ROLE OF SINGLETONS IN TOPOLOGY

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ABSTRACT

The purpose of this paper is to investigate some properties of singletons. The most useful results that proved in this paper are "In a topological space every singleton set is open or pre-closed" and "A topological space is pre-T_1 if and only if every singleton open set is closed". These lead to the investigation of singleton semi-pre-closed, rps-closed, gsp-closed and pre-semi-closed sets.

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1. Introduction

In1986, Andrijevic [1] defined the concept of semi-pre-closed sets. Dontchev [5] introduced the notion of generalized semi-pre-closed sets in 1995. Gnanambal [12] introduced and studied generalized pre-regular closed sets in 1997. Shyla Isac Mary and Thangavelu [19] defined the concept of regular pre-semiclosed sets in 2010. Recently, the authors [9] proved that "In a topological space every singleton is rg-open". This motivates us to concentrate on singletons in general topological spaces. The purpose of this paper is to investigate some characterization of singletons and discuss its role in particular spaces. The most useful results that proved in this paper are "In a topological space every singleton set is open or pre-closed" and "A topological space is pre-T_1 if and only if every singleton open set is closed". These lead to the investigation of singleton semi-pre-closed, rps-closed, gsp-closed and pre-semi-closed sets.

2. Preliminaries

Given any subset A in a topological space (X,τ), the closure, interior and complement of A are denoted by *cl*(A), *int*(A) and X\A respectively. Let us recall the following definitions, which we shall require later.

A subset A of a topological space (X,τ) is regular open [20] if A=*int*(*cl*(A)), regular closed if A=*cl*(*int*(A)), pre-open [17] if A$\subseteq$*int*(*cl*(A)), pre-closed if *cl*(*int*(A))$\subseteq$A, semi-open [14] if A$\subseteq$*cl*(*int*(A)), semi-closed if *int*(*cl*(A))$\subseteq$A, semi-pre-open [1] if A$\subseteq$*cl*(*int*(*cl*(A))) and semi-pre-closed if *int*(*cl*(*int*(A)))$\subseteq$A. The pre-interior of a subset A of X is the union of all pre-open sets contained in A and is denoted by *pint*(A). The pre-closure of a subset A of X is the intersection of all pre-closed sets containing A and is denoted by *pcl*(A). The semi-pre-closure of a subset A of X is analogously defined and is denoted by *spcl*(A).

Again a subset B of a topological space (X,τ) is called generalized closed (briefly g-closed) [15] if *cl*(B)$\subseteq$U whenever B$\subseteq$U and U is open in X and regular generalized closed (briefly rg-closed) [18] if *cl*(B)$\subseteq$U whenever B$\subseteq$U and U is regular open in X. The intersection of all rg-closed sets containing B is called the rg-closure of B and denoted by $cl_r^*(B)$. The complement of a rg-closed set is rg-open.

Definition 2.1: A subset B of a topological space (X,τ) is called generalized pre-closed (briefly gp-closed)[16] (resp. pre-generalized closed(briefly pg-closed)[16], generalized pre-regular-closed(briefly gpr-closed)[12] and pre-generalized pre-regular-closed (briefly pgpr-closed)[2]) if *pcl*(B)$\subseteq$U whenever B$\subseteq$U and U is open (resp. pre-open, regular open and rg-open) in X.

Definition 2.2: A subset B of a topological space (X,τ) is called pre-semi-closed [21] (resp. generalized semi-pre-closed (briefly gsp-closed)[5] and regular pre-semiclosed (briefly rps-closed) [19]) if *spcl*(A)$\subseteq$U whenever A$\subseteq$U and U is g-open (resp. open and rg-open).

The intersection of all gpr-closed sets containing A is called the gpr-closure of A and denoted by *gpr-cl*(A). The complement of a gpr-closed set is gpr-open and that of a pgpr-closed set is pgpr-open.

