# Operations on Intuitionistic Fuzzy Graphs

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## Abstract

The definition of operations "∪, + and Cartesian product" have been modified for Intuitionistic Fuzzy Graphs with one or more common vertices and their complement graphs are studied.

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**Keywords:** Complement intuitionistic fuzzy graphs, operations on IFG

## 1. Introduction

Euler (1736) introduced the concept of graph theory. Atanassov (1983) introduced the concept of intuitionistic fuzzy set. In his work, he defined a new concept, the degree of non membership in addition to the degree of membership as intuitionistic fuzzy sets, such that their sum is less than or equal to one. Rosenfeld (1975) introduced the concept of fuzzy graph. He developed the structure of fuzzy graphs and obtained several graph theoretical concepts. Later Bhattacharya gave some remarks on fuzzy graphs. Some operations on fuzzy graphs were introduced by Mordeson and Peng (1994). The complement of a fuzzy graph was defined by Mordeson and further studied by M.S. Sunitha and A. Vijaya Kumar (2002). Parvathi et. al (2009) gave a new definition for operations on intuitionistic fuzzy graphs.

In this article, we redefine the operations union, sum and cartesian product of intuitionistic fuzzy graph. Throughout this paper we consider the underlying set V as the finite set. Also the membership and non-membership of vertex set and edge set are chosen so as to satisfy the definitions. Also the operations on intuitionistic fuzzy graphs are considered for two intuitionistic fuzzy graphs with one or more vertices in common.

## 2. Preliminary Definitions:

**Definition: 2.1** [3]

A Minmax Intuitionistic fuzzy graph (IFG) is of the form G:(V, E) where

(i) $V = \{\nu_1, \nu_2, \ldots \nu_n\}$ such that $\mu_1 : V \to [0,1]$ and $\gamma : V \to [0,1]$ denotes the degree of membership and non membership of the elements $\nu_i \in V$ respectively and $0 \le \mu_1(\nu_i)+\gamma_1(\nu_i) \le 1$ for every $\nu_i \in V$; ( i = 1,2…..n).

(ii) $E \subset VxV$ where $\mu_2 : VxV \to [0,1]$ and $\gamma_2 : VxV \to [0,1]$ are such that

$$\mu_2 (\nu_i, \nu_j) \le \min\{\mu_1(\nu_i), \mu_1(\nu_j)\}$$

$$\gamma_2 (\nu_i, \nu_j) \le \max\{\gamma_1(\nu_i), \gamma_1(\nu_j)\}$$

and $0 \le \mu_2(\nu_i,\nu_j) + \gamma_2 (\nu_i, \nu_j) \le 1$ for every $(\nu_i, \nu_j) \in E$

Here the triple $(\nu_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and non membership of the vertex $\nu_i$. The triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and degree of non-membership of the edge relation $e_{ij} = (\nu_i, \nu_j)$ on V.

**Definition:2.2** [3]

Degree of the vertex $d(\nu_i) = \left\{\sum_{\nu_i,\nu_j \in E} \mu_2(\nu_i, \nu_j)\ ,\ \sum_{\nu_i,\nu_j \in E} \gamma_2(\nu_i, \nu_j)\right\}$ and $\mu_2(\nu_i,\nu_j) = \gamma_2(\nu_i,\nu_j) = 0$ for $\nu_i,\nu_j \notin E$.

**Definition:2.3** [3]

The min μ degree is $\delta_\mu(G) = \wedge\{d_\mu(\nu_i) / \nu_i \in V\}$

The max μ degree is $\Delta_\mu(G) = \vee\{d_\mu(\nu_i) / \nu_i \in V\}$

The min γ degree is $\delta_\gamma(G) = \wedge\{d_\gamma(\nu_i) / \nu_i \in V\}$

The max γ degree is $\Delta_\gamma(G) = \vee\{d_\gamma(\nu_i) / \nu_i \in V\}$

The max degree of G is $\Delta(G) = \vee\{d_\mu(\nu_i), d_\gamma(\nu_i) / \nu_i \in V\}$

**Definition:2.4** [5]

The complement of IFG $\overline{G}:(\overline{A}, \overline{B})$ of an IFG G:(A, B) is defined by

(i) $\overline{\mu}_A(x) = \mu_A(x)$; $\overline{\nu}_A(x) = \nu_A(x)\ \ \forall\ x \in A$

(ii) $\overline{\mu}_B(xy)= \begin{cases} [\mu_A(x) \wedge \mu_A(y)] - \mu_B(xy) & \forall xy \in B \\ [\mu_A(x) \wedge \mu_A(y)] & \forall xy \notin B \end{cases}$

$\overline{\nu}_B(xy)= \begin{cases} [\nu_A(x) \vee \nu_A(y)] - \nu_B(xy) & \forall xy \in B \\ [\nu_A(x) \vee \nu_A(y)] & \forall xy \notin B \end{cases}$

**Example:2.5** [5]

Consider the graph G with V={1,2,3,4,5} and E={12,13,23,24,35,45}. Let A be the intuitionistic fuzzy set on V and B be the intuitionistic fuzzy set on E.

