Orthogonal Generalized Higher Reverse Derivations on Semiprime Γ-Rings

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Abstract :

In this paper we present the concept of orthogonal generalized higher rever derivations on semiprime Γ-ring, also we prove the following resultes if D_n and G_n a orthogonal such that $x\alpha y\beta z = x\beta y\alpha z$, then the following relations hold:

(1) $d_n(x)\alpha\, G_n(y) = G_n(x)\alpha\, d_n(y) = 0.$

(2)$d_n(x)\alpha\, G_n(y) = \; G_n(y)\alpha d_n(x) = 0.$

(3)$d_n G_n = G_n d_n = 0$ and $g_n D_n = D_n g_n = 0.$

In addition to demonstration other.

Key words:

semiprime Γ-ring , generalized higher reverse derivations, orthogonal generaliz higher reverse derivations.

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1. Introduction:

The gamma ring is defined by Barnes in [2] as follows:

Let M and Γ be two additive abelian groups, if there exists mapping $M \times \Gamma \times M \to M$ (sending(x, α, y) into $x\alpha y$) for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, satisfying the following conditions:

(i) $(x + y)\alpha z = x\alpha z + y\alpha z$

$x(\alpha + \beta)y = x\alpha y + x\beta y$

$x\alpha(y + z) = x\alpha y + x\alpha z$

(ii) $(x\alpha y)\beta z = x\alpha(y\beta z)$

Then M is called a$\Gamma -$ring.

M is called a prime $\Gamma-$ring if $x\,\Gamma M\Gamma y=0$, implies $x=0$ or $y=0$, M is called a semiprime $\Gamma-$ring if $x\,\Gamma M\Gamma x=(0)$ with $x\in M$ implies $x=0$, M is called n- torsion free if $nx=0$ for $x\in M$implies $x=0$, where n is positive integer.

F.J. Jing in [5] defined a derivation and Jordan derivation on $\Gamma-$ring as follows: for all $x,y\in M$ and $\alpha\in\Gamma$, then the additive mapping $d: M\to M$ is called a derivation if $d(x\alpha y)=d(x)\alpha y+x\alpha d(y)$, and d is called Jordan derivation if $d(x\alpha x)=d(x)\alpha x+x\alpha d(x)$.

Y. Ceven and M.A.Özturk in [3] defined a generalized derivation on $\Gamma-$ring M as follows: for all $x,y\in M$ and $\alpha\in\Gamma$, then $D(x\alpha y)=D(x)\alpha y+x\alpha d(y)$ where d is assosiative derivation with D .

M.A shraf and M.R.Jamal in [8] presented the concepts of orthogonal derivations on $\Gamma-$ring as follows let d and g are said to be derivations on $\Gamma-$ring M then d and g are said to be orthogonal if :

$d(x)\Gamma M\,\Gamma g(y)=(0)=g(y)\Gamma M\,\Gamma d(x)$ for all $x,y\ \in M$

A.H. Majeed and N.N Suliman in [7] defined orthogonal generalized derivations on $\Gamma-$ring M as follows: let M be a $\Gamma-$ring two generalized derivations D and G of M associated with two derivations d and g of M, respectively are said to be orthogonal if

$$D(x)\Gamma M\Gamma G(y)=(0)=G(y)\Gamma M\Gamma D(x)\ \text{ for all } x,y\in M.$$

K.k.Dey, A.C. Paul and I.S. Rakhimov in [4] defined reverse derivation on $\Gamma-$ring M as follows for all $x,y\in M$ and $\alpha\in\Gamma$ the additive mapping $d: M\to M$ is called reverse derivation if $d(x\alpha y)=d(y)\alpha x+y\alpha d(x)$, and their defined the orthogonal reverse derivation as follows let d and g be two reverse derivations on M if

$$d(x)\Gamma M\Gamma g(y)=(0)=g(y)\Gamma M\Gamma d(x)\text{for all } x,y\in M.$$

S.M. Salih and M.R. Salih in [10] defined generalized reverse derivation on $\Gamma-$ring M as follows: let M be a $\Gamma-$ring and $f: M\to M$ be an additive mapping then f is called generalized reverse derivation on M if there exists a reverse derivation $d: M\to M$ such that $f(x\alpha y)=f(y)\alpha x+y\alpha d(x)$