Definition 2.3: A space (X,τ) is locally indiscrete if every open set is closed. [13]

Definition 2.4: A topological space (X,τ) is pre-T_1[3] (resp. pgpr-T_1[10]) if for any two distinct points x, $y\in X$, there exist pre-open (resp. pgpr-open) sets G and H such that $x\in G$ but $y\notin G$ and $y\in H$ but $x\notin H$.

Definition 2.5: A space (X,τ) is said to be pgpr-regular [11] if for every pgpr-closed set F and a point $x\notin F$, there exist disjoint open sets U and V such that $x\in U$ and $F\subseteq V$.

Definition 2.6: A space (X,τ) is said to be strongly rg-regular [8] if for every rg-closed set F and a point $x\notin F$, there exist disjoint open sets U and V such that $x\in U$ and $F\subseteq V$.

Lemma 2.7: (i) In a space (X,τ), $\{x\}$ is pre-open or pre-closed for each $x\in X$. [16]

(ii) A is pre-closed if and only if $pcl(A)=A$ for every subset A of X. [7]

(iii) A space (X,τ) is pre-T_1 if and only if $\{x\}$ is pre-closed for every $x\in X$. [3]

(iv) Every pre-T_1 space is pgpr-T_1. [10]

Lemma 2.8: For a topological space (X,τ), the following are equivalent:

(i) X is locally indiscrete.

(ii) Every subset of X is pre-open.

(iii) Every singleton in X is pre-open.

(iv) Every closed subset of X is pre-open. [6]

Theorem 2.9: A subset A of a space (X,τ) is

(i) g-open if and only if $F\subseteq int(A)$ for every closed set F with $F\subseteq A$. [15]

(ii) rg-open if and only if $F\subseteq int(A)$ for every regular closed set F with $F\subseteq A$. [18]

Theorem 2.10: A subset A of X is pgpr-open if and only if $F\subseteq pint(A)$ whenever $F\subseteq A$, F is rg-closed. [2]

Theorem 2.11: In a topological space (X,τ), the following hold:

(i) $\{x\}$ is rg-open for every $x\in X$.

(ii) $cl_r^*(A) = gpr\text{-}cl(A)=A$, for every subset A of X. [9]

§3. Characterizations of singletons

In this section, some of the special characterizations of singletons are studied. In general, a pgpr-closed set need not be pre-closed. Fortunately, every singleton pgpr-closed set is pre-closed as shown in the following theorem.

Theorem 3.1: In a topological space (X,τ) the following holds for $x\in X$:

(i) $\{x\}$ is pgpr-closed if and only if it is pre-closed.

(ii) {x} is pgpr-closed or pgpr-open.

(iii) {x} is open or pre-closed.

Proof: Let $x\in X$. Then by Theorem 2.11(i), {x} is rg-open. Suppose {x} is pgpr-closed. By Definition 2.1, $pcl(\{x\})\subseteq\{x\}$. Always $\{x\}\subseteq pcl(\{x\})$. That is $pcl(\{x\})=\{x\}$. By using Lemma 2.7(ii), {x} is pre-closed. Converse follows from the fact that every pre-closed set is pgpr-closed. This completes the proof of (i). If {x} is pgpr-closed, then the part (ii) is trivial. If not, we shall show that {x} is pgpr-open. By (i), {x} is not pre-closed and by using Lemma 2.7(i), {x} is pre-open. Since every pre-open set is pgpr-open, {x} is pgpr-open. This proves (ii). If {x} is open, then the part (iii) is trivial. If not, then $int(\{x\})=\emptyset$. Therefore $cl(int(\{x\}))=\emptyset\subseteq\{x\}$. It follows that {x} is pre-closed. This proves (iii).

Theorem 3.2: In a topological space (X,τ) the following holds for $x\in X$:

(i) {x} is pg-closed if and only if it is pre-closed.

(ii) {x} is pg-closed or pg-open.

(iii) {x} is gp-closed if and only if it is pre-closed.

(iv) {x} is gp-closed or gp-open.