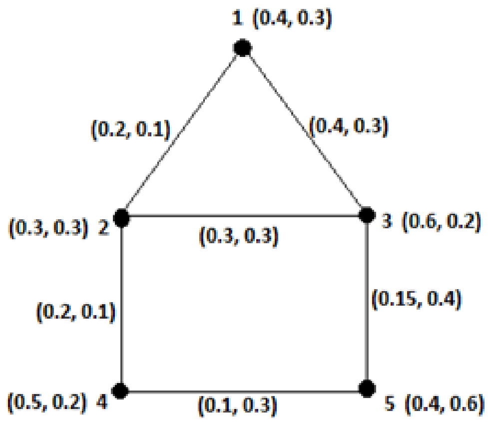


Fig. 1 G:(A, B)

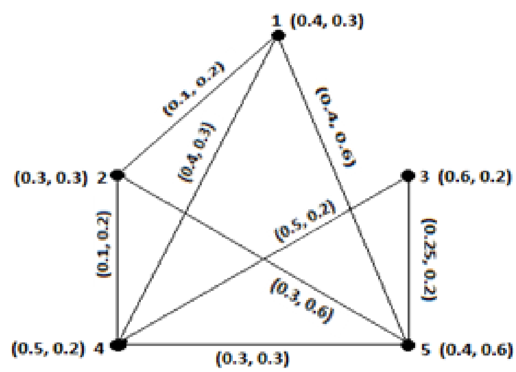


Fig. 2 $\overline{G}:(\overline{A}, \overline{B})$

By routine computation of $\overline{G}:(\overline{A}, \overline{B})$, it can be observed that $\overline{\overline{G}}:(\overline{\overline{A}}, \overline{\overline{B}})$ = G :(A, B ).

**Definition:2.4** [6]

A IFG is called strong IFG if $\mu_B(xy) = \mu_A(x) \wedge \mu_A(y)$ and $\nu_B(xy) = \nu_A(x) \vee \nu_A(y)$.

**Definition:2.4** [9]

An intuitionistic fuzzy graph is self-complementary if G is isomorphic to $\overline{G}$. (i.e.) $G \cong \overline{G}$.

## 3. Operations on Intuitionistic Fuzzy Graphs:

In this section, the operations Union, Sum and Cartesian product on IFG are redefined. In [6] and [9] the authors have analyzed on the operations of IFG with non- intersecting vertex sets of two graphs $G_1$ and $G_2$ [$\nu_1 \cap \nu_2 = \phi$]. In this section, we define the operations "$\cup$", "+" and Cartesian product between two IFGs $G_1:(A_1, B_1)$ and $G_2:(A_2, B_2)$ which has one or more vertices in common between $G_1$ and $G_2$. The operations "$\cup$" and "+" are found to be complementary to each other and are substantiated with relevant examples. Also it has been shown that Cartesian product is distributive over union.

**Definition: 3.1**

Let $G_1$:($A_1$, $B_1$) and $G_2$:($A_2$, $B_2$) be two IFGs with one or more vertices in common. Then the union of $G_1$ and $G_2$ is another IFG G:(A, B)=$G_1 \cup G_2$ defined by,

(i) $\mu_A(x) = \begin{cases} \mu_{1A}(x) & \forall\ x \in A_1 \\ \mu_{2A}(x) & \forall\ x \in A_2 \end{cases}$ and $\nu_A(x) = \begin{cases} \nu_{1A}(x) & \forall\ x \in A_1 \\ \nu_{2A}(x) & \forall\ x \in A_2 \end{cases}$

(ii) $\mu_B(xy) = \begin{cases} \mu_{1B}(xy) & \forall\ xy \in B_1 \\ \mu_{2B}(xy) & \forall\ xy \in B_2 \end{cases}$ and $\nu_B(xy) = \begin{cases} \nu_{1B}(xy) & \forall\ xy \in B_1 \\ \nu_{2B}(xy) & \forall\ xy \in B_2 \end{cases}$

**Definition: 3.2**

Let $G_1$:($A_1$, $B_1$) and $G_2$:($A_2$, $B_2$) be two IFGs with one or more vertices in common. The sum $G_1 + G_2$ is another IFG G:(A, B) defined by,

(i) $\mu_A(x) = \begin{cases} \mu_{1A}(x) & \forall\ x \in A_1 \\ \mu_{2A}(x) & \forall\ x \in A_2 \end{cases}$ and $\nu_A(x) = \begin{cases} \nu_{1A}(x) & \forall\ x \in A_1 \\ \nu_{2A}(x) & \forall\ x \in A_2 \end{cases}$

(ii) $\mu_B(xy) = \begin{cases} \mu_{1B}(xy) & \forall\ xy \in B_1 \\ \mu_{2B}(xy) & \forall\ xy \in B_2 \end{cases}$ and $\nu_B(xy) = \begin{cases} \nu_{1B}(xy) & \forall\ xy \in B_1 \\ \nu_{2B}(xy) & \forall\ xy \in B_2 \end{cases}$

(iii) There exists a strong edge between every pair of non-common vertices in $G_1$ and $G_2$.

