

## Some Public Tripled Coincidence Fixed Point Theorems for Continuous Mappings having Public Mixed $g$ -Monotone Property

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### **Abstract**

In this paper, we introduce the public fixed point, public coincidence point, public mixed  $g$ -monotone property and public commute. Also, we proved the existence and uniqueness of public coincidence fixed point and public fixed point for continuous mappings having public mixed  $g$ -monotone property without public commute in partially ordered metric space

Keywords: tripled fixed point, tripled coincidence point, mixed  $g$ -monotone property

### **1. Introduction**

The existence of fixed point for contraction type in partially ordered metric space was first considered by Ran and Reurings[1], they established new results in partially ordered metric space .Also , many researchers presented new results for contraction mapping in partially ordered metric space see([2]-[25] ). In 2006, Bhaskar and Lakshmikantham [5] introduced the concept mixed monotone property for contractive mappings they also established coupled fixed results for mappings has mixed monotone property.On other hand ,Sintunavarat et al. ([26], [27]) proved the existence and uniqueness of coupled fixed point theorem for nonlinear contractions without the mixed monotone property.lso ,Lakshmikantham and cric [12] introduced the concept of mixed  $g$ -monotone and proved some results for coupled coincidence fixed point and coupled common fixed point for commuting mappings, this results extend of results Bhaskar and Lakshmikantham[5].

Additionally ,Choudhury and Kundu[6] ,introduced the compatibility of mappings in partially ordered metric space and they established a coupled coincidence point results.Berinde and Borcut([28], [29]) introduced the concept of tripled fixed point and tripled coincidence fixed point ,they extended the results of Bhaskar and Lakshmikantham[5]and Cric and Lakshmikantham[12] to the tripled fixed point and coincidence fixed point.

Now , we recall the flowing definitions:

#### **Definition (1.1): [30]**

A set  $X$  with a binary operation  $\leq$  is called partially ordered set if for all  $p, q, r \in X$ .

- i.  $p \leq p$
- ii.  $p \leq q$  and  $q \leq p \Rightarrow p = q$
- iii.  $p \leq q$  and  $q \leq r \Rightarrow p \leq r$

**Definition (1.2):** [28]

Let  $f: X^3 \rightarrow X$  be any mapping. An element  $(x, y, z) \in X^3$  is called tripled fixed point of  $f$  iff  $f_{(x,y,z)} = x$  &  $f_{(y,x,y)} = y$  and  $f_{(z,y,x)} = z$

**Definition (1.3):** [29]

$X$  and  $g: X \rightarrow X$  be two mapping. An element  $(x, y, z)$  is called a tripled coincidence point of  $f$  and  $g$  if,

$$f_{(x,y,z)} = g_{(x)} \& f_{(y,x,y)} = g_{(y)} \quad \text{and} \quad f_{(z,y,x)} = g_{(z)}$$

**Definition (1.4):** [29]

Let  $f: X^3 \rightarrow X$  and  $g: X \rightarrow X$  be two mapping and  $(X, \leq)$  be a partially ordered set, then we say that  $f$  has mixed  $g$ -monotone property if  $f$  is monotone increasing in  $x$  and  $z$  and is monotone decreasing in  $y$ , i.e.,  $\forall x, y, z \in X$

$$x_1, x_2 \in X, g_{(x_1)} \leq g_{(x_2)} \rightarrow f_{(x_1, y, z)} \leq f_{(x_2, y, z)}$$

$$y_1, y_2 \in X, g_{(y_1)} \leq g_{(y_2)} \rightarrow f_{(x, y_1, z)} \leq f_{(x, y_2, z)}$$

$$\text{And } z_1, z_2 \in X, g_{(z_1)} \leq g_{(z_2)} \rightarrow f_{(x, y, z_1)} \leq f_{(x, y, z_2)}.$$

Now, we will give the following concepts

**Definition (1.5):**

Let  $X$  be a nonempty set. Then we say that the mappings

$f_1, f_2, \dots, f_n: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are public commuting if for each  $x, y, z \in X$ ,

$$\left( g \left( f_1 \left( f_2 \left( \dots \left( f_n(x, y, z) \right) \dots \right) \right) \right) \right) = f_1 \left( f_2 \left( \dots \left( f_n(g(x), g(y), g(z)) \right) \dots \right) \right)$$

**Definition (1.6):**

Let  $X$  be a nonempty and  $f_1, f_2, \dots, f_n: X^3 \rightarrow X$  be a given mappings. An element  $(x, y, z) \in X^3$  is called a public tripled fixed point of  $f_1, f_2, \dots, f_n$  if

$$f_1 \left( f_2 \left( \dots \left( f_n(x, y, z) \right) \dots \right) \right) = x, f_1 \left( f_2 \left( \dots \left( f_n(y, x, z) \right) \dots \right) \right) = y \text{ and}$$

$$f_1 \left( f_2 \left( \dots \left( f_n(z, y, x) \right) \dots \right) \right) = z$$

**Definition (1.7):**

Let  $(X, \leq)$  be a partially ordered set and  $f_1, f_2, \dots, f_n: X^3 \rightarrow X$  be a mapping. We say that  $f_1, f_2, \dots, f_n$  are public mixed monotone if  $f_1 \left( f_2 \left( \dots \left( f_n(x, y, z) \right) \dots \right) \right)$  is monotone increasing in  $x$  and  $z$  and is monotone decreasing in  $y$ , i.e.,

$$\forall x, y, z \in X$$

$$x_1, x_2 \in X, x_1 \leq x_2 \rightarrow f_1 \left( f_2 \left( \dots \left( f_n(x_1, y, z) \right) \dots \right) \right) \leq f_1 \left( f_2 \left( \dots \left( f_n(x_2, y, z) \right) \dots \right) \right)$$

$$y_1, y_2 \in X, y_1 \leq y_2 \rightarrow f_1 \left( f_2 \left( \dots \left( f_n(x, y_1, z) \right) \dots \right) \right) \geq f_1 \left( f_2 \left( \dots \left( f_n(x, y_2, z) \right) \dots \right) \right)$$

And

$$z_1, z_2 \in X, z_1 \leq z_2 \rightarrow f_1 \left( f_2 \left( \dots \left( f_n(x, y, z_1) \right) \dots \right) \right) \leq f_1 \left( f_2 \left( \dots \left( f_n(x, y, z_2) \right) \dots \right) \right)$$

**Definition (1.8):**

Let  $X$  be a nonempty set. Let  $f_1, f_2, \dots, f_3: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings. An element  $(x, y, z) \in X^3$  is called public tripled coincidence point of  $f_1, f_2, \dots, f_n$  and  $g$  if,

$$f_1(f_2(\dots(f_{n(x,y,z)})\dots)) = g(x), f_1(f_2(\dots(f_{n(y,x,y)})\dots)) = g(y)$$

$$f_1(f_2(\dots(f_{n(z,y,x)})\dots)) = g(z).$$
**Definition (1.9):**

Let  $(X, \leq)$  be a partially ordered set and  $f_1, f_2, \dots, f_n: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings, then we say that  $f_1, f_2, \dots, f_n$  are public mixed  $g$ -monotone property, If

