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HEATTRANSFERTHROUGHTHREE - DIMENSIONAL COUETTE FLOW BETWEEN STATIONARY AND MOVING POROUS PLATESWITH HEAT SOURCE

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ABSTRACT

This paper reports research on the effect of heat source on the 3-D couette flow in an isothermal condition. The variations are in injection and in constant suction, the flow becomes three dimensional. The governing equations are solved analytically and solutions for velocity and temperature fields are obtained and presented graphically.

Key words: Heat transfer, MHD, Porous medium, Couette flow, Heat Source.

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1.INTRODUCTION

MHD couetteflow between parallel plates is a classical one that has several applications in MHD accelerators, MHD pumps, MHD power generators and in many other industrial engineering designs. Thus such problems have been much investigated by many researchers. Transpiration cooling in three dimensional couette flow is analyzed by singh [1] and with variable permeability by Chaudhary andPawankumarsharma [2].

Some of the authors have studied the effect of suction and injection in their works, viz, Das [3], Das et al.[4], Govindarajan et al. [5] and Dileep and Vikas [6].Gireesha et al. [7] incorporated the mathematical modeling of heat transfer on dusty fluid in a coquette flow.Three dimensional fluctuating slip couette flow with thermo-diffusion through a porous channel were analyzed by Sumathi and Arunachalam[8]. JannathBegam et al.[9] have been investigated the the influence of hall current and thermal diffusion in MHD mass transfer flow with slip flow regime. More recently Ali et al. [10] presented an interesting study of the periodic injection / suction on MHD Maxwell fluid. Such studies were confined on the purely fluid regimes.

Heat source finds their use in the engineering applications where the prime concern is to dissipate as much heat as possible from a heated surface within a short time. Randall and Alexander [11] studied the free convection heat transfer with heat source. Song et al [12] examined the effects of heat source arrangements on marangoni convection in the electrostatically levitated droplets. Huo and Li [13] proposed a model to describe the three dimensional marangoni convection in electrostatically positioned droplets under microgravity.

On the other hand, Rafael cortellBataller [14], Krishnendu Bhattacharyya [15],Chung Liu [16],Yadav and Sharma [17], Kothandapani and Prakash [18],Srihari and Srinivas Reddy [19]were certain authors who have studied the effect of heat source/sink. More recently JannathBegam et al.[20] have been analyzed the MHD free convection flow with hall current and chemical reaction.

The preceding literature reveals that the problem on heat transfer through three dimensional couetteflow considering the impact of heat source in isothermal condition has not been considered. So the significance of three dimensional couetteflow with heat source using regular perturbation technique is investigated and presented graphically.

2.MATHEMATICAL FORMULATION

Consider the flow between two porous plates at a distance ‘d’ apart with isothermal condition. Lower plate is kept stable where as the upper plate is subjected to a uniform motion ‘U’. X*, Z* plane denotes lower plate horizontally and Y* is taken perpendicular to planes of plates. The fluid is sucked through upper plate with constant velocity 'V and injected at lower plate which is bounded by porous medium as,

$$V^*(Z^*) = V\left\{1+\epsilon\cos\pi\frac{Z^*}{d}\right\}(1) \qquad \text{where } 0 < \acute{\epsilon} << 1.$$

Continuity Equation:

$$\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0 \quad (2)$$

Momentum Equations:

$$\rho\left\{v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}\right\} = \mu\left(\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{\partial^2 u^*}{\partial z^{*2}}\right)-\frac{\mu u^*}{k^*}(3)$$

