Hexagonal Difference Prime Labeling of Some Tree Graphs

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ABSTRACT

The labeling of a graph, we mean assign some integers to the vertices or edges (or both) of the graph. Here the vertices of the graph are labeled with hexagonal numbers and the edges are labeled with absolute difference of the end vertex labels. Here the greatest common incidence number (g c i n) of a vertex of degree greater than one is defined as the g c d of the labels of the incident edges. If the g c i n of each vertex of degree greater than one is 1, then the graph admits hexagonal difference prime labeling. Here we characterize some tree graphs for hexagonal difference prime labeling.

Key words: Graph labeling, hexagonal numbers, g c i n, prime labeling, tree graph.

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INTRODUCTION

In this paper we deal with tree graphs that are undirected. The symbol V and E denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4]. Some basic concepts are taken from [1] and [2]. In this paper we investigated the hexagonal difference prime labeling of some tree graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number of a vertex of degree greater than or equal to 2, is the g c d of the labels of the incident edges.

Definition: 1.2 n^{th} hexagonal number is n(2n-1), where n is a positive integer. The hexagonal numbers are 1, 6, 15, 28, 45, 66--------------.

MAIN RESULTS

Definition 2.1 Let G be a graph with p vertices and q edges . Define a bijection
f : V(G) → {1,6,15,28 ,------------------,p(2p-1)} by f(v_i) = $i(2i-1)$, for every i from 1 to p and define a 1-1 mapping f_{hdpl}^{*} : E(G) → set of natural numbers N by $f_{hdpl}^{*}(uv)$ = | f(u)-f(v) |. The

induced function f^*_{hdpl} is said to be hexagonal difference prime labeling, if the g c i n of each vertex of degree at least 2, is one.

Definition 2.2 A graph which admits hexagonal difference prime labeling is called hexagonal difference prime graph.

Theorem: 2.1 The corona of path P_n admits hexagonal difference prime labeling.

Proof : Let $G = P_n \odot K_1$ and let $v_1,v_2,----------------,v_{2n}$ are the vertices of G.

Here $|V(G)| = 2n$ and $|E(G)| = 2n-1$.

Define a function f : V $\rightarrow$ {1,6,15,28 ,------------------,2n(4n-1)} by

$\qquad\qquad f(v_i) = i(2i-1)$, i = 1,2,------,2n.

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_i\ v_{i+1})$ $\qquad = (4i+1),$ $\qquad\qquad\qquad\qquad\qquad\qquad$ i = 1,2,------------,n+1

$f^*_{hdpl}(v_{i+2}\ v_{n+i+2})$ $\quad = 2n^2+7n+4ni,$ $\qquad\qquad\qquad\qquad\quad$ i = 1,2,------------,n-2.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_{i+1}) $\qquad = $ g c d of { $f^*_{hdpl}(v_i\ v_{i+1})$, $f^*_{hdpl}(v_{i+1}\ v_{i+2})$ }

$\qquad\qquad\qquad\qquad = $ g c d of { (4i+1) , (4i+5) } =1, $\qquad$ i = 1,2,----------------,n

So, g c i n of each vertex of degree greater than one is 1.

Hence $P_n \odot K_1$, admits hexagonal difference prime labeling.

Theorem: 2.2 Centipede graph admits hexagonal difference prime labeling.

Proof : Let $G = C(2,n)$ and let $v_1,v_2,----------------,v_{3n}$ are the vertices of G.

Here $|V(G)| = 3n$ and $|E(G)| = 3n-1$.

Define a function f : V $\rightarrow$ {1,6,15,28 ,------------------,3n(6n-1)} by

$\qquad\qquad f(v_i) = i(2i-1)$, i = 1,2,------,3n.

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_i\ v_{i+1})$ $\qquad = (4i+1),$ $\qquad\qquad\qquad\qquad\qquad\qquad$ i = 1,2,------------,n+1

$f^*_{hdpl}(v_{i+1}\ v_{n+i+2})$ $\quad = 2n^2+7n+5+4ni+4i,$ $\qquad\qquad\quad$ i = 1,2,------------,n-1.

$f^*_{hdpl}(v_{i+2}\ v_{2n+i+1})$ $\quad = 8n^2+6n-5+(8ni-4i),$ $\qquad\qquad$ i = 1,2,------------,n-1.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_{i+1}) $\qquad =1,$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ i = 1,2,----------------,n

So, g c i n of each vertex of degree greater than one is 1.

