F) \cup g^{-1}(F) \in g\alpha O(X)$[by assumption]. Therefore $\alpha^{-1}(F) \in g\alpha O(X)$. Hence α**:** $X \rightarrow Y$ is sl.gα.c.

4. Somewhat gα-continuous function:

Definition 4.1: A function f is said to be somewhat gα-continuous if for $U \in \sigma$ and $f^{-1}(U) \neq \varphi$, there exists a non-empty gα-open set V in X such that $V \subset f^{-1}(U)$.

It is clear that every continuous function is somewhat continuous and every somewhat continuous is somewhat gα-continuous. But the converses are not true by Example 1 of [8] and the following example.

Example 4.1: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{b\}, \{a, b\}, \{b, c\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{b, c\}, X\}$. The function f:$(X, \tau) \rightarrow (X, \sigma)$ defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ is somewhat gα-continuous.

Note 4: Every somewhat gα-continuous function is slightly gα-continuous.

Example 4.2: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{b, c\}, X\}$, $\sigma = \{\varphi, \{b\}, \{a, c\}, X\}$ and $\eta = \{\varphi, \{a\}, X\}$. Then the identity functions f:$(X, \tau) \rightarrow (X, \sigma)$ and g:$(X, \sigma) \rightarrow (X; \eta)$ and $g \bullet f$ are somewhat gα-continuous.
However, we have the following

Theorem 4.1: If f is somewhat gα-continuous and g is continuous, then $g\bullet f$ is somewhat gα-continuous.

Corollary 4.1:
(i) If f is somewhat gα-continuous and g is r-continuous, then $g\bullet f$ is somewhat gα-continuous.
(ii) If f is somewhat gα-continuous and g is r-irresolute, then $g\bullet f$ is somewhat gα-continuous.
(iii)If f is somewhat α-continuous and g is r-continuous, then $g\bullet f$ is somewhat gα-continuous.

Theorem 4.2: For a surjective function f, the following statements are equivalent:
(i) f is somewhat gα-continuous.
(ii) If C is a closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper gα-closed subset D of X such that $f^{-1}(C) \subset D$.
(iii)If M is a gα-dense subset of X, then $f(M)$ is a dense subset of Y.

Proof: (i) ⇒(ii): Let C be a closed subset of Y such that $f^{-1}(C) \neq X$. Then Y-C is an open set in Y such that $f^{-1}(Y\text{-}C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists a gα-open set $V\in g\alpha O(X)$ such that $V \neq \varphi$ and $V\subset f^{-1}(Y\text{-}C) = X - f^{-1}(C)$. This means that $X\text{-}V\supset f^{-1}(C)$ and X - V = D is a proper gα-closed set in X.
(ii) ⇒(i): Let $U\in\sigma$ and $f^{-1}(U) \neq \varphi$ Then Y-U is closed and $f^{-1}(Y\text{-}U) = X\text{-}f^{-1}(U) \neq X$. By (ii), there exists a proper gα-closed set D such that $D\supset f^{-1}(Y\text{-}U)$. This implies that $X\text{-}D \subset f^{-1}(U)$ and X-D is gα-open and $X\text{-}D \neq \varphi$.
(ii) ⇒(iii): Let M be a gα-dense set in X. Suppose that $f(M)$ is not dense in Y. Then there exists a proper closed set C in Y such that $f(M) \subset C\subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper gα-closed set D such that $M\subset f^{-1}(C) \subset D\subset X$. This is a contradiction to the fact that M is gα-dense in X.
(iii) ⇒(ii): Suppose (ii) is not true. there exists a closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper gα-closed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is gα-dense in X. But by (iii), $f(f^{-1}(C)) = C$ must be dense in Y, which is a contradiction to the choice of C.

Theorem 4.3: Let f be a function and $X = A\cup B$, where $A,B\in\tau(X)$. If the restriction functions $f_{/A}: (A; \tau_{/A}) \rightarrow (Y, \sigma)$ and $f_{/B}:(B; \tau_{/B}) \rightarrow (Y, \sigma)$ are somewhat gα-continuous, then f is somewhat gα-continuous.