**Remark:** The condition (iii) in above definition can be explained as follows:

If G = {$\nu_1$, $\nu_2$, $\nu_3$, $\nu_4$} and $G_2$ = {$\nu_3$, $\nu_4$, $\nu_5$, $\nu_6$}, the vertices $\nu_3$ and $\nu_4$ are common in $G_1$ and $G_2$. Hence there exist edges between vertex pairs ($\nu_1$, $\nu_5$), ($\nu_1$, $\nu_6$), ($\nu_2$, $\nu_5$) and ($\nu_2$, $\nu_6$). These edges are strong implies their membership is the minimum of memberships of their adjacent vertices and non-membership is the maximum of non-membership of their adjacent vertices.

**Example:3.3**

Consider two IFGs $G_1$:($A_1$, $B_1$) and $G_2$:($A_2$, $B_2$) with corresponding vertex and edge sets $A_1$ = {1, 2, 3}, $A_2$ = {1, 2, 4, 5}, $B_1$ = {12, 13, 23} and $B_2$ = {12, 14, 25, 45}.

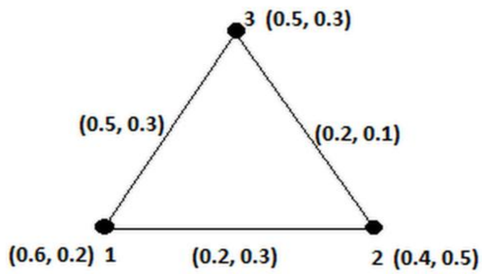


fig:1 $G_1:(A_1, B_1)$

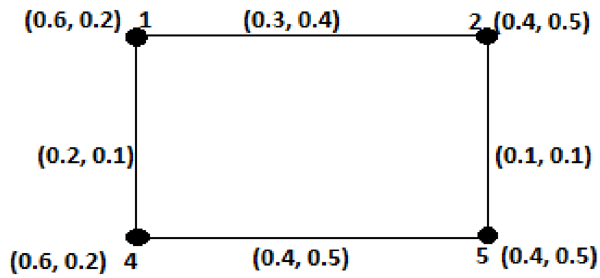


fig:2 $G_2:(A_2, B_2)$

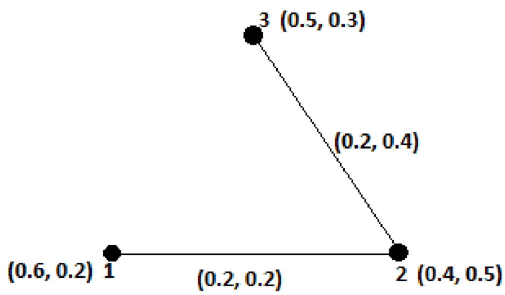


Fig. 3 $\overline{G}_1:(\overline{A}_1, \overline{B}_1)$

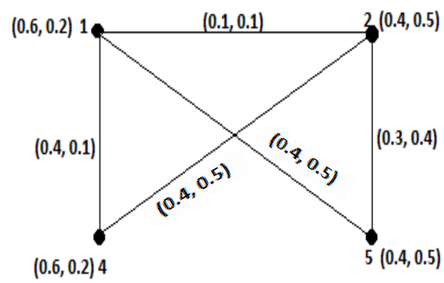


Fig. 4 $\overline{G}_2:(\overline{A}_2, \overline{B}_2)$

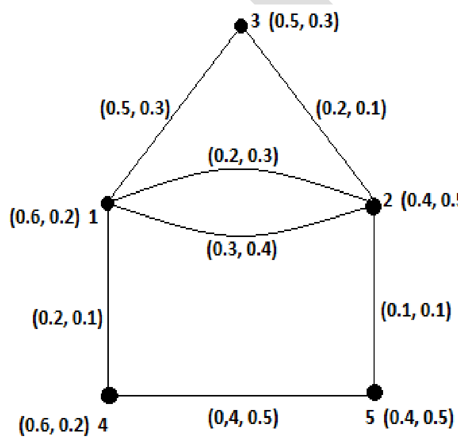


Fig. 5 $G_1 \cup G_2$

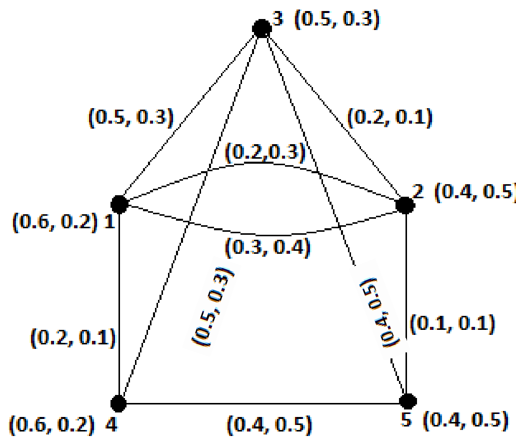


Fig. 6 $G_1 + G_2$

Issue 7 volume 3 May-June2017