M.R. Salih in [9] introduced the concept of higher revers derivations on $\Gamma-$ring M as follow: let $D=(d_i)_{i\in N}$ be a family of additive mappings of $\Gamma-$ring M into itself, then D is called a higher revers derivation of M if for every $x,y\in M$, and $\alpha\in\Gamma$ then $d_n(x\alpha y)=\sum_{i+j=n} d_i(y)\alpha d_j(x)$ and defined generalized higher revers derivation as follows: let M be a $\Gamma-$ring and $F=(f_i)_{i\in N}$ be a family of additive mappings of M such that $f_0=id_M$, then F is called generalized higher revers derivation no M if there exists higher revers derivation $D=(d_i)_{i\in N}$ on M such that forever $x,y\in M$, and $\alpha\in\Gamma$, $n\in N$

$$f_n(x\alpha y)=\sum_{i+j=n} f_i(y)\alpha d_j(x)$$

In this paper, the definition of orthogonal generalized higher reverse derivations on $\Gamma-$ring M and we also investigate conditions for two generalized higher reverse derivations to be orthogonal, also we will extend these results to orthogonal generalized higher revers derivations on $\Gamma-$ ring M and obtain some results parallel to these earlier obtained by N. Argac, A. Nakajima and E. Albas in [1].

2. orthogonal generalized higher reverse derivations on Γ-rings

In this section we present the definition of orthogonal generalized higher reverse derivations on gamma rings as follows:

Definition (2.1): let M be a $\Gamma-$ring, two generalized higher revers derivations $D = (D_i)_{i\in N}$ and $G = (G_i)_{i\in N}$ on M are called orthogonal if for every $n \in N$, $x, y \in M$ then

$$D_n(x)\Gamma M\Gamma G_n(y) = (0) = G_n(y)\Gamma M\Gamma D_n(x)$$

Where

$$D_n(x)\Gamma M\Gamma G_n(y) = \sum_{i=1}^{n} D_i(x)\Gamma M\Gamma G_i(y) = (0)$$

Example (2.2):

Let M be $\Gamma-$ring and d= $(d_i)_{i\in N}$, $g = (g_i)_{i\in N}$ be two higher revers derivations of M, we put $\acute{M} = M \oplus M$ and $\acute{\Gamma} = \Gamma \oplus \Gamma$ then the mappings $\acute{d} = (\acute{d}_i)_{i\in N}$ and $\acute{g} = (\acute{g}_i)_{i\in N}$ from $\acute{M}$ into itself which are defined by:

$\acute{d}_n\big((x,y)\big) = (d_n(x), 0)$ and $\acute{g}_n\big((x,y)\big) = (0, g_n(y))$, for all $(x,y) \in \acute{M}$ are higher revers derivations of $\acute{M}$ moreover if (D_n, d_n) and (G_n, g_n) are generalized higher revers derivations of M and we defined mappings $\acute{D} = \big(\acute{D}_i\big)_{i\in n}$, $\acute{G} = \big(\acute{G}_i\big)_{i\in n}$ on $\acute{M}$ by $\acute{D}_n\big((x,y)\big) = (D_n(x), 0)$ and $\acute{G}_n\big((x,y)\big) = \big(0, G_n(y)\big)$, for all $(x,y) \in \acute{M}$.

Then $\big(\acute{D}_n, \acute{d}_n\big)$ and $\big(\acute{G}_n, \acute{g}_n\big)$ are generalized higher revers derivations of $\acute{M}$ and $\acute{D}_n$, $\acute{G}_n$ are orthogonal.

Lemma (2.3):[6]

Let M be a 2- torsion free semiprime $\Gamma-$ring and a,b the elements of M if for all $\alpha, \beta \in \Gamma$, then the following conditions are equivalent:

(i) a α M β b = (0)
(ii) b αM β a = (0)
(iii) a α M β b+b α M β a = (0)

If one of these conditions are fulfilled then a α b = b α a = 0 for all $\alpha \in \Gamma$.

3. The Main Results:

The main results of the present paper states as follows:

Theorem (3.1):

Let $D = (D)_{i\in N}$ and $G = (G_i)_{i\in N}$ be generalized higher revers derivations of a 2-torsion free semiprime Γ-ring M, and d= $(d_i)_{i\in N}$, $g = (g_i)_{i\in N}$ are higher revers derivations, where D_n, G_n are commuting mappings if D_n *and* G_n are orthogonal such that $x\alpha y\beta z = x\beta y\alpha z$.