$f_1(f_2(\dots(f_{n(x,y,z)})\dots))$  is monotone increasing in  $x$  and  $z$  and is monotone decreasing in  $y$ ; i.e, for all  $x, y, z \in X$

$$x_1, x_2 \in X, g(x_1) \leq g(x_2) \Rightarrow$$

$$f_1(f_1(\dots(f_{n(x_1,y,z)})\dots)) \leq f_1(f_2(\dots(f_{n(x_2,y,z)})\dots))$$

$$y_1, y_2 \in X, g(y_1) \leq g(y_2) \Rightarrow$$

$$f_1(f_1(\dots(f_{n(x,y_1,z)})\dots)) \geq f_1(f_2(\dots(f_{n(x,y_2,z)})\dots))$$

Also

$$z_1, z_2 \in X, g(z_1) \leq g(z_2) \Rightarrow$$

$$f_1(f_1(\dots(f_{n(x,y,z_1)})\dots)) \geq f_1(f_2(\dots(f_{n(x,y,z_2)})\dots)).$$

**2. Main Results**

Consider,  $\emptyset$  be the set of all increasing mappings such that:

$$\emptyset_i : [0, \infty] \rightarrow [0, \infty] \text{ such that } \emptyset_{i(t)} = \begin{cases} t & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{cases}$$

and  $\lim_{n \rightarrow \infty} \emptyset_i^n(t) = 0 \forall i = 1, \dots, 7$ , where  $\emptyset^n$  denotes the  $n$ -th iterate of  $\emptyset$ . But  $\mathcal{M}$  is the set of all mappings  $f_1, \dots, f_n$  and  $g$  satisfy:

i.  $g(X)$  is complete of  $X$  containing  $f_1(f_2(\dots(f_n(X \times X \times X))\dots))$ .

ii.  $g$  is continuous mappings.

iii.  $f_1, f_2, \dots, f_n$  are public mixed  $g$ -monotone property

Now, we prove the following results

**Theorem (2.1):**

Let  $(X, d, \leq)$  be a partially ordered complete metric space and let  $f_1, f_2, \dots, f_n: X^3 \rightarrow X$  and  $g_1, g_2, \dots, g_n: X \rightarrow X$  are mappings lies in  $\mathcal{M}$ . If  $\emptyset_1, \emptyset_2, \dots, \emptyset_7 \in \emptyset$  such that

$$df_1(f_2(\dots(f_{n(x,y,z)})\dots)), f_1(f_2(\dots(f_{n(u,v,w)})\dots)) \leq$$

$$\max\{\emptyset_1 d(g(x), g(u)), \emptyset_2 d(g(y), g(v)), \emptyset_3 d(g(z), g(w)), \emptyset_4 d(f_1(f_2(\dots(f_{n(x,y,z)})\dots)), g(x)),$$

$$\emptyset_5 d(f_1(f_2(\dots(f_{n(z,y,x)})\dots)), g(z)), \emptyset_6 d(f_1(f_2(\dots(f_{n(u,v,w)})\dots)), g(u)),$$

$$\emptyset_7 d(f_1(f_2(\dots(f_{n(w,v,u)})\dots)), g(w))\}$$

For all  $x, y, z, u, v, w \in X$  with

$$g(x) \geq g(u), g(y) \leq g(v), g(z) \leq g(w)$$

Suppose that, if  $\langle x_n \rangle$  increasing sequence such that:  $x_n \rightarrow x$  then  $x_n \leq x$  for all  $n \in N$ . Also, if  $\langle y_n \rangle$  decreasing sequence such that:

$y_n \rightarrow y$  then  $y_n \geq y$  for all  $n \in N$ . If there exist  $x_0, y_0, z_0 \in X$  such that:

$$g(x_0) \leq f_1(f_2(\dots \dots (f_{n(x_0, y_0, z_0)}) \dots \dots))$$

$$g(y_0) \geq f_1(f_2(\dots \dots (f_{n(y_0, x_0, y_0)}) \dots \dots)) \quad (2.1)$$

$$g(z_0) \leq f_1(f_2(\dots \dots (f_{n(z_0, y_0, x_0)}) \dots \dots))$$

Then  $f_1, f_2, \dots, f_n$  and  $g$  having public tripled coincidence point.

**Proof:**

consider  $x_0, y_0, z_0 \in X$  satisfy (2.1). We can construct sequences as,

$$\begin{aligned} \text{Define: } & g(x_1) \leq f_1(f_2(\dots \dots (f_{n(x_0, y_0, z_0)}) \dots \dots)) \\ & g(y_1) \leq f_1(f_2(\dots \dots (f_{n(y_0, x_0, y_0)}) \dots \dots)) \\ & g(z_1) \leq f_1(f_2(\dots \dots (f_{n(z_0, y_0, x_0)}) \dots \dots)) \end{aligned}$$

And hence, we get:

$$g(x_0) \leq g(x_1), g(y_0) \geq g(y_1), g(z_0) \leq g(z_1)$$

As the same way define

$$g(x_2) = f_1(f_2(\dots \dots (f_{n(x_1, y_1, z_1)}) \dots \dots))$$

$$g(y_2) = f_1(f_2(\dots \dots (f_{n(y_1, x_1, y_1)}) \dots \dots))$$

$$g(z_2) = f_1(f_2(\dots \dots (f_{n(z_1, y_1, x_1)}) \dots \dots))$$

But  $f_1, f_2, \dots, f_n$  having public  $g$ \_mixed monotone property. Then, we get

$$g(x_0) \leq g(x_1) \leq g(x_2)$$

$$g(y_0) \geq g(y_1) \geq g(y_2)$$

$$g(z_0) \leq g(z_1) \leq g(z_2)$$

We continue operations where we get the sequences.