Proof: Let $U\in\sigma$ such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. Suppose $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is somewhat gα-continuous, there exists a gα-open set V in A such that $V\neq \varphi$ and $V\subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since V is g$\alpha$-open in A and A is r-open in X, V is gα-open in X. Thus f is somewhat gα-continuous.

The proof of other cases are similar.

Definition 4.2: If X is a set and τ and σ are topologies on X, then τ is said to be equivalent[resp: gα-equivalent] to σ provided if $U\in\tau$ and $U \neq \varphi$, then there is an open[resp: gα-open] set V in X such that $V \neq \varphi$ and $V \subset U$ and if $U\in\sigma$ and $U \neq \varphi$, then there is an open[resp: gα-open] set V in (X, τ) such that $V\neq \varphi$ and $U \supset V$.

Definition 4.3: $A\subset X$ is said to be gα-dense in X if there is no proper gα-closed set C in X such that $M\subset C\subset X$.

Now, consider the identity function f and assume that τ and σ are gα-equivalent. Then f and f^{-1} are somewhat gα-continuous. Conversely, if the identity function f is somewhat gα-continuous in both directions, then τ and σ are gα-equivalent.

Theorem 4.4: Let f:(X, τ)→ (Y, σ) be a somewhat gα-continuous surjection and τ* be a topology for X, which is gα-equivalent to τ. Then f:(X, τ*)→ (Y, σ) is somewhat gα-continuous.

Proof: Let V∈σ such that $f^{-1}(V) \neq \varphi$. Since f is somewhat gα-continuous, there exists a nonempty gα-open set U in (X, τ) such that U⊂ $f^{-1}(V)$. But by hypothesis τ* is gα-equivalent to τ. Therefore, there exists a gα-open set U*∈ (X; τ*) such that U*⊂ U. But U⊂ $f^{-1}(V)$. Then U*⊂ $f^{-1}(V)$; hence f:(X, τ*)→ (Y, σ) is somewhat gα-continuous.

Theorem 4.5: Let f:(X, τ)→ (Y, σ) be a somewhat gα-continuous surjection and σ* be a topology for Y, which is equivalent to σ. Then f:(X, τ)→ (Y, σ*) is somewhat gα-continuous.

Proof: Let V*∈σ* such that $f^{-1}(V^*) \neq \varphi$. Since σ* is equivalent to σ, there exists a nonempty open set V in (Y, σ) such that V⊂ V*. Now $\varphi = f^{-1}(V) \subset f^{-1}(V^*)$. Since f is somewhat gα-continuous, there exists a nonempty gα-open set U in (X, τ) such that U⊂ $f^{-1}(V)$. Then U⊂ $f^{-1}(V^*)$; hence f:(X, τ)→ (Y, σ*) is somewhat gα-continuous.

5. Somewhat gα-open function:

Definition 5.1: A function f is said to be somewhat gα-open provided that if U∈τ and U≠ φ , then there exists a non-empty gα-open set V in Y such that V⊂ f(U).

Example 5.1: Let X = {a, b, c}, τ = {φ, {a}, X} and σ = {φ, {a}, {b, c}, X}. The function f: (X, τ)→ (X, σ) defined by f(a) = a, f(b) = c and f(c) = b is somewhat gα-open, somewhat gα-open and somewhat open.

Example 5.2: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {a}, {b}, {a, b}, X}. The function f: (X, τ)→ (X, σ) defined by f(a) = c, f(b) = a and f(c) = b is not somewhat gα-open.

Theorem 5.1: Let f be an r-open function and g somewhat gα-open. Then g• f is somewhat gα-open.

Theorem 5.2: For a bijective function f, the following are equivalent:
(i) f is somewhat gα-open.
(ii) If C is a closed subset of X, such that f(C) ≠ Y, then there is a gα-closed subset D of Y such that D≠ Y and D⊃ f(C).