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$$\rho\left\{v^*\frac{\partial v^*}{\partial y^*}+ w^*\frac{\partial v^*}{\partial z^*}\right\} = \frac{\partial p^*}{\partial y^*}+\mu\left(\frac{\partial^2 v^*}{\partial y^{*2}}+\frac{\partial^2 v^*}{\partial z^{*2}}\right)-\frac{\mu u^*}{k^*}(4)$$

$$\rho\left\{v^*\frac{\partial w^*}{\partial y^*}+ w^*\frac{\partial w^*}{\partial z^*}\right\} = \frac{\partial p^*}{\partial z^*}+\mu\left(\frac{\partial^2 w^*}{\partial y^{*2}}+\frac{\partial^2 w^*}{\partial z^{*2}}\right)-\frac{\mu u^*}{k^*}(5)$$

Energy Equation:

$$\rho C_p\left\{v^*\frac{\partial T^*}{\partial y^*}+ w^*\frac{\partial T^*}{\partial z^*}\right\} = \alpha\left(\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}}\right)+$$

$$\mu\left\{(\frac{\partial u^*}{\partial y^*})^2+(\frac{\partial u^*}{\partial z^*})^2+(\frac{\partial w^*}{\partial y^*}+\frac{\partial v^*}{\partial z^*})^2+2\left[(\frac{\partial v^*}{\partial y^*})^2+(\frac{\partial w^*}{\partial z^*})^2\right]\right\}+s^*(T^*-T_0)(6)$$

The boundary conditions are

$$y^*=0,\frac{\partial T^*}{\partial y^*}=0,u^*=0,v^*(z^*)=-v\left\{1+\epsilon\cos\left(\pi\frac{z^*}{d}\right)\right\},w^*=0,T^*=T_0(7)$$

$$y^*=d,u^*=U,v^*=v,w^*=0,T^*=T_1 \qquad (8)$$

The non-dimensional parameters are introduced as follows:

$$y=\frac{y^*}{d};z=\frac{z^*}{d};u=\frac{u^*}{U};v=\frac{v^*}{V};w=\frac{w^*}{V};p=\frac{p^*}{\rho v^2};\theta=\frac{T^*-T_0}{T_1-T_0}(9)$$

$$k=\frac{k^*}{d^2};\lambda=\frac{V}{U};Ec=\frac{U^2}{C_p(T_1-T_0)};Re=\frac{Vd}{v};\Pr=\frac{\mu C_p}{\alpha};S=\frac{s^*d}{v}(10)$$

The reduced governing equations in non-dimensional form are

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (11)$$

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}-Re\left[v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]-\frac{u}{k}=0(12)$$

$$\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}-Re\left[v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]-Re\frac{\partial p}{\partial y}-\frac{v}{k}=0 \quad (13)$$

$$\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}-Re\left[v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right]-Re\frac{\partial p}{\partial z}-\frac{w}{k}=0 \quad (14)$$

$$\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}-Re\Pr\left[v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right]+\Pr Ec\left\{\left(v\frac{\partial u}{\partial y}\right)^2+\left(v\frac{\partial u}{\partial z}\right)^2\right\}+$$

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$$Pr.Ec.\lambda^2\left\{\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+2\left[\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]\right\}+S\theta=0 \quad (15)$$

The Corresponding boundary conditions in non-dimensional form are

$$y=0, u=0, v=1+\epsilon\cos(\pi z), w=0, \frac{\partial\theta}{\partial y}=0 \quad (16)$$

$$y=1, u=1, v=1, w=0, \theta=1 \quad (17)$$

3.RESULT AND DISCUSSION

The regular perturbation technique was used to obtain the asymptotic solutions of equations (11) to (15) using the boundary conditions (16) and (17). So the obtained values of velocity and temperature are distributed numerically and these values are plotted as graphs in figures 1-21.

The figures 1-5 depicts the main flow velocity u versus y for various parameters. The main flow velocity u is found to be increasing with pr and eothough increase in the magnitude of the velocity is negligible. The magnitude of the velocity is found to be considerably decreasing with decreasing Reynolds number, Permeability parameter and Heat source parameter.

The figures 6-10 depicts the cross flow velocity v for non dimentional parameters. It may be observed from these figures that Prandtl number and Eckert number decreases the non dimensional cross flow velocity v. Increasing Reynolds number, Permeability parameter and Heat source parameter enhance thenon dimensional flow velocity.