$\langle g(x_n) \rangle, \langle g(y_n) \rangle$  and  $\langle g(z_n) \rangle$  in  $g(X)$  and satisfy the following

$$g(x_n) = f_1(f_2(\dots \dots (f_{n(x_{n-1}, y_{n-1}, z_{n-1})}) \dots \dots)),$$

$$\leq g(x_{n+1}) = f_1(f_2(\dots \dots (f_{n(x_n, y_n, z_n)}) \dots \dots))$$

$$g(y_{n+1}) = f_1(f_2(\dots \dots (f_{n(y_n, x_n, y_n)}) \dots \dots)),$$

$$\leq g(y_n) = f_1(f_2(\dots \dots (f_{n(y_{n-1}, x_{n-1}, y_{n-1})}) \dots \dots))$$

$$g(z_{n+1}) = f_1(f_2(\dots \dots (f_{n(z_{n-1}, y_{n-1}, x_{n-1})}) \dots \dots)),$$

$$\leq g(z_n) = f_1(f_2(\dots \dots (f_{n(z_n, y_n, x_n)}) \dots \dots))$$

We will take two cases during the proof

**Case (1):** If,

$$(g(x_{n+1}), g(y_{n+1}), g(z_{n+1})) = (g(x_n), g(y_n), g(z_n))$$

For some  $n \in \mathbb{N}$ , then

$$f_1(f_2(\dots \dots (f_{n(x_n, y_n, z_n)}) \dots \dots)) = g(x_n)$$

$$f_1(f_2(\dots \dots (f_{n(y_n, x_n, y_n)}) \dots \dots)) = g(y_n)$$

$$f_1(f_2(\dots \dots (f_{n(z_n, y_n, x_n)}) \dots \dots)) = g(z_n)$$

Hence,  $(x_n, y_n, z_n)$  is a public tripled coincidence point of  $f_1, f_2, \dots, f_n$  and  $g$

**Case (2):If**

$$(g(x_{n+1}), g(y_{n+1}), g(z_{n+1})) \neq (g(x_n), g(y_n), g(z_n))$$

$\forall n \in N$ , either  $g(x_n) \neq g(x_{n+1})$  Or  $g(y_n) \neq g(y_{n+1})$  Or  $g(z_n) \neq g(z_{n+1})$

Now,

$$d(g(x_{n+1}), g(x_n))$$

$$= d\left(f_1\left(f_2(\dots \dots (f_{n(x_n, y_n, z_n)}) \dots \dots)\right), f_1\left(f_2(\dots \dots (f_{n(x_{n-1}, y_{n-1}, z_{n-1})}) \dots \dots)\right)\right)$$

$$\leq \max\{\emptyset_1 d(g(x_n), g(x_{n-1})), \emptyset_2 d(g(y_n), g(y_{n-1})), \emptyset_3 d(g(z_n), g(z_{n-1})), \emptyset_4 d\left(f_1\left(f_2(\dots \dots (f_{n(x_n, y_n, z_n)}) \dots \dots)\right), \emptyset_7 d\left(f_1\left(f_2(\dots \dots (f_{n(z_n, y_n, x_n)}) \dots \dots)\right), g(z_n)\right), \emptyset_6 d\left(f_1\left(f_2(\dots \dots (f_{n(x_{n-1}, y_{n-1}, z_{n-1})}) \dots \dots)\right), g(x_{n-1})\right), \emptyset_7 d\left(f_1\left(f_2(\dots \dots (f_{n(z_{n-1}, y_{n-1}, x_{n-1})}) \dots \dots)\right), g(z_{n-1})\right)\}$$

=

$$\max\{\emptyset_1 d\left(g_1\left(g_2(\dots \dots (g_{n(x_n)}) \dots \dots)\right), g(x_{n-1})\right), \emptyset_2 d(g(y_n), g(y_{n-1})), \emptyset_3 d(g(z_n), g(z_{n-1})), \emptyset_4 d(g(x_{n+1}), g(x_n)), \emptyset_7 d(g(z_n), g(z_{n-1}))\}$$

Let  $h_{1(t)} = \max\{\emptyset_{1(t)}, \emptyset_{6(t)}\}$ ,  $h_{2(t)} = \max\{\emptyset_{3(t)}, \emptyset_{7(t)}\}$ , and  $\emptyset_{(t)} \in \Phi$

$$\begin{aligned} &= \max\{h_1 d(g(x_n), g(x_{n-1})), \emptyset_2 d(g(y_n), g(y_{n-1})), h_2 d(g(z_n), g(z_{n-1})), \\ &\quad \emptyset_4 d(g(x_{n+1}), g(x_n)), \emptyset_5 d(g(z_{n+1}), g(z_n))\} \\ &\leq \max\{h_1 d(g(x_n), g(x_{n-1})), \emptyset_2 d(g(y_n), g(y_{n-1})), \\ &\quad h_2 d(g(z_n), g(z_{n-1})), \emptyset_4 d(g(x_{n+1}), g(x_n)), \\ &\quad \emptyset_5 d(g(y_{n+1}), g(y_n)), \emptyset_6 d(g(z_{n+1}), g(z_n))\} \end{aligned}$$

Now,

$$\begin{aligned} &d(g(y_n), g(y_{n+1})) \\ &= d\left(f_1\left(f_2(\dots \dots (f_{n(y_{n-1}, x_{n-1}, y_{n-1})}) \dots \dots)\right), f_1\left(f_2(\dots \dots (f_{n(y_n, x_n, y_n)}) \dots \dots)\right)\right) \\ &\leq \max\{h_3 d(g(y_{n-1}), g(y_n)), \emptyset_2 d(g(x_{n-1}), g(x_n)), \\ &\quad h_4 d\left(f_1\left(f_2(\dots \dots (f_{n(y_{n-1}, x_{n-1}, y_{n-1})}) \dots \dots)\right), g(y_{n-1})\right), \\ &\quad h_5 d\left(f_1\left(f_2(\dots \dots (f_{n(y_n, x_n, y_n)}) \dots \dots)\right), g(y_n)\right)\} \end{aligned}$$

where,  $h_{3(t)} = \max\{\emptyset_{1(t)}, \emptyset_{3(t)}\}$ ,  $h_{4(t)} = \max\{\emptyset_{4(t)}, \emptyset_{5(t)}\}$ ,

$h_{5(t)} = \max\{\emptyset_{6(t)}, \emptyset_{7(t)}\}$ ,  $h_{6(t)} = \max\{h_{3(t)}, h_{4(t)}\}$  and  $\emptyset_{(t)} \in \Phi$

$$d(g(y_n), g(y_{n+1})) = \max\{h_6 d(g(y_{n-1}), g(y_n)), \emptyset_2 d(g(x_{n-1}), g(x_n)), h_5 d(g(y_{n+1}), g(y_n))\}$$

$$\leq \max\{\emptyset_2 d(g(x_{n-1}), g(x_n)), h_6 d(g(y_{n-1}), g(y_n)), \emptyset d(g(z_{n-1}), g(z_n)), \\ \emptyset d(g(x_{n+1}), g(x_n)), h_5 d(g(y_{n+1}), g(y_n)), \emptyset d(g(z_{n+1}), g(z_n))\}$$