Then the following relations hold:

(i) $D_n(x)\alpha G_n(y) = G_n(x)\alpha D_n(y) = 0$

Hence: $D_n(x)\alpha G_n(y) + G_n(x)\alpha D_n(y) = 0$, for all $x, y \in M$ and $\alpha \in \Gamma$.

Proof: By the hypothesis, we have $D_n(x)\alpha z\beta\, G_n(y) = 0$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$ by Lemma (2.3)we get.

$$D_n(x)\alpha G_n(y) = G_n(x)\alpha D_n(y) = 0$$

Hence: $D_n(x)\alpha G_n(y) + G_n(x)\alpha D_n(y) = 0$

(ii) d_n and g_n are orthogonal higher revers derivations and $D_n(y)\alpha d_n(x) = G_n(x)\alpha d_n(y) = 0$, for all $x, y \in M$ and $\alpha \in \Gamma$.

proof:

since by (1) we get

$$D_n(x)\alpha G_n(y) = 0 = G_n(x)\alpha D_n(y)$$

$$\sum_{i=1}^{n} D_i(x)\alpha G_i(y) = 0$$

replace x by $x\beta z$ we get

$$\sum_{i=1}^{n} D_i(x\beta z)\alpha G_i(y) = 0$$

$$\sum_{i=1}^{n} D_i(z)\beta d_i(x)\alpha G_i(y) = 0$$

$$\sum_{i=1}^{n} d_i(x)\alpha G_i(y)\,\beta D_i(z) = 0$$

Replace $D_i(z)$ by $d_i(x)\alpha G_i(y)$

$$\sum_{i=1}^{n} d_i(x)\alpha G_i(y)\,\beta d_i(x)\alpha G_i(y) = 0$$

Replace β by βmy

$$\sum_{i=1}^{n} d_i(x)\alpha G_i(y)\,\beta my\, d_i(x)\alpha G_i(y) = 0$$

since M is semiprime

$\sum_{i=1}^{n} d_i(x)\alpha G_i(y) = 0 d_n(x)\alpha G_n(y) = 0$ ….. (1)

since G_n is commuting mapping therefore

$G_n(x)\alpha d_n(y) = 0$ …. (2)

$from\ (1)\ and\ (2)\ we\ get$

$$d_n(x)\alpha G_n(y) = G_n(y)\beta d_n(x) = 0$$

(iii)g_n and D_n are orthogonal higher revers derivations and $g_n(x)\alpha D_n(y) = D_n(y)\alpha g_n(x) = 0$, for all $x, y \in M$ and $\alpha \in \Gamma$.

Where g_n is commuting mapping.

proof:

since $D_n(x)\alpha G_n(y) = 0 = G_n(x)\alpha D_n(y)$ by(i)

Replace x by $x\beta z$ in $g_n(x)\alpha d_n(y) = 0$

$$\sum_{i=1}^{n} G_i(x\beta z)\alpha D_i(y) = 0$$

$$\sum_{i=1}^{n} G_i(z)\beta g_i(y)\alpha D_i(y) = 0$$

$$\sum_{i=1}^{n} D_i(y)\beta g_i(x)\alpha G_i(z) = 0$$

Replace $G_i(z)$ by $D_i(y)\beta G_i(x)$

$$\sum_{i=1}^{n} D_i(y)\beta G_i(x)\alpha D_i(y)\beta G_i(x) = 0$$

Replace α by $\alpha m\,\gamma$

$$\sum_{i=1}^{n} D_i(y)\beta g_i(x)\alpha m\gamma\, D_i(y)\beta\, g_i(x) = 0$$

since M is semiprime

$$\sum_{i=1}^{n} D_i(y)\beta g_i(x) = 0$$

$$\sum_{i=1}^{n} g_i(x)\beta D_i(y) = 0$$

$$g_n(x)\alpha D(y) = 0 \qquad \ldots(1)$$

since g_n is commuting mapping.

Therefor $D_n(y)\alpha g_n(x) = 0$ …(2)

From (1) and (2) we get

$$g_n(x)\alpha D_n(y) = D_n(y)\alpha g_n(x) = 0$$

(iv) d_n and g_n are orthogonal higher revers derivations.