Proof: (i) ⇒(ii): Let C be any closed subset of X such that f(C) ≠ Y. Then X-C is open in X and X-C ≠ φ. Since f is somewhat gα-open, there exists a gα-open set V≠ φ in Y such that V⊂ f(X-C). Put D = Y-V. Clearly D is gα-closed in Y and we claim D≠ Y. If D = Y, then V = φ, which is a contradiction. Since V⊂ f(X-C), D = Y-V ⊃ (Y- f(X-C)) = f(C).
(ii) ⇒(i): Let U be any nonempty open subset of X. Then C = X-U is a closed set in X and f(X-U) = f(C) = Y- f(U) implies f(C) ≠ Y. Therefore, by (ii), there is a gα-closed set D of Y such that D≠ Y and f(C) ⊂ D. Clearly V = Y-D is a gα-open set and V≠ φ . Also, V = Y-D ⊂ Y- f(C) = Y- f(X-U) = f(U).

Theorem 5.3: The following statements are equivalent:
(i) f is somewhat gα-open.
(ii)If A is a gα-dense subset of Y, then $f^{-1}(A)$ is a dense subset of X.

Proof: (i) ⇒(ii): Suppose A is a gα-dense set in Y. If $f^{-1}(A)$ is not dense in X, then there exists a closed set B in X such that $f^{-1}(A) \subset B \subset X$. Since f is somewhat gα-open and X-B is open, there exists a nonempty gα-open set C in Y such that $C \subset f(X\text{-}B)$. Therefore, $C \subset f(X\text{-}B) \subset f(f^{-1}(Y\text{-}A)) \subset Y\text{-}A$. That is, $A \subset Y\text{-}C \subset Y$. Now, Y-C is a g$\alpha$-closed set and $A \subset Y\text{-}C \subset Y$. This implies that A is not a gα-dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a dense set in X.

(ii) $\Rightarrow$(i): Suppose A is a nonempty open subset of X. We want to show that $g\alpha\,(f(A))^{o} \neq \varphi$. Suppose $g\alpha\,(f(A))^{o} = \varphi$. Then, $g\alpha cl\{(f(A))\} = Y$. Therefore, by (ii), $f^{-1}(Y - f(A))$ is dense in X. But $f^{-1}(Y- f(A)) \subset X\text{-}A$. Now, X-A is closed. Therefore, $f^{-1}(Y- f(A)) \subset X\text{-}A$ gives $X = cl\{(f^{-1}(Y- f(A)))\} \subset X\text{-}A$. This implies that $A = \varphi$, which is contrary to $A \neq \varphi$. Therefore, $g\alpha(f(A))^{o} \neq \varphi$. Hence f is somewhat gα-open.

Theorem 5.4: Let f be somewhat gα-open and A be any r-open subset of X. Then $f_{/A}:(A; \tau_{/A}) \rightarrow (Y, \sigma)$ is somewhat gα-open.

Proof: Let $U \in \tau_{/A}$ such that $U \neq \varphi$. Since U is r-open in A and A is r-open in X, U is r-open in X and since by hypothesis f is somewhat gα-open function, there exists a gα-open set V in Y, such that $V \subset f(U)$. Thus, for any open set U of A with $U \neq \varphi$, there exists a gα-open set V in Y such that $V \subset f(U)$ which implies $f_{/A}$ is a somewhat gα-open function.

Theorem 5.5: Let f be a function and $X = A \cup B$, where $A,B \in \tau(X)$. If the restriction functions $f_{/A}$ and $f_{/B}$ are somewhat gα-open, then f is somewhat gα-open.

Proof: Let U be any open subset of X such that $U \neq \varphi$. Since $X = A \cup B$, either $A \cap U \neq \varphi$ or $B \cap U \neq \varphi$ or both $A \cap U \neq \varphi$ and $B \cap U \neq \varphi$. Since U is open in X, U is open in both A and B.

Case (i): Suppose that $A \cap U \neq \varphi$, where $U \cap A$ is open in A. Since $f_{/A}$ is somewhat gα-open function, there exists a gα-open set V of Y such that $V \subset f(U \cap A) \subset f(U)$, which implies that f is a somewhat gα-open function.

Case (ii): Suppose that $B \cap U \neq \varphi$, where $U \cap B$ is r-open in B. Since $f_{/B}$ is somewhat gα-open function, there exists a gα-open set V in Y such that $V \subset f(U \cap B) \subset f(U)$, which implies that f is also a somewhat gα-open function.