Also we have:

$$\begin{aligned} & d(g(z_{n+1}), g(z_n)) \\ &= d\left(f_1\left(f_2(\dots \dots (f_{n(z_n, y_n, x_n)}) \dots \dots)\right), f_1\left(f_2(\dots \dots (f_{n(z_{n-1}, y_{n-1}, x_{n-1})}) \dots \dots)\right)\right). \\ &\leq \max\{\emptyset_1 d(g(z_n), g(z_{n-1})), \emptyset_2 d(g(y_n), g(y_{n-1})), \\ &\quad \emptyset_3 d(g(x_n), g(x_{n-1})), \emptyset_4 d\left(f_1\left(f_2(\dots \dots (f_{n(z_n, y_n, x_n)}) \dots \dots)\right), g(z_n)\right), \\ &\quad \emptyset_5 d\left(f_1\left(f_2(\dots \dots (f_{n(x_n, y_n, z_n)}) \dots \dots)\right), g(x_n)\right), \\ &\quad \emptyset_6 d\left(f_1\left(f_2(\dots \dots (f_{n(z_{n-1}, y_{n-1}, x_{n-1})}) \dots \dots)\right), g(z_{n-1})\right), \\ &\quad \emptyset_7 d\left(f_1\left(f_2(\dots \dots (f_{n(x_{n-1}, y_{n-1}, z_{n-1})}) \dots \dots)\right), g(x_{n-1})\right)\}. \\ &= \max\{\emptyset_1 d(g(z_n), g(z_{n-1})), \emptyset_2 d(g(y_n), g(y_{n-1})), \\ &\quad \emptyset_3 d(g(x_n), g(x_{n-1})), \emptyset_4 d(g(z_{n+1}), g(z_n)), \\ &\quad \emptyset_5 d(g(x_{n+1}), g(x_n)), \emptyset_6 d(g(z_n), g(z_{n-1})), \\ &\quad \emptyset_7 d(g(x_n), g(x_{n-1}))\}. \\ &\leq \max\{h_7 d(g(z_n), g(z_{n-1})), \emptyset_2 d(g(y_n), g(y_{n-1})), \\ &\quad h_8 d(g(x_n), g(x_{n-1})), \emptyset_4 d(g(z_{n+1}), g(z_n)), \emptyset_5 d(g(x_{n+1}), g(x_n))\} \end{aligned}$$

where,  $h_{7(t)} = \max\{\emptyset_{1(t)}, \emptyset_{6(t)}\}, h_{8(t)} = \max\{\emptyset_{3(t)}, \emptyset_{7(t)}\}$

$$\begin{aligned} d(g(z_{n+1}), g(z_n)) &\leq \max\{h_7 d(g(z_n), g(z_{n-1})), \emptyset_2 d(g(y_n), g(y_{n-1})), \\ &\quad h_8 d(g(x_n), g(x_{n-1})), \emptyset_4 d(g(z_{n+1}), g(z_n)), \\ &\quad \emptyset d(g(y_{n+1}), g(y_n)), \emptyset_5 d(g(x_{n+1}), g(x_n))\} \end{aligned}$$

Let  $\varphi_{(t)} = \max\{\emptyset_{(t)}, \emptyset_{1(t)}, \dots, \emptyset_{7(t)}, h_{1(t)}, \dots, h_{8(t)}\}$ , then we have

$$\begin{aligned} & \max\{d(g(x_{n+1}), g(x_n)), d(g(y_{n+1}), g(y_n)), d(g(z_{n+1}), g(z_n))\} \\ &\leq \max\{\varphi d(g(x_n), g(x_{n-1})), \varphi d(g(y_n), g(y_{n-1})), \\ &\quad \varphi d(g(z_n), g(z_{n-1})), \varphi d(g(x_{n+1}), g(x_n)), \\ &\quad \varphi d(g(y_{n+1}), g(y_n)), \varphi d(g(z_{n+1}), g(z_n))\} \quad (2.2) \\ &< \max\{d(g(x_n), g(x_{n-1})), d(g(y_n), g(y_{n-1})), d(g(z_n), g(z_{n-1})), \\ &\quad d(g(x_{n+1}), g(x_n)), d(g(y_{n+1}), g(y_n)), d(g(z_{n+1}), g(z_n))\} \end{aligned}$$

This leads

$$\begin{aligned} & \max\{d(g(x_{n+1}), g(x_n)), d(g(y_{n+1}), g(y_n)), d(g(z_{n+1}), g(z_n))\} \\ &< \max\{d(g(x_n), g(x_{n-1})), d(g(y_n), g(y_{n-1})), d(g(z_n), g(z_{n-1}))\} \end{aligned}$$

And hence, the equation (2.2) become

$$\begin{aligned} & \max\{d(g(x_{n+1}), g(x_n)), d(g(y_{n+1}), g(y_n)), d(g(z_{n+1}), g(z_n))\} \\ &\leq \max\{\varphi [d(g(x_n), g(x_{n-1})), d(g(y_n), g(y_{n-1})), d(g(z_n), g(z_{n-1}))]\} \\ &\leq \max\{\varphi^2 [d(g(x_{n-1}), g(x_{n-2})), d(g(y_{n-1}), g(y_{n-2})), d(g(z_{n-1}), g(z_{n-2}))]\} \\ &\quad \vdots \end{aligned}$$

$$\leq \max\{\varphi^n[d(g(x_1), g(x_o)), d(g(y_1), g(y_o)), d(g(z_1), g(z_o))]\}$$

$$\text{But} \lim_{n \rightarrow \infty} \max\{\varphi^n[d(g(x_1), g(x_o)), d(g(y_1), g(y_o)), d(g(z_1), g(z_o))]\} = 0$$

Then,  $\forall \epsilon > 0$ ;  $\varphi_{(\epsilon)} < \epsilon$ ,  $\exists n_o \in N$  such that:

$$\varphi^n\{d(g(x_1), g(x_o)), d(g(y_1), g(y_o)), d(g(z_1), g(z_o))\} < \epsilon - \varphi_{(\epsilon)} \quad \forall n \geq n_o$$

$$\begin{aligned} \max\{d(g(x_{n+1}), g(x_n)), d(g(y_{n+1}), g(y_n)), d(g(z_{n+1}), g(z_n))\} \\ &< \epsilon - \varphi_{(\epsilon)} \end{aligned} \quad (2.3)$$

Now, To prove that,  $\forall m \geq n \geq n_o$

$$\max\{d(g(x_n), g(x_m)), d(g(y_n), g(y_m)), d(g(z_n), g(z_m))\} < \epsilon \quad (2.4)$$

We will discuss the Cauchy sequence,

- i. For  $m = n + 1$  and by using (2.3) we get (2.4).
- ii. Suppose it is if  $m = k$ , i.e.