Where d_n and g_n are commuting mappings.

proof:

since by (i) we get

$D_n(x)\alpha G_n(y) = 0 = G_n(x)\alpha D_n(y)$

$$\sum_{i=1}^{n} D_i(x)\alpha G_i(y) = 0$$

Replace x by $x\beta z$ and y by $y\delta w$

$$\sum_{i=1}^{n} D_i(x\beta z)\alpha G_i(y\delta w) = 0$$

$$\sum_{i=1}^{n} D_i(z)\beta d_i(x)\alpha G_i(w)\delta\, g_i(y) = 0$$

Replace $D_i(z)$ by $g_i(y)$ and $G_i(w)\delta$ and by $d_i(x)\beta$

$$\sum_{i=1}^{n} g_i(y)\,\beta\, d_i(x)\alpha d_i(x)\beta g_i(y) = 0$$

Replace α by $\alpha m\gamma$

$$\sum_{i=1}^{n} d_i(x)\,\beta\, g_i(y)\alpha m\gamma\, d_i(x)\,\beta\, g_i(y) = 0$$

since M is semiprime

$$\sum_{i=1}^{n} d_i(x)\,\beta\, g_i(y) = 0$$

Hence d_n and g_n are orthogonal.

(v) $d_nG_n = G_nd_n = 0$ and $g_nD_n = D_ng_n = 0$

proof:

since by (ii) we get

$$d_n(x)\alpha G_n(y) = 0$$

$$\sum_{i=1}^{n} d_i(x)\alpha g_i(y) = 0$$

$$\sum_{i=1}^{n} G_i\big(d_i(x)\alpha G_i(y)\big) = 0$$

$$\sum_{i=1}^{n} G_i\big(G_i(y)\big)\alpha g_i\big(d_i(x)\big) = 0$$

Replace $G_i(y)$ by $d_i(x)$ and $d_i(x)$ by $G_i(x)$

$$\sum_{i=1}^{n} G_i\big(d_i(x)\big)\alpha g_i\big(G_i(x)\big) = 0$$

$$\sum_{i=1}^{n} G_i\big(d_i(x)\big)\alpha G_i\big(g_i(x)\big) = 0$$

Replace $g_i(x)$by $d_i(x)$ and α by $\alpha z\beta$

$$\sum_{i=1}^{n} G_i\big(d_i(x)\big)\alpha Z\beta\, G_i\big(d_i(x)\big) = 0$$

since M is semiprime

$$\sum_{i=1}^{n} G_i\big(d_i(x)\big) = 0 G_n d_n = 0$$

since by (ii) we get

$$G_n(x)\alpha d_n(y) = 0$$

$$\sum_{i=1}^{n} G_i(x)\alpha d_i(y) = 0$$

$$\sum_{i=1}^{n} d_i\big(G_i(x)\alpha d_i(y)\big) = 0$$

$$\sum_{i=1}^{n} d_i\big(d_i(y)\big)\alpha\Big(d_i\big(G_i(x)\big)\Big) = 0$$

Replace $d_i(y)$ by $G_i(x)$

$$\sum_{i=1}^{n} d_i\big(G_i(x)\big)\alpha d_i\big(G_i(x)\big) = 0$$

Replace α by $\alpha z\beta$ we get

$$\sum_{i=1}^{n} d_i\big(G_i(x)\big)\alpha z\beta\, d_i\big(G_i(x)\big) = 0$$

since M is semiprime

$$\sum_{i=1}^{n} d_i\left(G_i\quad(x)\right) = 0\, d_n G_n = 0$$

and

since by (ii) we get

$$g_n(x)\alpha D_n(y) = 0$$

$$\sum_{i=1}^{n} g_i(x)\alpha D_i(y) = 0$$

$$\sum_{i=1}^{n} D_i\big(g_i(x)\alpha D_i(y)\big) = 0$$

$$\sum_{i=1}^{n} D_i\big(D_i(y)\big)\alpha d_i\big(g_i(x)\big) = 0$$

Replace $D_i(y\ )$by $g_i(x)$and $g_i(x)$ by $D_i(x)$

$$\sum_{i=1}^{n} D_i\big(g_i(x)\big)\alpha d_i\big(D_i(x)\big) = 0$$

$$\sum_{i=1}^{n} D_i\big(g_i(x)\big)\alpha D_i\big(d_i(x)\big) = 0$$

Replace α by $\alpha z\beta$ and $d_i(x)$ by $g_i(x)$ we get

$$\sum_{i=1}^{n} D_i\big(g_i(x)\big)\alpha z\beta D_i\big(g_i(x)\big) = 0$$

since M is semiprime

$$\sum_{i=1}^{n} D_i\big(g_i(x)\big) = 0\ D_n g_n = 0$$

and

since $D_n(x)\alpha g_n(y) = 0$

$$\sum_{i=1}^{n} D_i(x)\alpha g_i(y) = 0$$

$$\sum_{i=1}^{n} g_i\big(D_i(x)\alpha g_i(y)\big) = 0$$

$$\sum_{i=1}^{n} g_i\big(g_i(y)\big)\alpha g_i\big(D_i(x)\big) = 0$$

Replace $g_i(y)$ by $D_i(x)$ we get

$$\sum_{i=1}^{n} g_i\big(D_i(x)\big)\alpha g_i\big(D_i(x)\big) = 0$$

Replace α by $\alpha z\beta$

$$\sum_{i=1}^{n} g_i\big(D_i(x)\big)\alpha z\beta\ g_i\big(D_i(x)\big) = 0$$

since M is semiprime

$$\sum_{i=1}^{n} g_i\big(D_i(x)\big) = 0\ g_n D_n = 0$$

(vi) $D_n G_n = G_n D_n = 0$

Proof:

since $D_n(x)\alpha G_n(y) = 0$

$$\sum_{i=1}^{n} D_i(x)\alpha G_i(y) = 0$$

$$\sum_{i=1}^{n} G_i\big(D_i(x)\alpha G_i(y)\big) = 0$$

$$\sum_{i=1}^{n} G_i\big(G_i(y)\big)\alpha g_i\big(D_i(x)\big) = 0$$

$$\sum_{i=1}^{n} G_i\big(G_i(y)\big)\alpha D_i\big(g_i(x)\big) = 0$$

Replace $G_i(y)$ by $D_i(x)$ and $g_i(x)$ by $G_i(x)$

$$\sum_{i=1}^{n} G_i\big(D_i(x)\big)\alpha D_i\big(G_i(x)\big) = 0$$

$$\sum_{i=1}^{n} G_i\big(D_i(x)\big)\alpha G_i\big(D_i(x)\big) = 0$$

Replace α by $\alpha\beta z$

$$\sum_{i=1}^{n} G_i\big(D_i(x)\big)\alpha z\beta\ G_i\big(D_i(x)\big) = 0$$

since M is semiprime

$$\sum_{i=1}^{n} G_i\big(D_i(x)\big) = 0$$

Therefore $G_n D_n = 0$

since $G_n(x)\alpha D_n(y) = 0$

$$\sum_{i=1}^{n} G_i(x)\,\alpha D_i(y) = 0$$

$$\sum_{i=1}^{n} D_i\big(G_i(x)\alpha D_i(y)\big) = 0$$

$$\sum_{i=1}^{n} D_i\big(D_i(y)\big)\alpha d_i\big(G_i(x)\big) = 0$$

$$\sum_{i=1}^{n} D_i\big(D_i(y)\big)\alpha G_i\big(d_i(x)\big) = 0$$

Replace $D_i(y)$ by $G_i(x)$ and $d_i(x)$ by $D_i(x)$

$$\sum_{i=1}^{n} D_i\big(G_i(x)\big)\alpha G_i\big(D_i(x)\big) = 0$$

$$\sum_{i=1}^{n} D_i\big(G_i(x)\big)\alpha D_i\big(G_i(x)\big) = 0$$

Replace α by $\alpha z\beta$

$$\sum_{i=1}^{n} D_i\big(G_i(x)\big)\alpha z\beta\, D_i\big(G_i(x)\big) = 0$$

since M is semiprime

$$\sum_{i=1}^{n} D_i\big(G_i(x)\big) = 0 D_n G_n = 0$$

Lemma(3.2):

Let M be a semiprime Γ-ring, let U be an ideal of M and V=Ann(U), if $D = (D_i)_{i\in N}$ is generalized higher revers derivations associated with $d = (d_i)_{i\in N}$ higher revers derivations of M and d_n is commuting mappings such that .