Proof: Let {x} be pg-closed. By using Lemma 2.7(i), {x} is pre-open or pre-closed. If {x} is pre-closed, then the part (i) is trivial. If not {x} is pre-open. By Definition 2.1, $pcl(\{x\})\subseteq\{x\}$. Always $\{x\}\subseteq pcl(\{x\})$. That is $pcl(\{x\})=\{x\}$. Now by Lemma 2.7(ii), {x} is pre-closed. Converse follows from the fact that every pre-closed set is pg-closed. This proves (i). If {x} is pg-closed and then the part (ii) is trivial. If not, we shall show that {x} is pg-open. By (i), {x} is not pre-closed and by Lemma 2.7(i), {x} is pre-open. Since every pre-open set is pg-open, {x} is pg-open. This proves (ii). Let {x} be gp-closed. By using Lemma 2.7(i), {x} is pre-open or pre-closed. If {x} is pre-closed, then the part (i) is trivial. If not, then by using Theorem 3.1(iii), {x} is open. By Definition 2.1, $pcl(\{x\})\subseteq\{x\}$. Always $\{x\}\subseteq pcl(\{x\})$. That is $pcl(\{x\})=\{x\}$. By using Lemma 2.7(ii), {x} is pre-closed. Converse is obvious. This proves (iii). If {x} is gp-closed, then it is nothing to prove. If not, then by using (iii), {x} is not pre-closed and by Lemma 2.7(i), {x} is pre-open. Since every pre-open set is gp-open, {x} is gp-open. This proves (iv).

Remark 3.3: From Theorem 3.1 and Theorem 3.2, we conclude that for $x\in X$,

{x}is pre-closed$\Leftrightarrow$ {x} is pg-closed$\Leftrightarrow$ {x} is gp-closed $\Leftrightarrow$ {x} is pgpr-closed and hence

$X\backslash\{x\}$is pre-open$\Leftrightarrow X\backslash\{x\}$ is pg-open$\Leftrightarrow X\backslash\{x\}$ is gp-open $\Leftrightarrow X\backslash\{x\}$ is pgpr-open.

Theorem 3.4: Let (X,τ) be a topological space. Then the following hold:

(a) $\{x\}$ is semi-pre-open or semi-pre-closed.

(b) $\{x\}$ is rps-closed if and only if it is semi-pre-closed.

(c) $\{x\}$ is rps-closed or rps-open.

(d) $\{x\}$ is pre-semi-closed if and only if it is semi-pre-closed.

(e) $\{x\}$ is pre-semi-closed or pre-semi-open.

Proof: Let (X,τ) be a topological space and let $x\in X$. Then by Theorem 3.1(iii), $\{x\}$ is open or pre-closed. If $\{x\}$ is open, then $\{x\}$ is semi-pre-open. If not, then $\{x\}$ is pre-closed and hence it is semi-pre-closed. This proves (a). Suppose $\{x\}$ is rps-closed. Then by using Theorem 2.11(i), $\{x\}$ is rg-open. By Definition 2.2, $spcl(\{x\})\subseteq\{x\}$. Always $\{x\}\subseteq spcl(\{x\})$. Therefore $spcl(\{x\})=\{x\}$ and hence $\{x\}$ is semi-pre-closed. Converse follows from the fact that every semi-pre-closed set is rps-closed. This proves (b). If $\{x\}$ is not rps-closed, then by (b) it is not semi-pre-closed. Now by using (a), $\{x\}$ is semi-pre-open. Since every semi-pre-open set is rps-open, $\{x\}$ is rps-open. This proves (c). Suppose $\{x\}$ is pre-semi-closed. If $\{x\}$ is pre-closed, then it is semi-pre-closed. If not, then by Theorem 3.1(iii), $\{x\}$ is open and hence g-open. By Definition 2.2, $spcl(\{x\})\subseteq\{x\}$. But always $\{x\}\subseteq spcl(\{x\})$. Therefore $spcl(\{x\})=\{x\}$ and hence $\{x\}$ is semi-pre-closed. Converse follows from the fact that every semi-pre-closed set is pre-semi-closed. This proves (d). If $\{x\}$ is not pre-semi-closed, then by (d) it is not semi-pre-closed. Now by using (a), $\{x\}$ is semi-pre-open. Since every semi-pre-open set is pre-semi-open, $\{x\}$ is pre-semi-open. This proves (e).