Hence $C(2,n)$, admits hexagonal difference prime labeling.

Theorem: 2.3 Twig graph admits hexagonal difference prime labeling.

Proof : Let $G = T_w(n)$ and let $v_1,v_2,----------------,v_{3n-4}$ are the vertices of G.

Here $|V(G)| = 3n-4$ and $|E(G)| = 3n-5$.

Define a function f : V $\rightarrow$ {1,6,15,28 ,------------------,(3n-4)(6n-9)} by

$\qquad\qquad f(v_i) = i(2i-1)$, i = 1,2,------,3n-4.

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_i\ v_{i+1})$ $\qquad\quad = (4i+1),$ $\qquad\qquad\qquad\qquad\qquad\quad$ i = 1,2,------------,n-1

$f^*_{hdpl}(v_{i+1}\ v_{n+i})$ $\qquad = (2n^2-n-1) +(4ni-4i),$ $\qquad\quad$ i = 1,2,------------,n-2.

$f^*_{hdpl}(v_{i+1}\ v_{2n+i-2})$ $\quad = (8n^2-18n+9) +(8ni-12i),$ $\qquad$ i = 1,2,------------,n-2.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_{i+1}) $\qquad =1,$ $\qquad\qquad\qquad\qquad\qquad\qquad$ i = 1,2,----------------,n-1.

So, g c i n of each vertex of degree greater than one is 1.

Hence $T_w(n)$, admits hexagonal difference prime labeling.

Theorem: 2.4 Coconut tree graph CT(m,n), admits hexagonal difference prime labeling.

Proof : Let G = CT(m,n) and let $v_1, v_2,$---------------,v_{m+n} are the vertices of G.

Here $|V(G)|$ = m+n and $|E(G)|$ = m+n-1.

Define a function f : V → {1,6,15,28 ,------------------,(m+n)(2m+2n-1)} by

$$f(v_i) = i(2i-1) , i = 1,2,------,m+n.$$

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_i \, v_{i+1})$ = (4i+1),   i = 1,2,------------,m-1

$f^*_{hdpl}(v_m \, v_{m+i})$ = 2i²+4mi-i,   i = 1,2,------------,n.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_{i+1}) =1,   i = 1,2,---------------,m-1.

So, g c i n of each vertex of degree greater than one is 1.

Hence CT(m,n), admits hexagonal difference prime labeling.

Theorem: 2.5 Bistar B(m,n), admits hexagonal difference prime labeling, when m,n > 2.

Proof : Let G = B(m,n) and let $v_1, v_2,$---------------,v_{m+n+2} are the vertices of G.

Here $|V(G)|$ = m+n+2 and $|E(G)|$ = m+n+1.

Define a function f : V → {1,6,15,28 ,------------------,(m+n+2)(2m+2n+3)} by

$$f(v_i) = i(2i-1) , i = 1,2,------,m+n+2.$$

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_1 \, v_{i+2})$ = 2i²+7i+5,   i = 1,2,------------,m.

$f^*_{hdpl}(v_2 \, v_{m+i+2})$ = (m+i+2)(2m+2i+3) − 6,   i = 1,2,------------,n.

$f^*_{hdpl}(v_1 \, v_2)$ = 5.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_1) = 1.

g c i n of (v_2) = 2.

So, g c i n of each vertex of degree greater than one is 1.

Hence B(m,n), admits hexagonal difference prime labeling.

Theorem: 2.6 Star $K_{1,n}$, admits hexagonal difference prime labeling, when n > 2.

Proof : Let G = $K_{1,n}$ and let $v_1, v_2,$---------------,v_{n+1} are the vertices of G.

Here $|V(G)|$ = n+1 and $|E(G)|$ = n.

Define a function f : V → {1,6,15,28 ,------------------,(n+1)(2n+1)} by

$$f(v_i) = i(2i-1) , i = 1,2,------,n+1.$$

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_1 \, v_{i+1})$ = 2i²+3i,   i = 1,2,------------,n.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_1) =1.

So, g c i n of each vertex of degree greater than one is 1.

Hence $K_{1,n}$, admits hexagonal difference prime labeling.

CONCLUSION

In this paper we proved that some trees admit hexagonal difference prime labeling. We did not consider all possible trees for labeling. So this labeling is open for other researchers to prove that remaining trees also admit hexagonal difference prime labeling.

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