$D_n(M), d_n(M) \subset U,$ then $D_n(V) = d_n(V) = 0$

Proof:

If $\in V$, then $x\alpha U = (0)$

by the hypothesis we have

$d_n(M) \subset U d_n(U)$, hence for all $y \in U$

$$0 = D_n(x\alpha y) = \sum_{i=1}^{n} D_i(y)\,\alpha d_i(x)$$

Replace y by $y\beta x$ we get

$$\sum_{i=1}^{n} D_i(y\beta x)\alpha d_i(x) = 0$$

$$\sum_{i=1}^{n} D_i(x)\,\beta d_i(y)\alpha d_i(x) = 0$$

Replace

$d_i(x)$ by $D_i(x)$ we get

$$\sum_{i=1}^{n} D_i(x)\beta\, d_i(y)\, \alpha D_i(x) = 0$$

since M is semiprime $\sum_{i=1}^{n} D_i(x) = 0\; D_n(x) = 0$

since $D_n(x) \in U \cap V$ we get $D_n(V) = 0$

Similarly:

If $\in V$, then $x\alpha U = (0)$

$$D_n(M) \subset UD_n(u) \subset U$$

Hence $d_n(x\alpha y) = 0$

$$\sum_{i=1}^{n} d_i(y)\, \alpha d_i(x) = 0$$

Replace y by $y\beta x$ we get

$$\sum_{i=1}^{n} d_i(y\beta z)\alpha d_i(x) = 0$$

$$\sum_{i=1}^{n} d_i(x)\, \beta d_i(y)\alpha d_i(x) = 0$$

since M is semiprime $\sum_{i=1}^{n} d_i(x) = 0$

$d_n(x) \in U \cap V$ we get

$$d_n(x) = 0 \;\; then \;\; d_n(V) = 0$$

Lemma(3.3):

Let D $=(D_i)_{i\in N}$ be generalized higher revers derivations of semiprime Γ-ring M associative with d = $(d_i)_{i\in N}$ higher reverse derivations.If $D_n(x)\alpha D_n(y) = 0$, for all $x, y \in M$, $\alpha \in \Gamma$.Then $D_n = d_n = 0$

Proof: by the hypothesis , we have

$\sum_{i=1}^{n} D_i(x)\, \alpha\, D_i(y) = 0$, replace y by $x\beta y$

$$\sum_{i=1}^{n} D_i(x)\, \alpha\, D_i(x\beta y) = 0$$

$$\sum_{i=1}^{n} D_i(x)\,\alpha\, D_i(y)\beta d_i(x) = 0$$

Replace $D_i(x)$ by $d_i(x)$ we get

$$\sum_{i=1}^{n} d_i(x)\,\alpha\, D_i(y)\beta d_i(x) = 0$$

Since M is semiprime

$$\sum_{i=1}^{n} d_i(x) = 0\, d_n = 0$$

by hypothesis , we have

$$\sum_{i=1}^{n} D_i(x)\alpha D_i(y) = 0$$

replace x by $x\beta y$ we get

$$\sum_{i=1}^{n} D_i(x\beta y)\alpha D_i(y) = 0$$

$$\sum_{i=1}^{n} D_i(y)\,\beta\, d_i(x)\,\alpha\, D_i(y) = 0$$

since M is semiprime

$$\sum_{i=1}^{n} D_i(y) = 0\, D_n(y) = 0$$

We get $D_n = d_n = 0$

Theorem (3.4):

Let $D = (D_i)_{i \in N}$and $G = (G_i)_{i \in N}$ be generalized higher revers derivations of a2- torsion free semiprime Γ-ring M, and $= (d_i)_{i \in N}$, $g = (g_i)_{i \in N}$ are higher revers derivations. Then D_n andG_n are orthogonal iff for all $, y \in M$, then

(a) $D_n(x)\alpha G_n(y) + G_n(x)\alpha D_n(x) = (0)$
(b) $d_n(x)\alpha G_n(y) + g_n(x)\alpha D_n(x) = (0)$

Where D_n , G_n are commuting mappings.

Proof:

Suppose that D_n , G_n are orthogonal.