From the figures 11-15, it may be seen that rise in pr and Eckert number reduces the temperature distribution at the stationary plate and increases the temperature distribution at the moving plate. Permeability parameter and Heat source parameter decrease the temperature.

The figures 16-21 shows the effect of various parameters over the wall shear stress. At the stationary plate y=0, when the heat source parameter is small, skin friction increases. For large Heat source parameter, skin friction remains constant. eo increases S_h. Prandtl number , Heat source term has no significant effect on the skin friction.

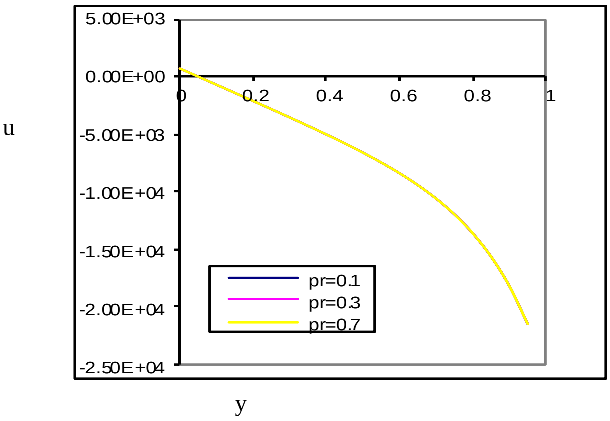

Fig.1 Effect of pron the velocity profile u

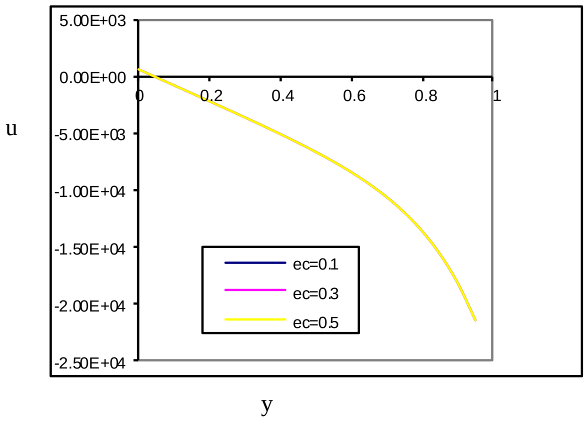

Fig. 2 Effect of eoon the velocity profile u

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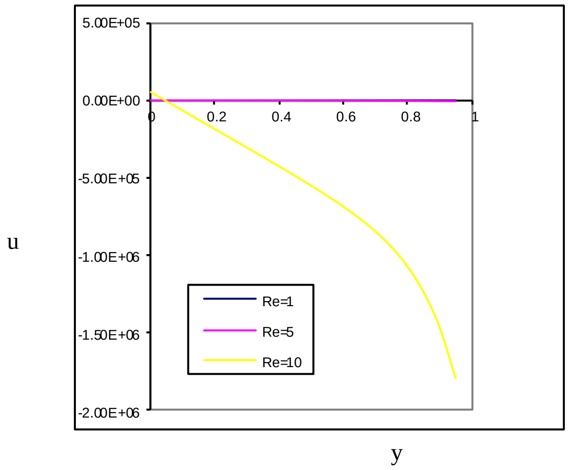