Case (iii): Suppose that both $A \cap U \neq \varphi$ and $B \cap U \neq \varphi$. Then by case (i) and (ii) f is a somewhat gα-open function.

Remark 3: Two topologies τ and σ for X are said to be gα-equivalent if and only if the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is somewhat gα-open in both directions.

Theorem 5.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a somewhat almost open function. Let τ^* and σ^* be topologies for X and Y, respectively such that τ^* is equivalent to τ and σ^* is gα-equivalent to σ. Then $f: (X; \tau^*) \rightarrow (Y; \sigma^*)$ is somewhat gα-open.

6.Covering and Separation properties of sl.gα.c. functions:

Theorem 6.1: If f**:**$X \rightarrow Y$ is sl.gα.c., surjection and X is gα-compact, then Y is compact.

Proof: Let $\{G_i : i \in I\}$ be any open cover for Y. Then each G_i is open in Y and hence each G_i is clopen in Y. Since f**:**$X \rightarrow Y$ is sl.gα.c., $f^{-1}(G_i)$ is gα-open in X. Thus $\{f^{-1}(G_i)\}$ forms a gα-open cover for X and hence have a finite subcover, since X is gα-compact. Since f is surjection, $Y = f(X) = \cup^{n}_{i=1} G_i$. Therefore Y is compact.

Corollary 6.1: If f**:**$X \rightarrow Y$ is sl.α.c.[resp**:** sl.r.c] surjection and X is gα-compact, then Y is compact.

Theorem 6.2: If f**:**X→ Y is sl.α.c., surjection and X is gα-compact[gα-lindeloff] then Y is mildly compact[mildly lindeloff].

Proof: Let $\{U_i : i\in I\}$ be clopen cover for Y. For each x in X, $\exists\ \alpha_x\in I$ such that $f(x) \in U_{\alpha x}$ and $\exists\ V_x\in$ gαO(X, x) such that $f(V_x)\subset U_{\alpha x}$. Since the family $\{V_i : i\in I\}$ is a cover of X by gα-open sets of X, ∃ a finite subset I_0 of I such that $X\subset \{V_x : x\in I_0\}$. Therefore $Y\subset \cup \{f(V_x) : x\in I_0\}\subset \cup \{U_{\alpha x} : x\in I_0\}$. Hence Y is mildly compact.

Corollary 6.2:
(i) If f**:**X→ Y is sl.gα.c[resp**:** sl.α.c.; sl.r.c] surjection and X is gα-compact[gα-lindeloff] then Y is mildly compact[mildly lindeloff].
(ii) If f**:**X→ Y is sl.gα.c.[resp**:** sl.c; sl.α.c.; sl.r.c] surjection and X is locally gα-compact{resp**:** gα-Lindeloff; locally gα-lindeloff}, then Y is locally compact{resp**:** Lindeloff; locally lindeloff}.
(iii) If f**:**X→ Y is sl.gα.c.[sl.r.c.], surjection and X is locally gα-compact{resp**:** gα-lindeloff; locally gα-lindeloff} then Y is locally mildly compact{resp**:** locally mildly lindeloff}.

Theorem 6.3: If f**:**X→ Y is sl.gα.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Proof: Let $\{V_i : V_i\in CO(Y); i\in I\}$ be a cover of Y, then $\{f^{-1}(V_i) : i\in I\}$ is gα-open cover of X[by Thm 3.1] and so there is finite subset I_0 of I, such that $\{f^{-1}(V_i) : i\in I_0\}$ covers X. Therefore $\{V_i : i\in I_0\}$ covers Y since f is surjection. Hence Y is mildly compact.

Corollary 6.3: If f**:**X→ Y is sl.α.c[resp**:** sl.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Theorem 6.4: If f**:**X→ Y is sl.gα.c.,[resp**:** sl.α.c; sl.r.c.] surjection and X is *gr*-connected, then Y is connected.

Proof: If Y is disconnected, then Y = A∪ B where A and B are disjoint clopen sets in Y. Since f is sl.gα.c. surjection, $X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ $f^{-1}(B)$ are disjoint gα-open sets in X, which is a contradiction for X is gα-connected. Hence Y is connected.