Remark 3.5: Clearly, if a set is semi-closed or pre-closed, then it is semi-pre-closed. However, the converse is not true as shown in the following example.

Example 3.6: Let $X=\{a,b,c,d\}$ with $\tau=\{\varnothing,\{a\},\{b,c\},\{a,b,c\},X\}$. Then $\{a,b\}$is semi-pre-closed, but neither semi-closed nor pre-closed.

The next theorem shows that the converse is true for singletons.

Theorem 3.7: Let (X,τ) be a topological space and let $x\in X$. Then

(i) $\{x\}$ is semi-pre-closed if and only if it is semi-closed or pre-closed.

(ii) $\{x\}$ is semi-pre-closed if and only if it is regular open or pre-closed.

Proof: Let $\{x\}$ be semi-pre-closed. Then by Theorem 3.1(iii), $\{x\}$ is open or pre-closed. If $\{x\}$ is not pre-closed, then $\{x\}$ is open. Since $\{x\}$ is semi-pre-closed, $int(cl(int(\{x\})))\subseteq\{x\}$. Since $\{x\}$ is open, $int(cl(\{x\}))\subseteq\{x\}$. Therefore, $\{x\}$ is semi-closed. Converse is obvious. This proves (i). Suppose $\{x\}$ is semi-pre-closed. If $\{x\}$ is not pre-closed, then by (i), $\{x\}$ is semi-closed. That is $int(cl(\{x\}))\subseteq\{x\}$. Now by Theorem 3.1(iii), $\{x\}$ is open and hence $\{x\}=int(\{x\})\subseteq int(cl(\{x\}))\subseteq\{x\}$. That is $int(cl(\{x\}))=\{x\}$. Therefore, $\{x\}$ is regular open.

Theorem 3.8: Let (X,τ) be a topological space and let $x\in X$. Then

(i) $\{x\}$ is gsp-closed if and only if it is semi-pre-closed.

(ii) $\{x\}$ is gsp-closed or gsp-open.

(iii) $\{x\}$ is gpr-closed if and only if it is pre-closed or rg-closed.

Proof: Suppose $\{x\}$ is gsp-closed. Then by Theorem 3.1(iii), $\{x\}$ is open or pre-closed. If $\{x\}$ is open, then by using Definition 2.2, $spcl(\{x\})\subseteq\{x\}$ and hence $spcl(\{x\})=\{x\}$. That is $\{x\}$ is semi-pre-closed. If not, then $\{x\}$ is pre-closed and hence it is semi-pre-closed. Converse follows from the fact that every semi-pre-closed set is gsp-closed. This proves (i). If $\{x\}$ is not gsp-closed, then by (i), $\{x\}$ is not semi-pre-closed and by Theorem 3.4(a), $\{x\}$ is semi-pre-open. Since every semi-pre-open set is gsp-open, $\{x\}$ is gsp-open. This proves (ii). Suppose $\{x\}$ is gpr-closed. If $\{x\}$ is not pre-closed, then by Theorem 3.1(iii), $\{x\}$ is open. Let U be any regular open set containing x. Since $\{x\}$ is gpr-closed, by using Definition 2.1, $pcl(\{x\})\subseteq U$. That is $\{x\}\cup cl(int(\{x\})\subseteq U$. Since $\{x\}$ is open, $\{x\}\cup cl(\{x\})\subseteq U$ and hence $cl(\{x\})\subseteq U$. Therefore, $\{x\}$ is rg-closed. This proves (iii).

Remark 3.9: From Theorem 3.4 and Theorem 3.8, we conclude that for $x\in X$,
$\{x\}$ is semi-pre-closed$\Leftrightarrow\{x\}$ is rps-closed$\Leftrightarrow\{x\}$ is gsp-closed$\Leftrightarrow\{x\}$ is pre-semi-closed and $X\backslash\{x\}$ is semi-pre-open$\Leftrightarrow X\backslash\{x\}$ is rps-open$\Leftrightarrow X\backslash\{x\}$ is gsp-open$\Leftrightarrow X\backslash\{x\}$ is pre-semi-open.