Fig. 3 Effect of Reon the velocity profile u

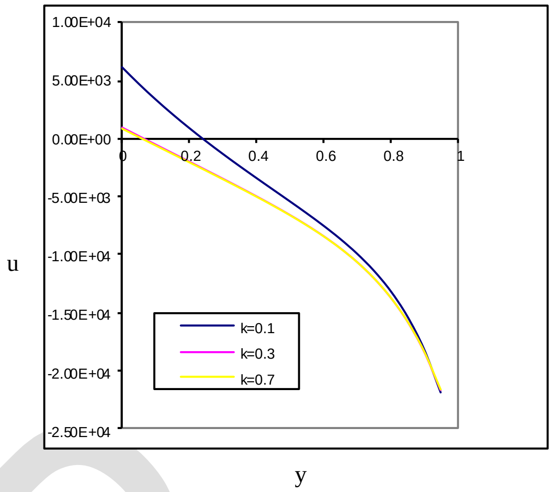

Fig. 4 Effect of k on the velocity profile u

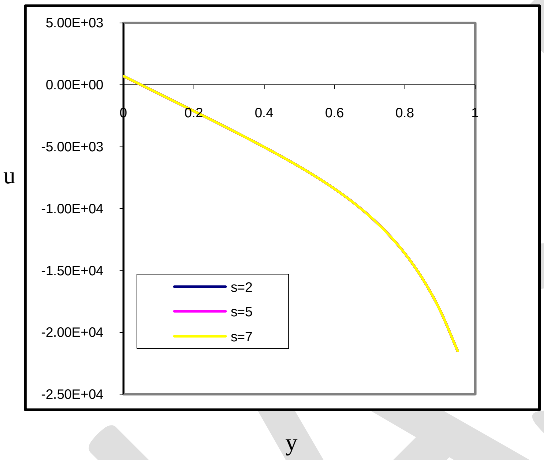

Fig. 5 Effect of son the velocity profile u

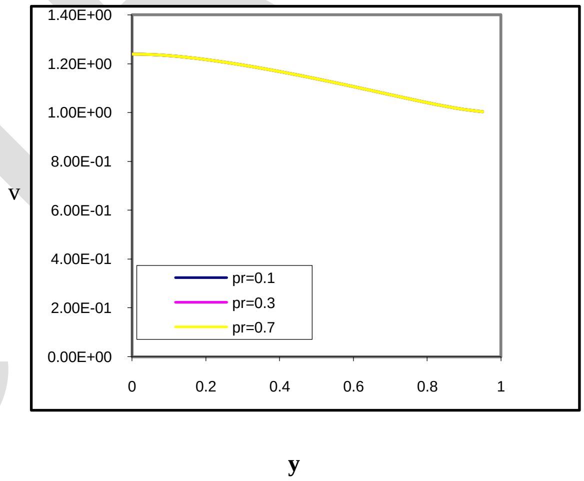

Fig. 6 Effect of pr on the velocity profile v

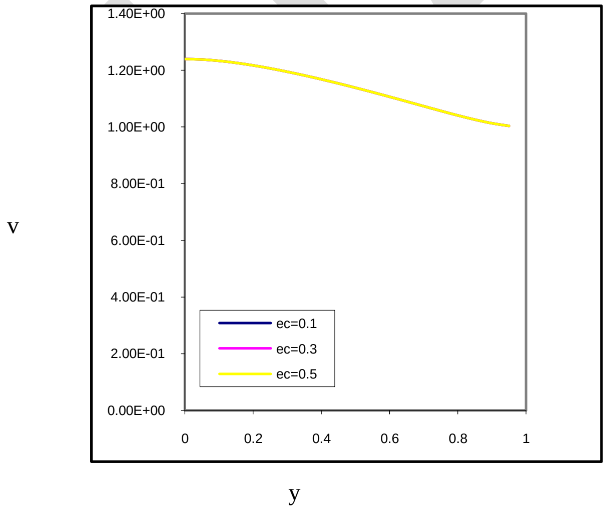