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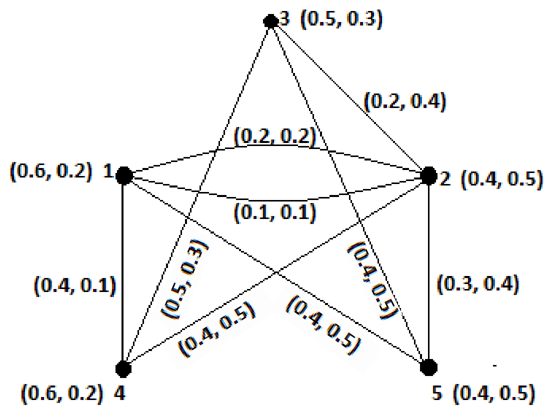


Fig. 7 $\overline{G_1 \cup G_2}$

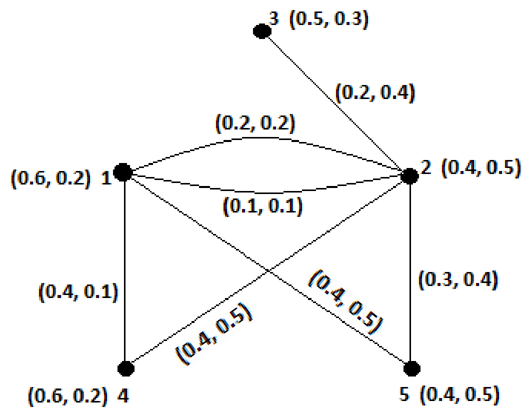


Fig. 8 $\overline{G_1 + G_2}$

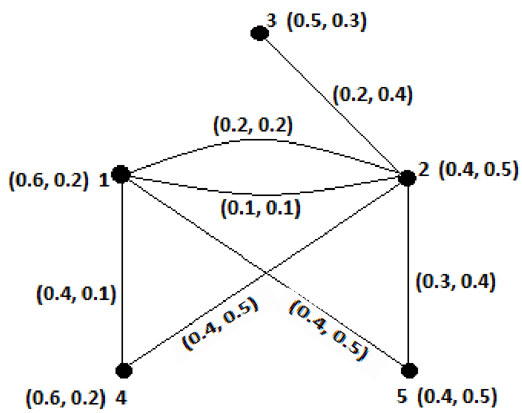


Fig. 9 $\overline{G}_1 \cup \overline{G}_2$

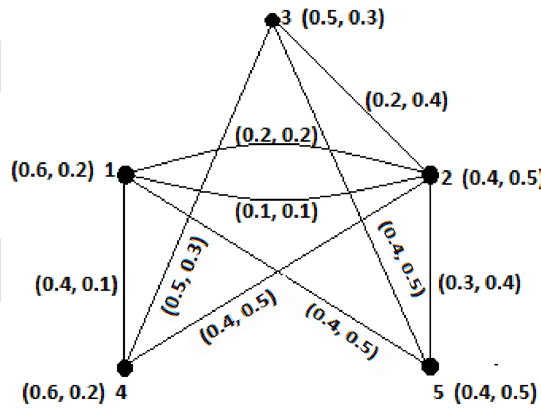


Fig. 10 $\overline{G}_1 + \overline{G}_2$

From the Fig. 7 and Fig. 10, $\overline{G_1 \cup G_2} = \overline{G}_1 + \overline{G}_2$ and from Fig: 8 and Fig. 9, $\overline{G_1 + G_2} = \overline{G}_1 \cup \overline{G}_2$. This result holds true for all types of IFGs. Hence "$\cup$" and "+" are complementary to each other according to our definition for union and sum of IFGs with one or more common vertices. As a special case of the above statement, if both the IFGs have the same set of vertices, then the following theorem holds true.

**Theorem: 3.4**

If $G_1:(A_1, B_1)$ and $G_2:(A_2, B_2)$ are two IFGs, then $G_1 \cup G_2 = G_1 + G_2$ if and only if $A_1 \subseteq A_2$ or $A_2 \subseteq A_1$.

**Proof:** Let $G_1 \cup G_2 = G_1 + G_2$ then, condition (iii) of definition: 4.16 is zero.

 

$\Rightarrow$ There doesn't exist any non-common vertex between $G_1$ and $G_2$. Hence either of the vertex set should be the subset of the other. $\therefore$ $A_1 \subseteq A_2$ or $A_2 \subseteq A_1$.