Corollary 6.4: The inverse image of a disconnected space under a sl.gα.c.,[resp**:** sl.α.c; sl.r.c.] surjection is gα-disconnected.

Theorem 6.5: If f**:** X→ Y is sl.gα.c.[resp**:** sl.α.c.], injection and Y is UT_i, then X is $g\alpha_i$ i = 0, 1, 2.

Proof: Let $x_1 \neq x_2\in X$. Then $f(x_1) \neq f(x_2)\in Y$ since f is injective. For Y is UT_2 $\exists\ V_j\in CO(Y)$ such that $f(x_j)\in V_j$ and $\cap V_j = \phi$ for j = 1,2. By Theorem 3.1, $x_j\in f^{-1}(V_j)\in$ gαO(X) for j = 1,2 and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is $g\alpha_2$.

Theorem 6.6: If f**:**X→ Y is sl.gα.c.[resp**:** sl.α.c.], injection; closed and Y is UT_i, then X is $g\alpha g_i$ i = 3, 4.

Proof:(i) Let x in X and F be disjoint closed subset of X not containing x, then $f(x)$ and $f(F)$ be disjoint closed subset of Y not containing $f(x)$, since f is closed and injection. Since Y is ultraregular, $f(x)$ and $f(F)$ are separated by disjoint clopen sets U and V respectively. Hence $x\in f^{-1}(U)$; $F\subseteq f^{-1}(V)$, $f^{-1}(U)$; $f^{-1}(V)\in$ gαO(X) and $f^{-1}(U)\cap f^{-1}(V) = \phi$. Thus X is $g\alpha g_3$.

(ii) Let F_j and $f(F_j)$ are disjoint closed subsets of X and Y respectively for j = 1,2, since f is closed and injection. For Y is ultranormal, $f(F_j)$ are separated by disjoint clopen sets V_j respectively for j = 1,2. Hence $F_j \subseteq f^{-1}(V_j)$ and $f^{-1}(V_j) \in g\alpha O(X)$ and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is $g\alpha g_4$.

Theorem 6. 7: If f**:**X$\rightarrow$ Y is sl.gα.c.[resp**:** sl.α.c.], injection and
(i) Y is UC_i[resp**:** UD_i] then X is $g\alpha C_i$[resp**:** $g\alpha D_i$] i = 0, 1, 2.
(ii)Y is UR_i, then X is $g\alpha$-R_i i = 0, 1.

Theorem 6.8: If f**:**X$\rightarrow$Y is sl.gα.c.[resp**:** sl.α.c; sl.r.c] and Y is UT_2, then the graph G(f) of f is gα-closed in the product space X$\times$ Y.

Proof: Let $(x_1, x_2) \notin G(f)$ implies $y \neq f(x)$ implies $\exists$ disjoint V; $W \in CO(Y)$ such that $f(x) \in V$ and $y \in W$. Since f is sl.gα.c., $\exists$ $U \in g\alpha O(X)$ such that $x \in U$ and $f(U) \subset W$ and $(x, y) \in U \times V \subset X \times Y - G(f)$. Hence G($f$) is g$\alpha$-closed in X$\times$Y.

Theorem 6.9: If f**:**X$\rightarrow$Y is sl.gα.c.[resp**:** sl.α.c; sl.r.c] and Y is UT_2, then $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$ is gα-closed in the product space X$\times$ X.

Proof: If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2)$ implies $\exists$ disjoint $V_j \in CO(Y)$ such that $f(x_j) \in V_j$, and since f is sl.gα.c., $f^{-1}(V_j) \in g\alpha O(X, x_j)$ for j = 1,2. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in g\alpha O(X \times X)$ and $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X - A$. Hence A is g$\alpha$-closed.

Theorem 6.10: If f**:** X$\rightarrow$ Y is sl.r.c.[resp**:** sl.c.]; g**:** X$\rightarrow$ Y is sl.gα.c[resp**:** sl.gα.c]; and Y is UT_2, then E = {x in X **:** $f(x) = g(x)$} is gα-closed in X.
Following definitions 3.1; 4.1 and Note 4, we have the following consequences of theorems 6.1 to 6.10:

Theorem 6.11: If f**:**X$\rightarrow$ Y is swt.gα.c., surjection and X is gα-compact, then Y is compact.