Fig. 7 Effect of eoon the velocity profile v

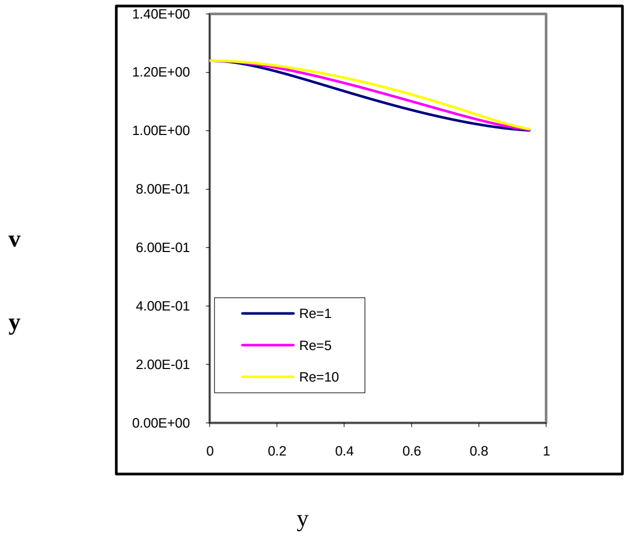

Fig.8 Effect of Reon the velocity profile v

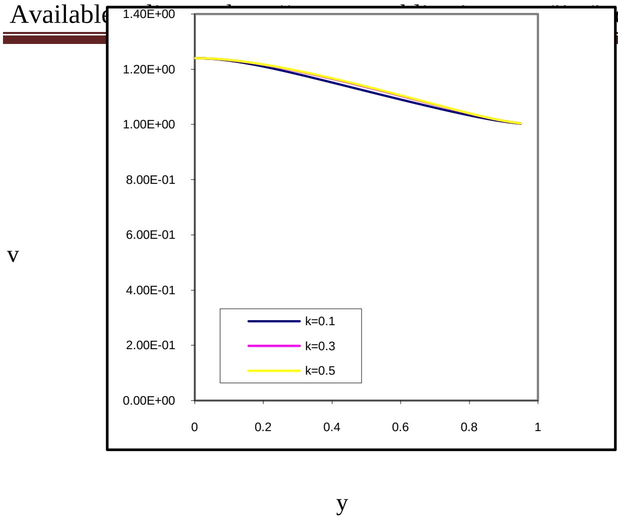

Fig. 9 Effect of k on the velocity profile v

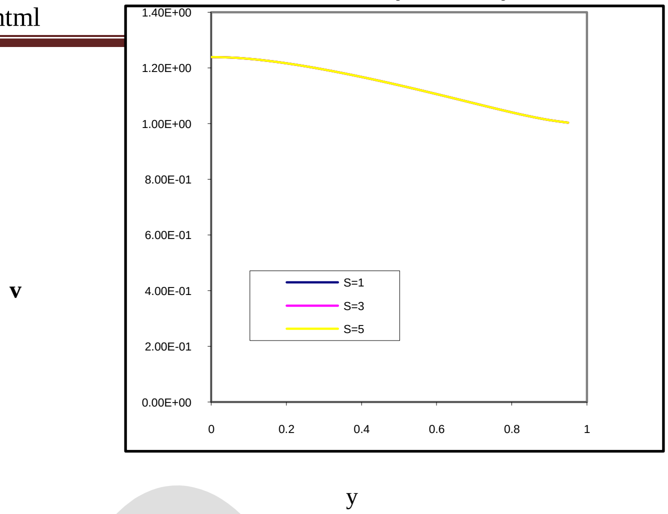

Fig. 10 Effect of son the velocity profile v

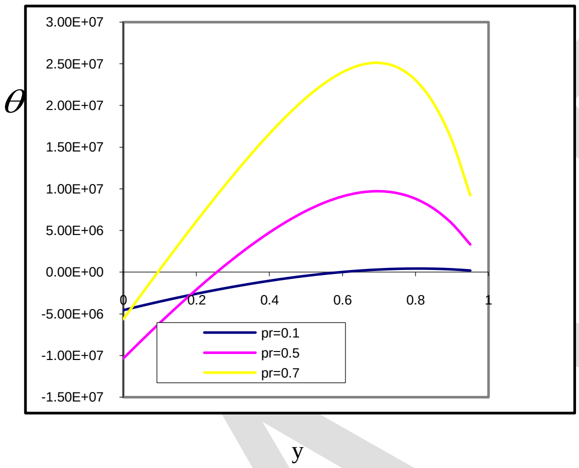

Fig. 11 Effect of pron the temperature profile θ