Conversely, if $A_1 \subseteq A_2$ or $A_2 \subseteq A_1$ then from definition: 4.15 and definition: 4.16 it is obvious that $G_1 \cup G_2 = G_1 + G_2$.

**Theorem: 3.5**

Let $G_1$:($A_1$, $B_1$) and $G_2$:($A_2$, $B_2$) be two IFGs with one or more common vertices in $G_1$ and $G_2$ then (a) $\overline{G_1 + G_2} \cong \overline{G}_1 \cup \overline{G}_2$

(b) $\overline{G_1 \cup G_2} \cong \overline{G}_1 + \overline{G}_2$.

**Proof:** Consider the identity map I: $V_1 \cup V_2 \rightarrow V_1 \cup V_2$.

To prove (a) we have to show that,

(1) (i) $(\overline{\mu_{1A} + \mu_{2A}})(x) = (\overline{\mu}_{1A} \cup \overline{\mu}_{2A})(x)$ (ii) $(\overline{\nu_{1A} + \nu_{2A}})(x) = (\overline{\nu}_{1A} \cup \overline{\nu}_{2A})(x)$

(2) (i) $(\overline{\mu_{1B} + \mu_{2B}})(xy) = (\overline{\mu}_{1B} \cup \overline{\mu}_{2B})(xy)$ (ii) $(\overline{\nu_{1B} + \nu_{2B}})(xy) = (\overline{\nu}_{1B} \cup \overline{\nu}_{2B})(xy)$

(1), (i): consider $(\overline{\mu_{1A} + \mu_{2A}})(x) = (\mu_{1A} + \mu_{2A})(x)$ [ by definition of complement]

$$= \begin{cases} \mu_{1A}(x) & \forall\ x \in A_1 \\ \mu_{2A}(x) & \forall\ x \in A_2 \end{cases}$$

$$= \mu_{1A}(x) \cup \mu_{2A}(x)$$

$$= \overline{\mu}_{1A}(x) \cup \overline{\mu}_{2A}(x)$$

$$= (\overline{\mu}_{1A} \cup \overline{\mu}_{2A})(x).$$

(ii): Similar to above proof it can be shown that $(\overline{\nu_{1A} + \nu_{2A}})(x) = (\overline{\nu}_{1A} \cup \overline{\nu}_{2A})(x)$.

(2), (i): Consider

$$(\overline{\mu_{1B} + \mu_{2B}})(xy) = \begin{cases} (\mu_{1A} + \mu_{2A})(x) \wedge (\mu_{1A} + \mu_{2A})(y) - (\mu_{1B} + \mu_{2B})(xy) & ; xy \in B \\ (\mu_{1A} + \mu_{2A})(x) \wedge (\mu_{1A} + \mu_{2A})(y) & ; xy \notin B \end{cases}$$

$$= \begin{cases} [\mu_{1A}(x) \cup \mu_{2A}(x)] \wedge [\mu_{1A}(y) \cup \mu_{2A}(y)] - (\mu_{1B} + \mu_{2B})(xy) & ; xy \in B \\ [\mu_{1A}(x) \cup \mu_{2A}(x)] \wedge [\mu_{1A}(y) \cup \mu_{2A}(y)] & ; xy \notin B \end{cases}$$

$$= \begin{cases} [\mu_{1A}(x) \cup \mu_{2A}(x)] \wedge [\mu_{1A}(y) \cup \mu_{2A}(y)] - \mu_{1B}(xy) & ; xy \in B_1 \\ [\mu_{1A}(x) \cup \mu_{2A}(x)] \wedge [\mu_{1A}(y) \cup \mu_{2A}(y)] - \mu_{2B}(xy) & ; xy \in B_2 \\ [\mu_{1A}(x) \cup \mu_{2A}(x)] \wedge [\mu_{1A}(y) \cup \mu_{2A}(y)] - [\mu_{1A}(x) \wedge \mu_{2A}(y)] & ; x \in A_1 \text{ and } y \in A_2 \\ [\mu_{1A}(x) \cup \mu_{2A}(x)] \wedge [\mu_{1A}(y) \cup \mu_{2A}(y)] & ; xy \notin B \end{cases}$$

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$$= \begin{cases} [\mu_{1A}(x) \wedge \mu_{1A}(y)] - \mu_{1B}(xy) & ;xy \in B_1 \\ [\mu_{2A}(x) \wedge \mu_{2A}(y)] - \mu_{2B}(xy) & ;xy \in B_2 \\ [\mu_{1A}(x) \wedge \mu_{2A}(y)] - [\mu_{1A}(x) \wedge \mu_{2A}(y)] & ;x \in A_1 \text{ and } y \in A_2 \\ [\mu_{1A}(x) \wedge \mu_{1A}(y)] & ;xy \notin B_1 \\ [\mu_{2A}(x) \wedge \mu_{2A}(y) & ;xy \notin B_2 \end{cases}$$