$$\max\{d(g(x_n), g(x_k)), d(g(y_n), g(y_k)), d(g(z_n), g(z_k))\} < \epsilon$$

- iii. Now, to prove it is true when  $m = k + 1$

$$\begin{aligned} d(g(x_n), g(x_{k+1})) &\leq d(g(x_n), g(x_{n+1})) + d(g(x_{n+1}), g(x_{k+1})) \\ &< \epsilon - \varphi_{(\epsilon)} + df_1(f_2(\dots (f_{n(x_n, y_n, z_n)}) \dots)), f_1(f_2(\dots (f_{n(x_k, y_k, z_k)}) \dots)) \\ &\leq \epsilon - \varphi_{(\epsilon)} + \max\{\emptyset_1 d(g(x_n), g(x_k)), \emptyset_2 d(g(y_n), g(y_k)), \emptyset_3 d(g(z_n), g(z_k)), \\ &\quad \emptyset_4 d(f_1(f_2(\dots (f_{n(x_n, y_n, z_n)}) \dots))), g(x_n)), \\ &\quad \emptyset_5 d(f_1(f_2(\dots (f_{n(z_n, y_n, x_n)}) \dots))), g(z_n)), \\ &\quad \emptyset_6 d(f_1(f_2(\dots (f_{n(x_k, y_k, z_k)}) \dots))), g(x_n)), \\ &\quad \emptyset_7 d(f_1(f_2(\dots (f_{n(z_k, y_k, x_k)}) \dots))), g(z_n))\} \\ &\leq \epsilon - \varphi_{(\epsilon)} + \max\{\varphi d(g(x_n), g(x_k)), \varphi d(g(y_n), g(y_k)), \\ &\quad \varphi d(g(z_n), g(z_k)), \varphi d(g(x_{n+1}), g(x_n)), \varphi d(g(z_{n+1}), g(z_n)), \\ &\quad \varphi d(g(x_{k+1}), g(x_k)), \varphi d(g(z_{k+1}), g(z_k))\} \\ &\leq \epsilon - \varphi_{(\epsilon)} \\ &+ \varphi \max\{d(g(x_n), g(x_k)), d(g(y_n), g(y_k)), d(g(z_n), g(z_k)), \\ d(g(x_{n+1}), g(x_n)), (g(z_{n+1}), g(z_n)), d(g(x_{k+1}), g(x_k)), \\ &\quad d(g(z_{k+1}), g(z_k))\} \\ &\leq \epsilon - \varphi_{(\epsilon)} + \varphi \max\{d(g(x_n), g(x_k)), d(g(y_n), g(y_k)), \\ &\quad d(g(z_n), g(z_k)), d(g(x_{n+1}), g(x_n)), d(g(y_{n+1}), g(y_n)), \\ &\quad d(g(z_{n+1}), g(z_n)), d(g(x_{k+1}), g(x_k)), d(g(y_{k+1}), g(y_k)), \\ &\quad d(g(z_{k+1}), g(z_k))\} \\ &< \epsilon - \varphi_{(\epsilon)} + \varphi \max\{\epsilon, \epsilon - \varphi_{(\epsilon)}\} \quad \text{by (i) and (ii)} \\ &< \epsilon - \varphi_{(\epsilon)} + \varphi_{(\epsilon)} = \epsilon \end{aligned}$$

This leads,  $d(g(x_n), g(x_{k+1})) < \epsilon$

As the same way, we get  $d(g(y_n), g(y_{k+1})) < \epsilon$

and  $d(g(z_n), g(z_{k+1})) < \epsilon$

And hence,

$$\max\{d(g(x_n), g(x_{k+1})), d(g(y_n), g(y_{k+1})), d(g(z_n), g(z_{k+1}))\} < \epsilon.$$

For all  $m \geq n$ , (iii) holds, therefor

$$< g(x_n) >, < g(y_n) > \text{ and } < g(z_n) >$$

are Cauchy sequences in  $g(X)$  such that:

- $g(x_1) \leq g(x_2) \leq \dots \leq g(x_n) \leq \dots$  and  

$$g(x_n) = f_1(f_2(\dots (f_{n(x_{n-1}, y_{n-1}, z_{n-1})) \dots))$$
- $g(y_1) \geq g(y_2) \geq \dots \geq g(y_n) \geq \dots$  and  

$$g(y_n) = f_1(f_2(\dots (f_{n(y_{n-1}, x_{n-1}, y_{n-1})) \dots))$$
- $g(z_1) \leq g(z_2) \leq \dots \leq g(z_n) \leq \dots$  and  

$$g(z_n) = f_1(f_2(\dots (f_{n(z_{n-1}, y_{n-1}, x_{n-1})) \dots))$$

But  $g(X)$  complete, then there exist  $L_1, L_2, L_3 \in g(X)$  such that

$$g(x_n) \rightarrow L_1 = g(x) \in g(X)$$

$$g(y_n) \rightarrow L_2 = g(y) \in g(X)$$

$$g(z_n) \rightarrow L_3 = g(z) \in g(X)$$

This leads,  $g(x_n) \leq g(x), g(y_n) \geq g(y), g(z_n) \leq g(z)$

Now,

$$\begin{aligned} & d(f_1(f_2(\dots (f_{n(x,y,z)}) \dots)), g(x_{n+1})) \\ &= d(f_1(f_2(\dots (f_{n(x,y,z)}) \dots)), f_1(f_2(\dots (f_{n(x_n, y_n, z_n)}) \dots))) \\ &\leq \max\{\emptyset_1 d(g(x), g(x_n)), \emptyset_2 d(g(y), g(y_n)), \emptyset_3 d(g(z), g(z_n)), \\ &\quad \emptyset_4 d(f_1(f_2(\dots (f_{n(x,y,z)}) \dots)), g(x)), \\ &\quad \emptyset_5 d(f_1(f_2(\dots (f_{n(z,y,x)}) \dots)), g(z)), \\ &\quad \emptyset_6 d(g(x_{n+1}), g(x_n)), \emptyset_7 d(g(z_{n+1}), g(z_n))\} \\ &\leq \max\{\varphi d(g(x), g(x_n)), \\ &\quad \varphi d(g(y), g(y_n)), \varphi d(g(z), g(z_n)), \\ &\quad \varphi d(f_1(f_2(\dots (f_{n(x,y,z)}) \dots)), g(x)), \\ &\quad \varphi d(f_1(f_2(\dots (f_{n(z,y,x)}) \dots)), g(z)), \\ &\quad \varphi d(f_1(f_2(\dots (f_{n(y,x,y)}) \dots)), g(y)), \\ &\quad \varphi d(g(x_{n+1}), g(x_n)), \varphi d(g(y_{n+1}), g(y_n)), \\ &\quad \varphi d(g(z_{n+1}), g(z_n))\} \end{aligned}$$

where  $\varphi = \max\{\emptyset_1, \emptyset_2, \dots, \emptyset_7\}$

$$\begin{aligned} & d(g(y_{n+1}), f_1(f_2(\dots (f_{n(y,x,y)}) \dots))) \\ &= d(f_1(f_2(\dots (f_{n(y_n, x_n, y_n)}) \dots)), f_1(f_2(\dots (f_{n(y, x, y)}) \dots))) \\ &\leq \max\{\varphi d(g(y_n), g(y)), \varphi d(g(x_n), g(x)), \varphi d(g(y_{n+1}), g(y_n)), \\ &\quad \varphi d(f_1(f_2(\dots (f_{n(y,x,y)}) \dots)), g(y))\} \end{aligned}$$

$$\begin{aligned} &\leq \max\{\varphi d(g(x), g(x_n)), \varphi d(g(y), g(y_n)), \varphi d(g(z), g(z_n)), \\ &\quad \varphi d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), g(x)), \\ &\quad \varphi d(f_1(f_2(\dots \dots (f_{n(z,y,x)}) \dots \dots)), g(z)), \\ &\quad \varphi d(f_1(f_2(\dots \dots (f_{n(y,x,y)}) \dots \dots)), g(y)), \\ &\quad \varphi d(g(x_{n+1}), g(x_n)), \varphi d(g(y_{n+1}), g(y_n)), \\ &\quad \varphi d(g(z_{n+1}), g(z_n))\} \end{aligned}$$