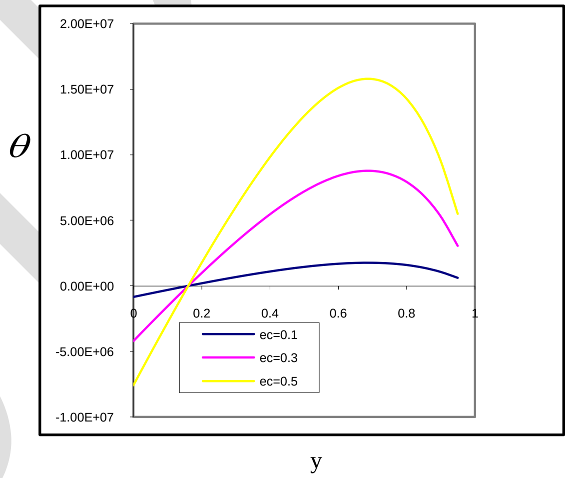

Fig. 12 Effect of eoon the temperature profile

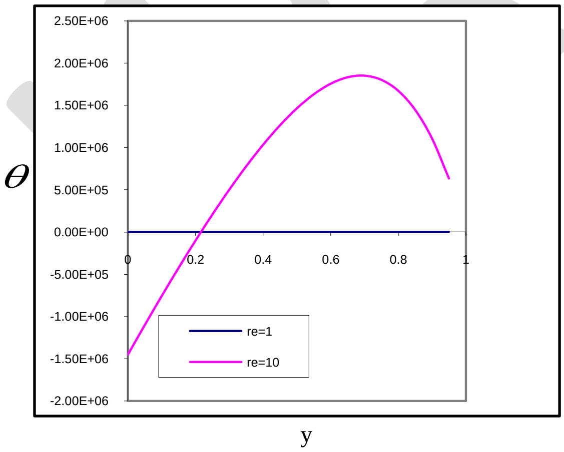

Fig. 13 Effect of Re on the temperature profile θ

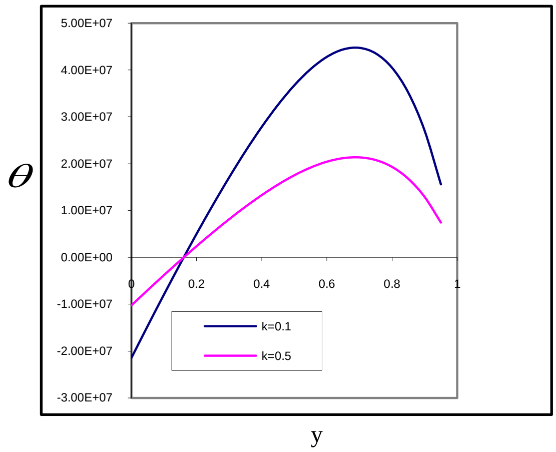

Fig. 14 Effect of kon the temperature profile θ

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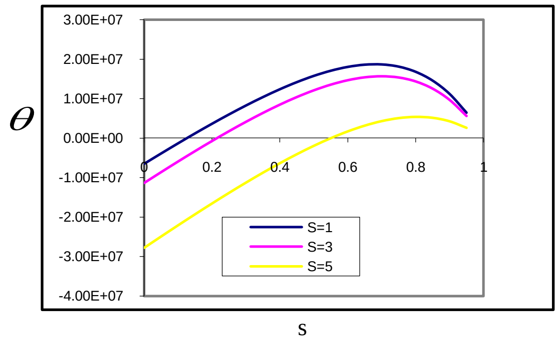