§4. Role of Singletons

In this section, some roles of singletons in particular spaces are studied. The following theorem gives a necessary and sufficient condition for pre-T_1 spaces.

Theorem 4.1: A space (X,τ) is pre-T_1 if and only if every singleton open set is closed.

Proof: Suppose (X,τ) is pre-T_1. Let $\{x\}$ be open in X. Then by using Lemma 2.7(iii), $\{x\}$ is pre-closed. That is $cl(int(\{x\})\subseteq\{x\}$. Since $\{x\}$ is open, we have $cl(\{x\})=cl(int(\{x\})\subseteq\{x\}$and hence $cl(\{x\})=\{x\}$. This proves that $\{x\}$ is closed. Conversely, suppose every singleton open set is closed. If $\{x\}$ is open then by assumption,$\{x\}$ is closed and hence $\{x\}$ is pre-closed. If $\{x\}$ is not open, then by Theorem 3.1(iii), $\{x\}$ is pre-closed. Thus, every singleton is pre-closed. Now by using Lemma 2.7(iii), (X,τ) is pre-T_1.

Corollary 4.2: Let (X,τ) be a topological space. If every singleton open set in X is closed, then it is pgpr-T_1.

Proof: Follows from Theorem 4.1 and Theorem 2.7(iv).

Theorem 4.3: If (X,τ) is pgpr-regular, then $\{x\}$ is open or closed for every $x\in X$.

Proof: Let (X,τ) be a pgpr-regular space and let $x\in X$. If $\{x\}$ is not open, then by using Theorem 3.1(iii), $\{x\}$ pre-closed and hence $\{x\}$ is pgpr-closed. Since (X,τ) is pgpr-regular, for each $y\in X\setminus\{x\}$, there exists an open set V_y such that $y\in V_y$ and $x\notin V_y$. This implies that $z\in \bigcup_{y\in X\setminus\{x\}} V_y$ for all $z\in X\setminus\{x\}$ and $x\notin \bigcup_{y\in X\setminus\{x\}} V_y$. That is $\bigcup_{y\in X\setminus\{x\}} V_y = X\setminus\{x\}$. Since each V_y is open, we have $X\setminus\{x\}= \bigcup_{y\in X\setminus\{x\}} V_y$ is open and hence $\{x\}$ is closed.

The converse of Theorem 4.3 is not true as shown in the following example.

Example 4.4: Let $X=\{a,b,c,d\}$ with $\tau=\{\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\},X\}$. Then $\{a\},\{b\},\{c\}$ are open and $\{d\}$ is closed. But (X,τ) is not pgpr-regular, since the pgpr-closed set $\{b,c,d\}$ and the point 'a' are cannot be separated by disjoint open sets.

Remark 4.5: Clearly, every clopen subset of a space (X,τ) is regular closed. However, the converse is not true as the regular closed set $\{b,c\}$ is not clopen in (X,τ) where $X=\{a,b,c\}$ and $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},X\}$. Nevertheless, the converse is true for singletons. That is every singleton regular closed set is clopen. This is proved in the following theorem.

Theorem 4.6: Let (X,τ) be a topological space. Then $\{x\}$ is regular closed if and only if it is clopen for every $x\in X$.

Proof: If $\{x\}$ is clopen, then $cl(int(\{x\}))=cl(\{x\})=\{x\}$ and hence $\{x\}$ is regular closed. Conversely, suppose $\{x\}$ is regular closed. Then $cl(int(\{x\}))=\{x\}$. If $\{x\}$ is not open, then $\{x\}=cl(int(\{x\}))=cl(\emptyset)=\emptyset$, which is absurd. Therefore, $\{x\}$ is open. Now $\{x\}=cl(int(\{x\}))$ $=cl(\{x\})$, implies that $\{x\}$ is closed. Hence, $\{x\}$ is clopen.