To prove (a)

Then $\sum_{i=1}^{n} D_i(x)\alpha z\beta\, G_i\,(y) = 0$, for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

By Lemma(2.3) we get

$$\sum_{i=1}^{n} D_i(x)\alpha\, G_i\,(y) = \sum_{i=1}^{n} G_i(x)\alpha\ D_i(y) = (0)$$

Thus $\sum_{i=1}^{n} D_i(x)\alpha\, G_i\,(y) + \sum_{i=1}^{n} G_i(x)\alpha\ D_i(y) = (0)$

Hence $\sum_{i=1}^{n} D_i(x)\alpha G_i(y) + G_i(x)\alpha D_i(x) = (0)$

Now, to prove (b)

since $D_n(x)\alpha G_n(y) = (0)$

$$\sum_{i=1}^{n} D_i(x)\alpha\, G_i\,(y) = (0)$$

$$\sum_{i=1}^{n} d_i\big(D_i(x)\alpha\, G_i(y)\big) = (0)$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big)\,\alpha\, d_i\big(D_i(x)\big) = (0)$$

Replace $D_i(x)$ by $G_i(x)$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big)\,\alpha\, d_i\big(G_i(x)\big) = (0)$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big)\,\alpha\, G_i\big(d_i(x)\big) = (0)$$

Replace $G_i(y)$ by x and $d_i(x)$ by y we get

$$\sum_{i=1}^{n} d_i(x)\,\alpha\, G_i(y) = (0)$$

$$d_n(x)\alpha G_n(y) = (0)$$

and

since $G_n(x)\alpha\, D_n(y) = (0)$

$$\sum_{i=1}^{n} G_i(x)\,\alpha\, D_i(y) = (0)$$

$$\sum_{i=1}^{n} g_i\big(G_i(x)\alpha\, D_i(y)\big) = (0)$$

$$\sum_{i=1}^{n} g_i\big(D_i(y)\big)\alpha g_i\big(G_i(x)\big) = 0$$

Replace: $G_i(x)$ by $D_i(y)$

$$\sum_{i=1}^{n} g_i\big(D_i(y)\big)\alpha g_i\big(D_i(y)\big) = 0$$

$$\sum_{i=1}^{n} g_i\big(D_i(y)\big)\alpha D_i\big(g_i(y)\big) = 0$$

Replace: $D_i(y)$ by x and $g_i(y)$ by y we get

$$\sum_{i=1}^{n} g_i(x)\,\alpha\, D_i(y) = (0)$$

Hence $g_n(x)\,\alpha\, D_n(y) = (0)$, we get

$$d_n(x)\alpha G_n(y) + g_n(x)\,\alpha\, D_n(y) = (0)$$

Conversely:

Suppose that (a)we get

$$D_n(x)\alpha G_n(y) + G_n(x)\,\alpha\, D_n(y) = (0)$$

Replace$x$ by $y\beta x$ we get

$$\sum_{i=1}^{n} D_i(y\beta x)\,\alpha\, G_i(y) + G_i(y\beta x)\alpha\, D_i(y) = (0)$$

$$\sum_{i=1}^{n} D_i(x)\,\beta\, d_i(y)\alpha G_i(y) + G_i(x)\beta\, g_i(y)\alpha\, D_i(y) = (0)$$

Replace: $g_i(y)$by $d_i(y)$ we get

$$\sum_{i=1}^{n} D_i(x)\,\beta\, d_i(y)\alpha G_i(y) + G_i(x)\beta\, d_i(y)\alpha\, D_i(y) = (0)$$

By lemma (2-3) we get

$$\sum_{i=1}^{n} D_i(x)\,\beta\, d_i(y)\alpha\, G_i(y) = 0$$

$$\sum_{i=1}^{n} G_i(x)\,\beta\, d_i(y)\alpha\, D_i(y) = 0$$

D_n , G_n are orthogonal.

Theorem (3.5):

Let $D = (D_i)_{i\in N}$and $G = (G_i)_{i\in N}$ be generalized higher revers derivations of a 2- torsion free semiprime Γ-ring M, and d = $(d_i)_{i\in N}$, $g = (g_i)_{i\in N}$ are higher revers derivations, then D_n , G_n are orthogonal if and only if $D_n(x)\alpha G_n(y) = d_n(x)\,\alpha\, G_n(y) = (0)$ for all $x, y \in M$ and $\alpha \in$ Γ.Where D_n , G_n are commuting mappings.