As the same way, we get

$$\begin{aligned} &d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), f_1(f_2(\dots \dots (f_{n(z_{n+1})}) \dots \dots))) \\ &\leq \max\{\varphi d(g(x), g(x_n)), \varphi d(g(y), g(y_n)), \varphi d(g(z), g(z_n)), \\ &\quad \varphi d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), g(x)), \\ &\quad \varphi d(f_1(f_2(\dots \dots (f_{n(z,y,x)}) \dots \dots)), g(z)), \\ &\quad \varphi d(f_1(f_2(\dots \dots (f_{n(y,x,y)}) \dots \dots)), g(y)), \\ &\quad \varphi d(g(x_{n+1}), g(x_n)), \varphi d(g(y_{n+1}), g(y_n)), \varphi d(g(z_{n+1}), g(z_n))\} \end{aligned}$$

Now,

$$\begin{aligned} &\max\{d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), g(x_{n+1})), \\ &\quad d(f_1(f_2(\dots \dots (f_{n(y,x,y)}) \dots \dots)), g(y_{n+1})), \\ &\quad d(f_1(f_2(\dots \dots (f_{n(z,y,x)}) \dots \dots)), g(z_{n+1}))\} \\ &\leq \max\{\varphi d(g(x), g(x_n)), \varphi d(g(y), g(y_n)), \varphi d(g(z), g(z_n)), \\ &\quad \varphi d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), g(x)), \varphi d(f_1(f_2(\dots \dots (f_{n(z,y,x)}) \dots \dots)), g(z)), \\ &\quad \varphi d(f_1(f_2(\dots \dots (f_{n(y,x,y)}) \dots \dots)), g(y)), \varphi d(g(x_{n+1}), g(x_n)), \\ &\quad \varphi d(g(y_{n+1}), g(y_n)), \varphi d(g(z_{n+1}), g(z_n))\} \end{aligned}$$

We claim that

$$\begin{aligned} M &= \max\{d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), g(x)), \\ &\quad d(f_1(f_2(\dots \dots (f_{n(y,x,y)}) \dots \dots)), g(y)), \\ &\quad d(f_1(f_2(\dots \dots (f_{n(z,y,x)}) \dots \dots)), g(z))\} = 0 \end{aligned}$$

Since, if not

$$\begin{aligned} &\max\{d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), g(x)), \\ &\quad d(f_1(f_2(\dots \dots (f_{n(y,x,y)}) \dots \dots)), g(y)), \\ &\quad d(f_1(f_2(\dots \dots (f_{n(z,y,x)}) \dots \dots)), g(z))\} \neq 0 \end{aligned}$$

i.e.,  $M \neq 0$ . Let  $M = \epsilon > 0$

Since,  $d(g(x), g(x_n)) \rightarrow 0, d(g(y), g(y_n)) \rightarrow 0, d(g(z), g(z_n)) \rightarrow 0$

$d(g(x_n), g(x_{n+1})) \rightarrow 0, d(g(y_n), g(y_{n+1})) \rightarrow 0$  and  $d(g(z_n), g(z_{n+1})) \rightarrow 0$

Then,

$$\begin{aligned} & \max\{\max\{d(f_1(f_2(\dots(f_{n(x,y,z)})\dots)), g(x_{n+1})), \\ & \quad d(f_1(f_2(\dots(f_{n(y,x,y)})\dots)), g(y_{n+1})), d(f_1(f_2(\dots(f_{n(z,y,x)})\dots)), g(z_{n+1}))\}, \\ & \quad \varphi d(f_1(f_2(\dots(f_{n(x,y,z)})\dots)), g(x)), \\ & \quad \varphi d(f_1(f_2(\dots(f_{n(z,y,x)})\dots)), g(z)), \\ & \quad \varphi d(f_1(f_2(\dots(f_{n(y,x,y)})\dots)), g(y))\} \end{aligned}$$

when  $n \rightarrow \infty$ , we have

$$\begin{aligned} & \max\{\max\{d(f_1(f_2(\dots(f_{n(x,y,z)})\dots)), g(x)), \\ & \quad d(f_1(f_2(\dots(f_{n(y,x,y)})\dots)), g(y)), \\ & \quad d(f_1(f_2(\dots(f_{n(z,y,x)})\dots)), g(z))\}, \\ & \quad \varphi d(f_1(f_2(\dots(f_{n(x,y,z)})\dots)), g(x)), \\ & \quad \varphi d(f_1(f_2(\dots(f_{n(z,y,x)})\dots)), g(z)), \\ & \quad \varphi d(f_1(f_2(\dots(f_{n(y,x,y)})\dots)), g(y))\} \\ & < \max\{\max\{d(f_1(f_2(\dots(f_{n(x,y,z)})\dots)), g(x)), \\ & \quad d(f_1(f_2(\dots(f_{n(z,y,x)})\dots)), g(z)), \\ & \quad d(f_1(f_2(\dots(f_{n(y,x,y)})\dots)), g(y))\} \end{aligned}$$

which is a contradiction, and hence  $M = 0$ . That is,

$$\begin{aligned} & \max\{\max\{d(f_1(f_2(\dots(f_{n(x,y,z)})\dots)), g(x)), \\ & \quad d(f_1(f_2(\dots(f_{n(y,x,y)})\dots)), g(y)), \\ & \quad d(f_1(f_2(\dots(f_{n(z,y,x)})\dots)), g(z))\}\} = 0 \end{aligned}$$

This leads

$$\begin{aligned} & d(f_1(f_2(\dots(f_{n(x,y,z)})\dots)), g(x)) = 0 \\ & d(f_1(f_2(\dots(f_{n(y,x,y)})\dots)), g(y)) = 0 \\ & d(f_1(f_2(\dots(f_{n(z,y,x)})\dots)), g(z)) = 0 \end{aligned}$$