Corollary 4.7: Let (X,τ) be a topological space. Then $X\setminus\{x\}$ is regular open if and only if it is clopen for every $x\in X$.

Proof: Let (X,τ) be a topological space and let $x\in X$.

$X\setminus\{x\}$ is regular open $\Leftrightarrow$ $\{x\}$ is regular closed.

$\Leftrightarrow$ $\{x\}$ is clopen by using Theorem 4.6.

$\Leftrightarrow$ $X\setminus\{x\}$ is clopen.

Theorem 4.8: A topological space (X,τ) is strongly rg-regular if and only if for any $x\in X$, $\{x\}$ and $X\setminus\{x\}$ are open in X.

Proof: Let (X,τ) be strongly rg-regular and $x\in X$. Then by Theorem 2.11(i), $\{x\}$ is rg-open and hence $X\setminus\{x\}$ is rg-closed and $x\notin X\setminus\{x\}$. By the definition of strongly rg-regular, we can find disjoint open sets containing the point x and $X\setminus\{x\}$. But the only sets containing $\{x\}$ and $X\setminus\{x\}$ are itself. Hence $\{x\}$ and $X\setminus\{x\}$ are open. Conversely assume that $\{x\}$ and $X\setminus\{x\}$ are open for every $x\in X$. Let F be any rg-closed set and $y\notin F$. Then by assumption, $\{y\}$ and $X\setminus\{y\}$ are open. Also $F\subseteq X\setminus\{y\}$. Therefore, $\{y\}$ and $X\setminus\{y\}$ are disjoint open sets containing y and F respectively. Thus (X,τ) is strongly rg-regular.

Clearly, every pre-open set is pg-open, gp-open and pgpr-open. However, the converse is not true. In particular $\{x\}$ is pgpr-open does not imply $\{x\}$ is pre-open as shown in the following example.

Example 4.9: Let $X=\{a,b,c,d\}$ endowed with $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},X\}$. Then $\{c\}$ is pgpr-open, but not pre-open.

Proposition 4.10: If (X,τ) is locally indiscrete, then every subset of X is rg-closed.

Proof: Let (X,τ) be a locally indiscrete space and let A be any subset of X. Since $cl(A)$ is closed and (X,τ) is locally indiscrete, $cl(A)$ is open. Therefore $int(cl(A))=cl(A)$. Hence A is closed if and only if $A=cl(A)=int(cl(A))$. That is A is closed if and only if A is regular open. Let B be any subset of X and let U be regular open in X containing B. Then U is a closed set containing B. Therefore $cl(B)\subseteq U$ and hence B is rg-closed. This shows that every subset of X is rg-closed.

The next theorem shows that, in a locally indiscrete space, $\{x\}$ is both pgpr-open and pre-open for every $x\in X$.

Theorem 4.11: For a topological space (X,τ), the following are equivalent:

(i) X is locally indiscrete.

(ii) Every singleton in X is both rg-closed and pgpr-open.

(iii) Every singleton in X is both pre-open and pgpr-open.

Proof: (i)$\Rightarrow$(ii): Suppose (X,τ) is locally indiscrete. Let $x\in X$. Then by Proposition 4.10, $\{x\}$ is rg-closed for every $x\in X$. By using Lemma 2.8, $\{x\}$ is pre-open for every $x\in X$. Since every pre open set is pgpr-open, we have $\{x\}$ is pgpr-open for every $x\in X$. Hence every singleton in X is both rg-closed and pgpr-open.

(ii) $\Rightarrow$ (iii): By (ii),{x} is both rg-closed and pgpr-open for every $x \in X$. Now by using Theorem 2.10, $\{x\} \subseteq pint(\{x\})$ for every $x \in X$. Always, $pint(\{x\}) \subseteq \{x\}$ for every $x \in X$. Therefore $\{x\} = pint(\{x\})$ for every $x \in X$. Hence {x} is pre-open for every $x \in X$.

(iii) $\Rightarrow$ (i): Follows from Lemma 2.8.

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