$$= \begin{cases} [\mu_{1A}(x) \wedge \mu_{1A}(y)] - \mu_{1B}(xy) & ;xy \in B_1 \\ [\mu_{1A}(x) \wedge \mu_{1A}(y)] & ;xy \notin B_1 \\ [\mu_{2A}(x) \wedge \mu_{2A}(y)] - \mu_{2B}(xy) & ;xy \in B_2 \\ [\mu_{2A}(x) \wedge \mu_{2A}(y) & ;xy \notin B_2 \end{cases}$$

$$= \begin{cases} \overline{\mu}_{1B}(xy) & ; xy \in B_1 \\ \overline{\mu}_{2B}(xy) & ; xy \in B_2 \\ 0 & ; xy \notin B_1 \text{ and } B_2 \end{cases}$$

$$= (\overline{\mu}_{1B} \cup \overline{\mu}_{2B})(xy).$$

(ii): $(\overline{\nu_{1B} + \nu_{2B}})(xy) = (\overline{\nu}_{1B} \cup \overline{\nu}_{2B})(xy)$ can be proved similar to (2), (i).

**Definition: 3.6**

Let $G_1$: $(V_1, E_1)$ and $G_2$: $(V_2, E_2)$ be two intuitionistic fuzzy graphs then their Cartesian product $G_1 \times G_2$ is an intuitionistic fuzzy graph G:(V, E) with

$V = \{(x_y) \mid \text{for all } x \in V_1 \text{ and } y \in V_2\}$ and

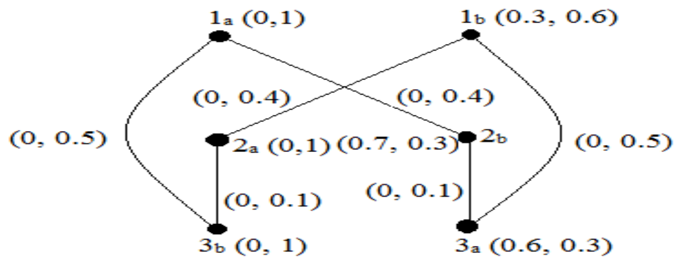
$E = \{(xu, yv), (xv, yu) \mid \text{for all } xy \in E_1 \text{ and } uv \in E_2\}$ where

i. $\mu_A(x_y) = \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x_y) = \max\{\nu_A(x), \nu_A(y)\}$

ii. $\mu_B(xu, yv) = \mu_B(xv, yu) = \min\{\mu_B(xy), \mu_B(uv)\}$ and
$\nu_B(xu, yv) = \nu_B(xv, yu) = \max\{\nu_B(xy), \nu_B(uv)\}$.

**Example: 3.7**


$G_1:(V_1, E_1)$ $G_2:(V_2, E_2)$




$G_1 \times G_2$

**Proposition: 3.8**

Cartesian product is distributive over union. [i.e. $(G_1 \cup G_2)\times G_3 = (G_1\times G_3)\cup(G_2\times G_3)$ ].

**Proof**

i. Let $x_y \in (G_1\cup G_2)\times G_3$

$\Rightarrow x\in G_1\cup G_2$ and $y\in G_3$

$\Rightarrow x\in G_1$ or $G_2$ and $y\in G_3$

$\Rightarrow x_y\in G_1\times G_3$ or $x_y\in G_2\times G_3$

$\therefore\ x_y\in(G_1\times G_3)\cup(G_2\times G_3)$

If $x_y\in(G_1\times G_3)\cup(G_2\times G_3) \Rightarrow x_y\in(G_1\times G_3)$ or $x_y\in(G_2\times G_3)$

$\Rightarrow x\in G_1$ and $y\in G_3$ or $x\in G_2$ and $y\in G_3$

$\therefore\ x\in G_1$ or $G_2$ and $y\in G_3$

$\therefore\ x_y\in(G_1\cup G_2)\times G_3$

$\therefore (G_1\cup G_2)\times G_3= (G_1\times G_3)\cup(G_2\times G_3)$

ii. Let $(xu, yv)\in (G_1\cup G_2)\times G_3 \Rightarrow xy\in G_1\cup G_2$ and $uv\in G_3$

$\Rightarrow xy\in G_1$ or $xy\in G_2$ and $uv\in G_3$

$\therefore (xu, yv)\in G_1\times G_3$ or $(xu, yv)\in G_2\times G_3$

$\therefore (xu, yv)\in(G_1\times G_3)\cup(G_2\times G_3)$

If $(xu, yv)\in(G_1\times G_3)\cup(G_2\times G_3)$

$(xu, yv)\in G_1\times G_3$ or $(xu, yv)\in G_2\times G_3$

$\Rightarrow xy\in G_1$ and $uv\in G_3$ or $xy\in G_2$ and $uv\in G_3$

$\therefore\ xy\in(G_1\cup G_2)$ and $uv\in G_3$

$\therefore (xu, yv)\in (G_1\cup G_2)\times G_3$

$\therefore (G_1\cup G_2)\times G_3= (G_1\times G_3)\cup(G_2\times G_3)$.