That is,  $f_1(f_2(\dots(f_{n(x,y,z)})\dots)) = g(x)$

$$\begin{aligned} & f_1(f_2(\dots(f_{n(y,x,y)})\dots)) = g(y) \\ & f_1(f_2(\dots(f_{n(z,y,x)})\dots)) = g(z) \end{aligned}$$

Therefore,  $(x, y, z)$  is public tripled coincidence point of  $f_1, f_2, \dots, f_n$  and  $g$

### Corollary(2.2)

Let  $(X, d, \leq)$  be a partially ordered metric space. If  $f_1, f_2, \dots, f_n: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings lies in  $\mathcal{M}$  such that  $f_1, f_2, \dots, f_n$  having public mixed  $g$ -monotone property. Suppose that, for all  $x, y, z, u, v, w \in X$  with  $x \geq u, y \leq v$  and  $z \geq w$ ,

$d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), f_1(f_2(\dots \dots (f_{n(u,v,w)}) \dots \dots))) \leq$   
 $\max\{k_1d(g(x), g(u)), k_2d(g(y), g(v)), k_3d(g(z), g(w)),$   
 $k_4d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), g(x)), k_5d(f_1(f_2(\dots \dots (f_{n(z,y,x)}) \dots \dots)), g(z)),$   
 $k_6d(f_1(f_2(\dots \dots (f_{n(u,v,w)}) \dots \dots)), g(u)), k_7d(f_1(f_2(\dots \dots (f_{n(w,v,u)}) \dots \dots)), g(w))\}$   
 where  $k_1, k_2, \dots, k_7 \in [0,1]$  and  $\sum_{i=1}^7 k_i < 1$ . If there exist  $x^\circ, y^\circ, z^\circ \in X$ , such that  
 $g(x^\circ) \leq f_1(f_2(\dots \dots (f_{n(x^\circ,y^\circ,z^\circ)}) \dots \dots))$   
 $g(y^\circ) \geq f_1(f_2(\dots \dots (f_{n(y^\circ,x^\circ,y^\circ)}) \dots \dots))$  and  
 $g(z^\circ) \leq f_1(f_2(\dots \dots (f_{n(z^\circ,y^\circ,x^\circ)}) \dots \dots))$   
 Then,  $f_1, f_2, \dots, f_n$  and  $g$  having a public tripled coincidence point

### Corollary(2.3)

Let  $(X, d, \leq)$  be a partially ordered metric space. If  $f_1, f_2, \dots, f_n: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings lies in  $\mathcal{M}$  such that  $f_1, f_2, \dots, f_n$  are public mixed  $g_-$  monotone. Suppose that, for all  $x, y, z, u, v, w \in X$  with  $x \geq u, y \leq v$  and  $z \geq w$ ,

$d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), f_1(f_2(\dots \dots (f_{n(u,v,w)}) \dots \dots))) \leq$   
 $\max\{\emptyset\{d(g(x), g(u)), d(g(y), g(v)), d(g(z), g(w)), d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), g(x)),$   
 $d(f_1(f_2(\dots \dots (f_{n(z,y,x)}) \dots \dots)), g(z)), d(f_1(f_2(\dots \dots (f_{n(u,v,w)}) \dots \dots)), g(u)),$   
 $d(f_1(f_2(\dots \dots (f_{n(w,v,u)}) \dots \dots)), g(w))\}$   
 where  $\emptyset \in \Phi$ . If there exist  $x^\circ, y^\circ, z^\circ \in X$  such that  
 $g(x^\circ) \leq f_1(f_2(\dots \dots (f_{n(x^\circ,y^\circ,z^\circ)}) \dots \dots)), g(y^\circ) \geq f_1(f_2(\dots \dots (f_{n(y^\circ,x^\circ,y^\circ)}) \dots \dots))$  and  
 $g(z^\circ) \leq f_1(f_2(\dots \dots (f_{n(z^\circ,y^\circ,x^\circ)}) \dots \dots))$ . Then,  $f_1, f_2, \dots, f_n$  and  $g$  having a public tripled coincidence point

### Corollary(2.4)

Let  $(X, d, \leq)$  be a partially ordered metric space. If  $f_1, f_2, \dots, f_n: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings lies in  $\mathcal{M}$  such that  $f_1, f_2, \dots, f_n$  having modified  $g$ \_mixed monotone. Suppose that, for all  $x, y, z, u, v, w \in X$  with  $x \geq u, y \leq v$  and  $z \geq w$ ,

$d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), f_1(f_2(\dots \dots (f_{n(u,v,w)}) \dots \dots))) \leq$   
 $k \max\{d(g(x), g(u)), d(g(y), g(v)), d(g(z), g(w)),$   
 $d(f_1(f_2(\dots \dots (f_{n(x,y,z)}) \dots \dots)), g(x)), d(f_1(f_2(\dots \dots (f_{n(z,y,x)}) \dots \dots)), g(z))$   
 $d(f_1(f_2(\dots \dots (f_{n(u,v,w)}) \dots \dots)), g(u)), d(f_1(f_2(\dots \dots (f_{n(w,v,u)}) \dots \dots)), g(w))\}$   
 where  $k \in [0,1]$

If there exist  $x^\circ, y^\circ, z^\circ \in X$  such that

$g(x^\circ) \leq f_1(f_2(\dots \dots (f_{n(x^\circ,y^\circ,z^\circ)}) \dots \dots)), g(y^\circ) \geq f_1(f_2(\dots \dots (f_{n(y^\circ,x^\circ,y^\circ)}) \dots \dots))$  and  
 $g(z^\circ) \leq f_1(f_2(\dots \dots (f_{n(z^\circ,y^\circ,x^\circ)}) \dots \dots))$

Then,  $f_1, f_2, \dots, f_n$  and  $g$  having a public tripled coincidence point

### Corollary(2.5)

Let  $(X, d, \leq)$  be a partially ordered metric space. If  $f_1, f_2, \dots, f_n: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings lies in  $\mathcal{M}$  such that  $f_1, f_2, \dots, f_n$  having modified  $g$ -mixed monotone of. Suppose that, for all  $x, y, z, u, v, w \in X$  with  $x \geq u, y \leq v$  and  $z \geq w$ ,

$$\begin{aligned} & d(f_1(f_2(\dots (f_{n(x,y,z)} \dots \dots)), f_1(f_2(\dots (f_{n(u,v,w)} \dots \dots)) \dots \dots)) \leq \\ & k_1 d(g(x), g(u)) + k_2 d(g(y), g(v)) + k_3 d(g_1(g_2(\dots (g_{n(z)} \dots \dots))), g(w)) + \\ & k_4 d(f_1(f_2(\dots (f_{n(x,y,z)} \dots \dots)), g(x)) + k_5 d(f_1(f_2(\dots (f_{n(z,y,x)} \dots \dots)), g(z)) \\ & + k_6 d(f_1(f_2(\dots (f_{n(u,v,w)} \dots \dots)), g(u)) \\ & + k_7 d(f_1(f_2(\dots (f_{n(w,v,u)} \dots \dots)), g(w)) \end{aligned}$$

where  $k_1, k_2, \dots, k_7 \in [0,1]$  and  $\sum_{i=1}^7 k_i < 1$

If there exist  $x^\circ, y^\circ, z^\circ \in X$  such that

$$\begin{aligned} g(x^\circ) &\leq f_1(f_2(\dots (f_{n(x^\circ, y^\circ, z^\circ)} \dots \dots)), g(y^\circ) \geq f_1(f_2(\dots (f_{n(y^\circ, x^\circ, y^\circ)} \dots \dots)) \text{ and} \\ g(z^\circ) &\leq f_1(f_2(\dots (f_{n(z^\circ, y^\circ, x^\circ)} \dots \dots)) \end{aligned}$$