Proof: Suppose that D_n , G_n are orthogonal.

since by Theorem (3.1) (i)we get

$$D_n(x)\alpha G_n(y) = (0)$$

$$\sum_{i=1}^{n} D_i(x)\,\alpha\, G_i(y) = (0)$$

$$\sum_{i=1}^{n} d_i\big(D_i(x)\alpha\, G_i(y)\big) = (0)$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big)\,\alpha\, d_i\big(D_i(x)\big) = (0)$$

Replace: $D_i(x)$ by $G_i(x)$ we get

$$\sum_{i=1}^{n} d_i\left(G_i(x)\alpha\, d_i\big(G_i(x)\big)\right) = (0)$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big)\,\alpha\, G_i\big(d_i(x)\big) = (0)$$

Replace: $G_i(y)$ by x and $d_i(x)$ by y

$$\sum_{i=1}^{n} d_i(x)\,\alpha\, G_i(y) = (0)d_n(x)\alpha G_n(y) = (0)$$

Conversely:

since $D_n(x)\,\alpha\, G_n(y) = (0)$

Replace x by $y\beta x$ we get

$$\sum_{i=1}^{n} D_i(y\beta x)\,\alpha\, G_i(y) = (0)$$

$$\sum_{i=1}^{n} D_i(x)\,\beta\, d_i(y)\alpha\, G_i(y) = (0)$$

$$\sum_{i=1}^{n} G_i(x)\,\beta\, d_i(y)\alpha\, D_i(y) = (0)$$

Hence

$$\sum_{i=1}^{n} D_i(x)\,\beta\, d_i(y)\alpha\, G_i(y) = (0) = \sum_{i=1}^{n} G_i(x)\,\beta\, d_i(y)\alpha\, D_i(y)$$

Thus D_n and $G_n$are orthogonal.

Theorem (3.6):

Let $D = (D_i)_{i\in N}$and $G = (G_i)_{i\in N}$ be generalized higher revers derivations of a 2- torsion free semiprime Γ-ring M, associated respectively with higher reverse derivations $d = (d_i)_{i\in N}$ and $g = (g_i)_{i\in N}$, then D_n , G_n are orthogonal if and only if $D_n(x)\alpha G_n(y) = (0)$ for all $x, y \in M$ and $d_n G_n = d_n g_n = 0$, where $G_n$commuting mappings.

Proof: Suppose that D_n , G_n are orthogonal.

By Theorem(3.1) (i) we get $D_n(x)\alpha G_n(y) = (0)$ and by(ii) $G_n(x)\,\alpha\, d_n(y) = (0)$

$$\sum_{i=1}^{n} d_i\big(G_i(x)\alpha\, d_i(y)\big) = (0)$$

$$\sum_{i=1}^{n} d_i\big(d_i(y)\big)\,\alpha d_i\big(G_i(x)\big) = (0)$$

Replace: $d_i(y)$ by $G_i(x)$and x by $\alpha z\beta$ we get

$$\sum_{i=1}^{n} d_i\big(G_i(x)\big)\alpha z\beta\, d_i\big(G_i(x)\big) = (0)$$

Replace: z by $d_i(z)$

$$\sum_{i=1}^{n} d_i\big(G_i(x)\big)\alpha\, d_i(z)\,\beta\, d_i\big(G_i(x)\big) = (0)$$

since M is semiprime we get $\sum_{i=1}^{n} d_i\big(G_i(x)\big) = 0$

$$d_n G_n = (0)$$

by Theorem (3.1) (iv)

d_n , g_n are orthogonal higher revers derivations.

$$\text{Thus } d_n g_n = (0)$$

Conversely:

Suppose that $d_n G_n = 0$

$$\sum_{i=1}^{n} d_i\, G_i(x\alpha y) = 0$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\alpha\, g_i(x)\big) = 0$$

$$\sum_{i=1}^{n} d_i\big(g_i(x)\big)\,\alpha d_i\big(G_i(y)\big) = 0$$

$$\sum_{i=1}^{n} d_i\big(g_i(x)\big)\,\alpha\, G_i\big(d_i(y)\big) = 0$$

Replace: $g_i(x)$ by x and $d_i(y)$ by y

$$\sum_{i=1}^{n} d_i(x)\,\alpha G_i(y) = 0$$

$d_n(x)\,\alpha G_n(y) = 0$, by(ii) we get

D_n , G_n are orthogonal.

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