Corollary 6.5: If f**:**X$\rightarrow$ Y is swt.α.c.[resp**:** swt.r.c] surjection and X is gα-compact, then Y is compact.

Theorem 6.12: If f**:**X$\rightarrow$ Y is swt.α.c., surjection and X is gα-compact[gα-lindeloff] then Y is mildly compact[mildly lindeloff].

Corollary 6.6:
(i) If f**:**X$\rightarrow$ Y is swt.gα.c[resp**:** swt.α.c.; swt.r.c] surjection and X is gα-compact[gα-lindeloff] then Y is mildly compact[mildly lindeloff].
(ii) If f**:**X$\rightarrow$ Y is swt.gα.c.[resp**:** swt.c; swt.α.c.; swt.r.c] surjection and X is locally gα-compact{resp**:** gα-Lindeloff; locally gα-lindeloff}, then Y is locally compact{resp**:** Lindeloff; locally lindeloff}.
(iii) If f**:**X$\rightarrow$ Y is swt.gα.c.[swt.r.c.], surjection and X is locally gα-compact{resp**:** gα-lindeloff; locally gα-lindeloff} then Y is locally mildly compact{resp**:** locally mildly lindeloff}.

Theorem 6.13: If f**:**X$\rightarrow$ Y is swt.gα.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Corollary 6.7: If f**:**X$\rightarrow$ Y is swt.α.c[resp**:** swt.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Theorem 6.14: If f**:**X$\rightarrow$ Y is swt.gα.c.,[resp**:** swt.α.c; swt.r.c.] surjection and X is *gr*-connected, then Y is connected.

Corollary 6.8: The inverse image of a disconnected space under a swt.gα.c.,[resp**:** swt.α.c; swt.r.c.] surjection is gα-disconnected.

Theorem 6.15: If *f***:** X→ Y is swt.gα.c.[resp**:** swt.α.c.], injection and
(i) Y is UT_i, then X is $g\alpha_i$ i = 0, 1, 2.
(ii) Y is T_i, then X is $g\alpha_i$ i = 0, 1, 2.

Theorem 6.16: If *f***:**X→ Y is swt.gα.c.[resp**:** swt.α.c.], injection; closed and
(i) Y is UT_i, then X is $g\alpha g_i$ i = 3, 4.
(ii) Y is T_i, then X is $g\alpha g_i$ i = 3, 4.

Theorem 6.17: If *f***:**X→ Y is swt.gα.c.[resp**:** swt.α.c.], injection and
(i) Y is UC_i[resp**:** UD_i] then X is $g\alpha C_i$[resp**:** $g\alpha D_i$] i = 0, 1, 2.
(ii)Y is R_i, then X is gα-R_i i = 0, 1.
(iii) Y is C_i[resp**:** D_i] then X is $g\alpha C_i$[resp**:** $g\alpha D_i$] i = 0, 1, 2.

Theorem 6.18: If *f***:**X→Y is swt.gα.c.[resp**:** swt.α.c; swt.r.c] and
(i) Y is UT_2, then the graph G(*f*) of *f* is gα-closed in the product space X× Y.
(ii) Y is UT_2, then A = $\{(x_1, x_2) | f(x_1) = f(x_2)\}$ is gα-closed in the product space X× X.
(iii) Y is T_2, then the graph G(*f*) of *f* is gα-closed in the product space X× Y.
(iv) Y is T_2, then A = $\{(x_1, x_2) | f(x_1) = f(x_2)\}$ is gα-closed in the product space X× X.

Theorem 6.19: If *f***:** X→ Y is swt.r.c.[resp**:** swt.c.]; *g***:** X→ Y is swt.gα.c[resp**:** swt.gα.c]; and
(i) Y is UT_2, then E = {x in X **:** *f*(x) = *g*(x)} is gα-closed in X.
(ii) Y is T_2, then E = {x in X **:** *f*(x) = *g*(x)} is gα-closed in X.

CONCLUSION: In this paper we defined slightly-gα-continuous functions, studied its properties and their interrelations with other types of slightly-continuous functions.

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