Then,  $f_1, f_2, \dots, f_n$  and  $g$  having a public tripled coincidence point

Also, you can get others results :

if  $g(x) = x$  and  $f_1, f_2, \dots, f_n$  having public mixed monotone in above results, then we get the public tripled fixed point theorems.

### Corollary(2.6)

Let  $(X, d, \leq)$  be a partially ordered metric space. If  $f: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings lies in  $\mathcal{M}$  such that  $f$  having mixed  $g$ -monotone property. Suppose that, for all  $x, y, z, u, v, w \in X$  with  $x \geq u, y \leq v$  and  $z \geq w$ ,

$$\begin{aligned} & d(f(x, y, z), f(u, v, w)) \leq \\ & \max\{\emptyset_1 d(g(x), g(u)), \emptyset_2 d(g(y), g(v)), \emptyset_3 d(g(z), g(w)), \emptyset_4 d(f(x, y, z), g(x)), \\ & \emptyset_5 d(f(z, y, x), g(z)), \emptyset_6 d(f(u, v, w), g(u)), \emptyset_7 d(f(w, v, u), g(w))\} \end{aligned}$$

where  $\emptyset_1, \emptyset_2, \dots, \emptyset_7 \in \Phi$

If there exist  $x^\circ, y^\circ, z^\circ \in X$  such that

$$x_0 \leq f(x_0, y_0, z_0), y_0 \geq f(y_0, x_0, y_0) \text{ and}$$

Then, there exists a tripled coincidence fixed point of  $f$  and  $g$

### Corollary(2.7)

Let  $(X, d, \leq)$  be a partially ordered metric space. If  $f: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings lies in  $\mathcal{M}$  such that  $f$  has mixed  $g$ -monotone property. Suppose that, for all  $x, y, z, u, v, w \in X$  with  $x \geq u, y \leq v$  and  $z \geq w$ ,

$$d(f(x, y, z), f(u, v, w)) \leq$$

$$\max\{k_1 d(g(x), g(u)), k_2 d(g(y), g(v)), k_3 d(g(z), g(w)), k_4 d(f(x, y, z), g(x)),$$

$k_5 d(f(z, y, x), g(z)), k_6 d(f(u, v, w), g(u)), k_7 d(f(w, v, u), g(w))\}$

where  $k_1, k_2, \dots, k_7 \in [0,1]$  and  $\sum_{i=1}^7 k_i < 1$

If there exist  $x_0, y_0, z_0 \in X$  such that

$x_0 \leq f(x_0, y_0, z_0), y_0 \geq f(y_0, x_0, y_0)$  and  $z_0 \leq f(z_0, y_0, x_0)$

Then, there exists a tripled coincidence fixed point of  $f$  and  $g$ .

### Corollary(2.8)

Let  $(X, d, \leq)$  be a partially ordered metric space. If  $f: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings lies in  $\mathcal{M}$  such that  $f$  has mixed  $g_-$  monotone property. Suppose that, for all  $x, y, z, u, v, w \in X$  with  $x \geq u, y \leq v$  and  $z \geq w$ ,

$$\begin{aligned} d(f(x, y, z), f(u, v, w)) \\ \leq \max\{d(g(x), g(u)), d(g(y), g(v)), d(g(z), g(w)), d(f(z, y, x), g(z)), \\ d(f(x, y, z), g(x)), d(f(u, v, w), g(u)), d(f(w, v, u), g(w))\}, \text{where } \emptyset \in \Phi \end{aligned}$$

If there exist  $x_0, y_0, z_0 \in X$  such that

$x_0 \leq f(x_0, y_0, z_0), y_0 \geq f(y_0, x_0, y_0)$  and  $z_0 \leq f(z_0, y_0, x_0)$ .

Then, there exists a tripled coincidence fixed point of  $f$  and  $g$ .

### Corollary(2.9)

Let  $(X, d, \leq)$  be a partially ordered metric space. If  $f: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings lies in  $\mathcal{M}$  such that  $f$  has mixed  $g_-$  monotone property. Suppose that, for all  $x, y, z, u, v, w \in X$  with  $x \geq u, y \leq v$  and  $z \geq w$ ,

$$\begin{aligned} d(f(x, y, z), f(u, v, w)) \leq \\ k_1 d(g(x), g(u)) + k_2 d(g(y), g(v)) + k_3 d(g(z), g(w)) + k_4 d(f(x, y, z), g(x)) + \\ k_5 d(f(z, y, x), g(z)), k_6 d(f(u, v, w), g(u)) + k_7 d(f(w, v, u), g(w))), \end{aligned}$$

where  $k \in [0,1]$ . If there exist  $x_0, y_0, z_0 \in X$  such that

$x_0 \leq f(x_0, y_0, z_0), y_0 \geq f(y_0, x_0, y_0)$  and  $z_0 \leq f(z_0, y_0, x_0)$

Then, there exists a tripled coincidence fixed point of  $f$  and  $g$ .

### Corollary(2.10)

Let  $(X, d, \leq)$  be a partially ordered metric space. If  $f: X^3 \rightarrow X$  and  $g: X \rightarrow X$  are mappings lies in  $\mathcal{M}$  such that  $f$  has mixed  $g$ -monotone property. Suppose that, for all  $x, y, z, u, v, w \in X$  with  $x \geq u, y \leq v$  and  $z \geq w$ ,

$$\begin{aligned} d(f(x, y, z), f(u, v, w)) \leq \\ k (\max\{d(g(x), g(u)), d(g(y), g(v)), d(g(z), g(w)), d(f(x, y, z), g(x)), \\ d(f(u, v, w), g(u)), d(f(w, v, u), g(w))\}), \text{where } k \in [0,1] \end{aligned}$$

If there exist  $x_0, y_0, z_0 \in X$  such that

$x_0 \leq f(x_0, y_0, z_0), y_0 \geq f(y_0, x_0, y_0)$  and  $z_0 \leq f(z_0, y_0, x_0)$ .

Then, there exists a tripled coincidence fixed point of  $f$  and  $g$ .

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