Fig. 15 Effect of s on the temperature profile θ

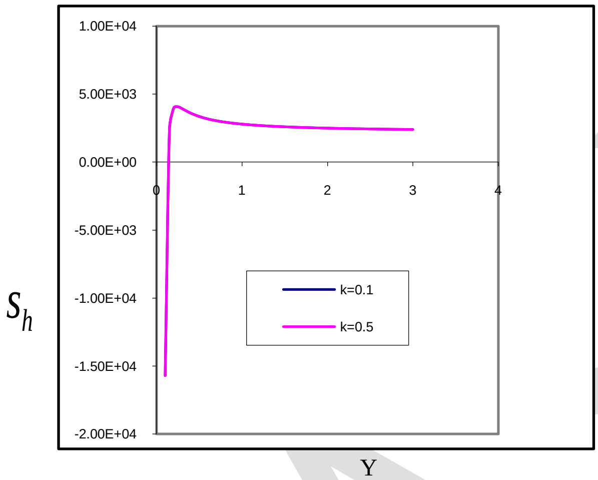

Fig. 16 Effect of kon the Skin friction $S_h$at y =0

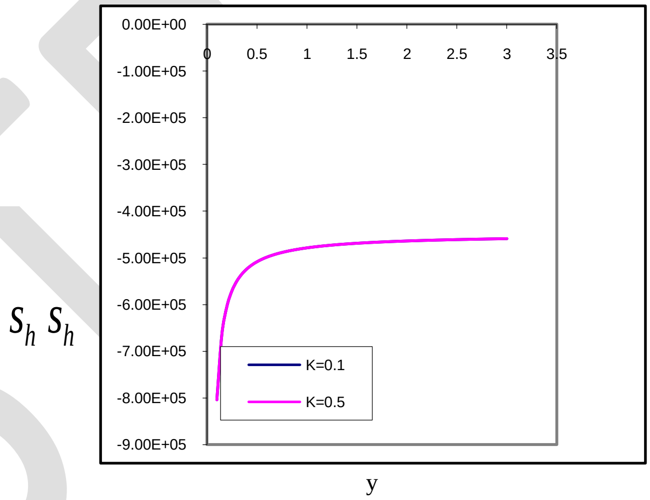

Fig. 17 Effect of kon the Skin friction $S_h$at y =1

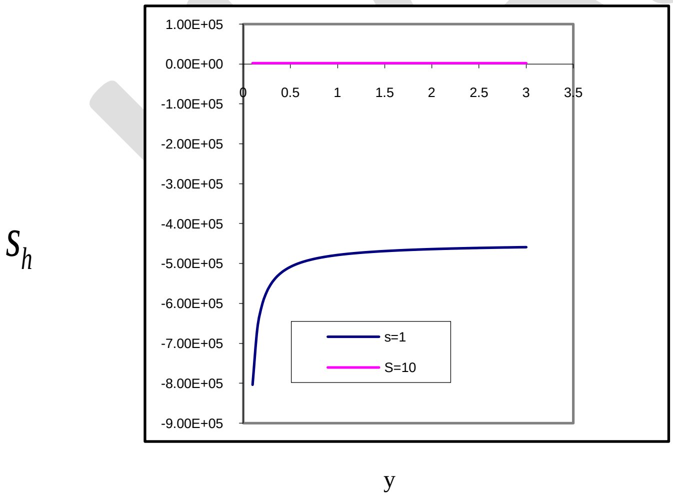

Fig.18 Effect of son the Skin friction $S_h$at y =0

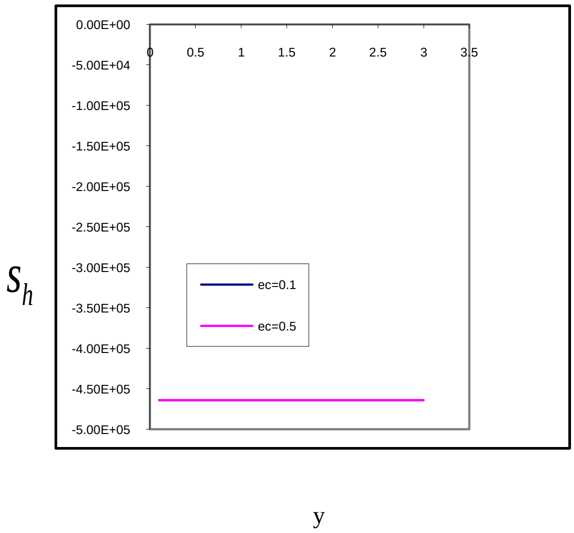

Fig.19 Effect of eoon the Skin friction $S_h$at y =1

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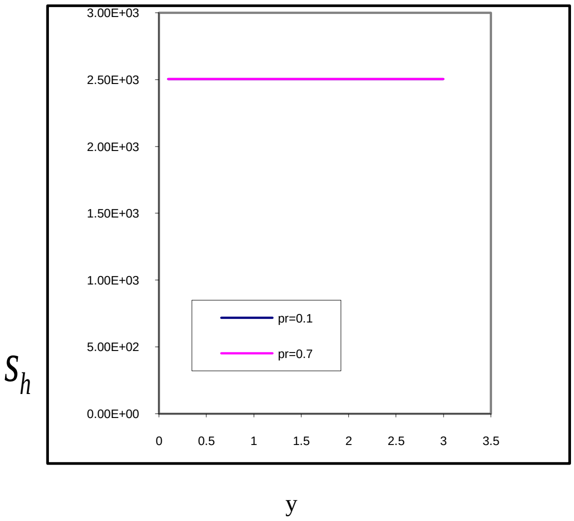

Fig.20 Effect of pron the Skin friction $S_h$at y =0

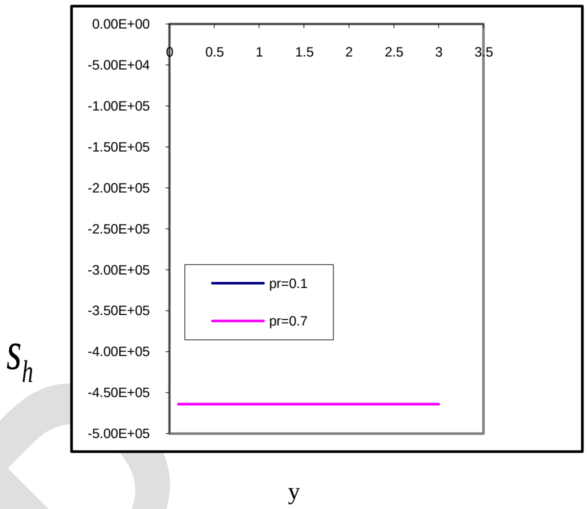

Fig. 21 Effect of pron the Skin friction S_h at y =1

4.CONCLUSION

Three dimensional coquette flow and heat transfer with the effect of heat source is studied theoretically. The regular perturbation technique is used to obtain asymptotic solutions of velocity and temperature fields .Numerical results are obtained for various parameters of interest and results are discussed with the help of graphs .

- Main flow velocity u is found to be increasing with the increment in pr and eothough increase in the magnitude of the velocity is negligible.
- The magnitude of the velocity is found to be considerably decreasing with decreasing Reynolds number, Permeability parameter and Heat source parameter.
- As Prandtl number and Eckert number increases, the temperature distribution decreases at stationary plate and increases at the moving plate.
- Permeability parameter and Heat source parameter decrease the temperature.

5.NOMENCLATURE

μ	Viscosity
ρ	Density
$p*$	Pressure
α	Thermal conductivity
$k*$	Permeability of porous medium

C_p	Specific heat at constant pressure
λ	Velocity ratio parameter
ν	Kinematic viscosity
Pr	Prandtl number
Ec	Eckert number
Re	Reynolds number
S	Heat source parameter.

$k$permeability parameter

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