 Issue 7 volume 3 May-June2017
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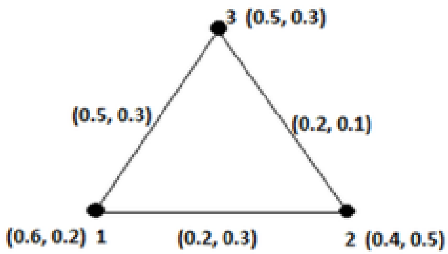
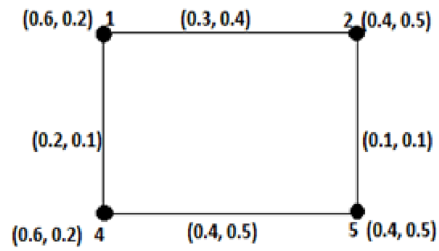
**Example: 3.9**

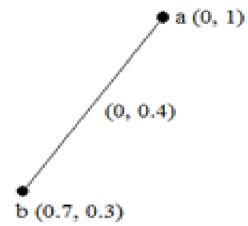
Consider the following intuitionistic fuzzy graphs $G_1$, $G_2$ and $G_3$ given below.



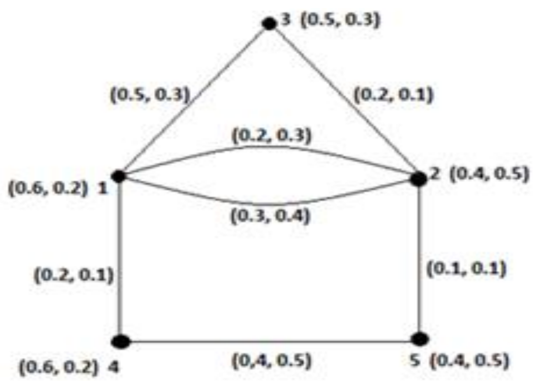

$G_1$: ($V_1$, $E_1$)

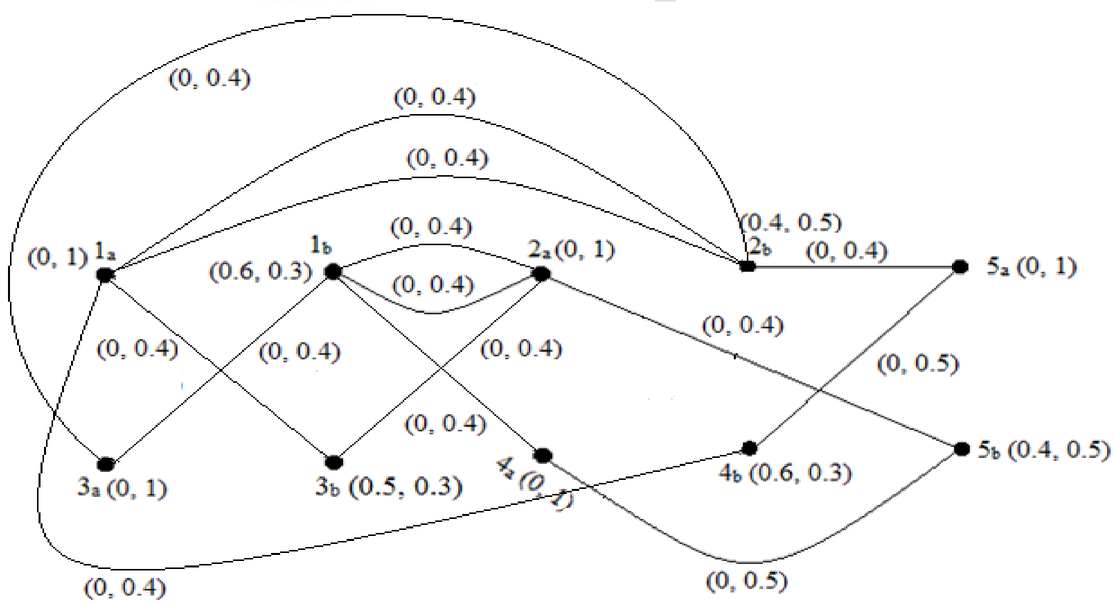



$G_2$: ($V_2$, $E_2$)

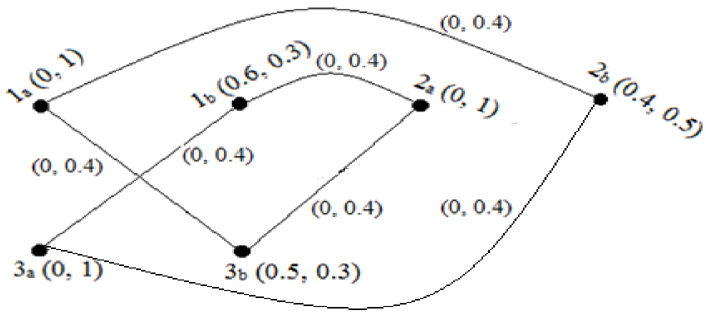



$G_3$: ($V_3$, $E_3$)

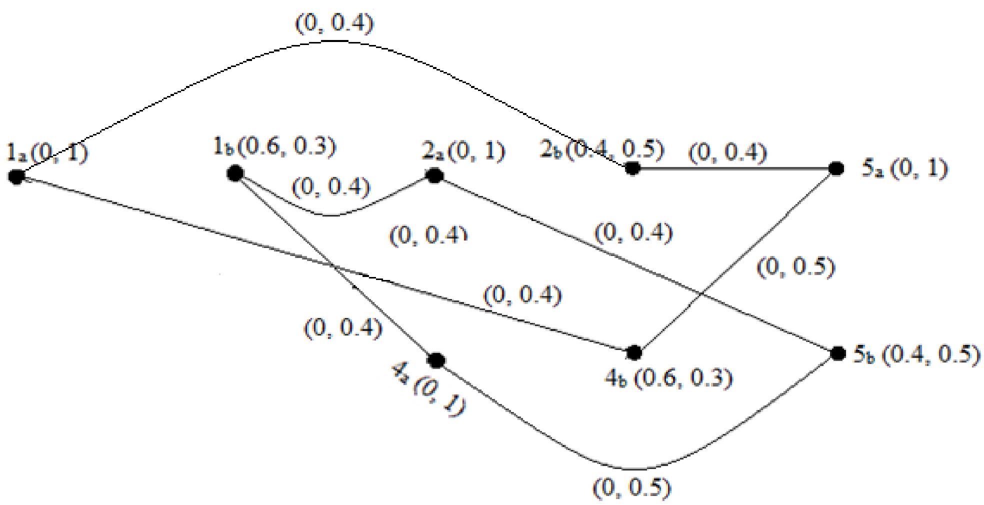


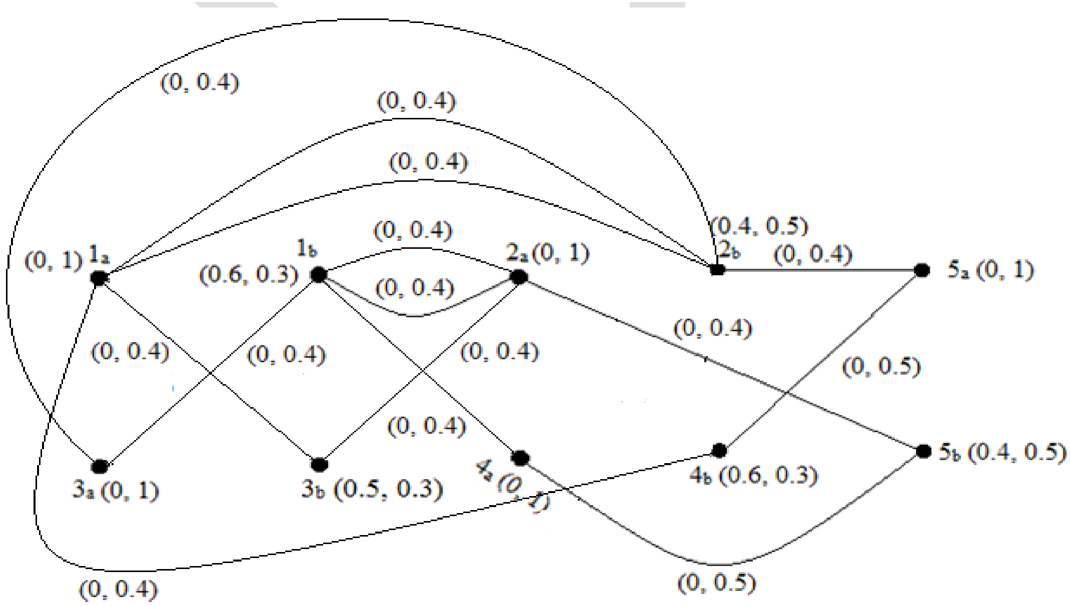

$(G_1 \cup G_2) \times G_3$




$G_1 \times G_3$




$G_2 \times G_3$




$(G_1 \times G_3) \cup (G_2 \times G_3)$

From the above graphs it can be seen that $(G_1 \cup G_2) \times G_3 = (G_1 \times G_3) \cup (G_2 \times G_3)$.

## 4. Conclusion

In this paper, the operation Cartesian product has been modified and the operations "∪" and "+" have been analyzed for the sets with one or more common vertices with relevant examples. Further extension can be made for composition and intersection of two intuitionistic fuzzy graphs. Also intuitionistic fuzzy lines, complete intuitionistic fuzzy graphs etc can be studied based on our modified definition.

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