Peristaltic Flow of Couple-Stress Fractional Burger’s Fluid Through an Inclined Uniform Tubes with Heat Transfer

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Abstract: Peristaltic flow of a fractional Burger’s fluid through an inclined uniform tubes with heat transfer under the assumption of long wavelength and low Rynold’s number approximations are used to linearise the governing equations. Analytical solution is obtained by using fractional complex transform definition. The expressions for fractional parameter, material constant, heat transfer and amplitude ratio, temperature and friction force are discussed. The computational results are presented in graphical form.

Keywords: Peristalsis; Fractional Burgers’ fluid; Couple Stress; fractional complex transform.

1. Introduction

Peristaltic transport is a form of material transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube, mixing and transporting the fluid in the direction of the wave propagation. It plays an indispensable role in transporting many physiological fluids in the body such as the movement of chime in the gastrointestinal tracts, the swallowing of food through esophagus and the vasomotion of small blood vessels. Many modern mechanical devices have been designed on the principle of peristaltic pumping for transporting fluids without internal moving parts. The idea of peristaltic transport in mathematical point of view was first coined by Latham [1]. The initial mathematical model of peristalsis obtained by train of sinusoidal waves in an infinitely long symmetric channel or tube has been investigated by Shapiro et al [2], and Fung and Yahi [3]. Subba Reddy et.al [4] has been studied on Slip effects on the peristaltic motion of a Jeffrey fluid through a porous medium in an asymmetric channel under the effect magnetic field.

Studies pertaining to the couple-stress fluid behavior are very useful, such studies bear the potential to better explain the behavior of rheological complex fluids, such as liquid crystals, polymeric suspensions that have long chain molecules, lubrication as well as human/sub-human blood. Couple-stress fluid is a special type of a non-Newtonian fluid, whose particle sizes are taken into account. While the classical continuum theory does not study the particle size effects, a micro-continuum theory given by Stokes [5] has been considered to take into account the particle size effects. The Stokes micro-continuum theory is the generalization of the classical theory of fluids allowing for polar effects such as the presence of couple-stresses, body couples, and an anti-symmetric stress tensor. In this fluid model, the couple-stress effects are considered as a consequence of the action of a deforming body on its neighborhood. Srivastava [6], El-Shehawey and El-Sebai [7], Mekheimer [8], Mekheimer [9], and Ali et al. [10] have reported the investigations based on the peristaltic flow of the couple-stress fluids through uniform and non-uniform channel/tube. They discussed
the effects of couple-stress parameter and magnetic field on pressure and frictional force.

Consideration of porosity is very much necessary to properly explain the fluid dynamical process that occur in different parts of the body, such as vascular beds, lungs, kidneys, and tumorous vessels. Moreover, in many bio-mechanical studies, porosity of the media has significant influence on the transport of fluids. Many technical processes involve parallel flow of fluids of different viscosity and density through porous media. Such parallel flows exist in packed bed reactors in the chemical industry, in petroleum production engineering, and in many other processes as well. Flow through porous medium occurs in filtration of fluids and seepage of water in river beds. Movement of underground water and oils, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones, and small blood vessels are some important examples of flow through porous medium. Some important studies on peristaltic flow of Newtonian fluid with constant viscosity/variable viscosity and non-Newtonian fluids through the porous medium have been presented by El-Shehawey and Husseyen [11], Afifi and Gad [12], Mekheimer [13], El-Shehawey et al. [14], Hayat et al. [15], and Srinivas and Kothandapani [16]. The features of Newtonian, non-Newtonian, and porosity on flow pattern have been discussed.

Fractional calculus has encountered much success in the description of viscoelasticity. The starting point of the fractional derivative model of non-Newtonian model is usually a classical differential equation which is modified by replacing the time derivative of an integer order by the so-called Riemann–Liouville fractional calculus operators. This generalization allows one to define precisely non-integer order integrals or derivatives. Fractional Burgers’ model is the model of viscoelastic fluid. In general, fractional Burgers’ model is derived from well known Burgers’ model by replacing the ordinary time derivatives to fractional order time derivatives and this plays an important role to study the valuable tool of viscoelastic properties. Some authors [17-31] have investigated unsteady flows of viscoelastic fluids with fractional Maxwell model, fractional generalized Maxwell model fractional second grade fluid, fractional Oldroyed-B model, fractional Burgers’ model and fractional generalized Burgers’ model through channel (annulus) tube and solutions for velocity field and the associated shear stress are obtained by using Laplace transform, Fourier transform, Weber transform, Hankel transform, and discrete Laplace transform.

Motivated by the above facts, in this paper the authors have studied the peristaltic transport of viscoelastic fluid with fractional Burgers’ model through an inclined uniform tubes with porus space under the assumption of long-wavelength and low Reynolds number. The effects of fractional parameters, material constants, parameter the pressure difference, pressure gradient, temperature, velocity, and friction force across one wavelength are discussed and presented through graphs.

2. Mathematical model with solution

This geometry of two wall surfaces are mathematically modeled [see Fig. 1] as

\[ \tilde{r}_1 = a_1, \]  
\[ \tilde{r}_2 = a_2 + b \sin \left( \frac{2\pi}{\lambda} (\tilde{Z} - c\tilde{t}) \right), \]
Where $a_1$, $a_2$ are the radii of the inner and outer tubes, $b$ is the amplitude, $\lambda$ is the wavelength, $c$ is the wave velocity and $\tilde{t}$ is the time.

The viscolastic fluid is modeled as a fractional Burgers’ model given by:

$$\left(1 + \tilde{\lambda}_1^a \tilde{D}_\tilde{t}^a + \tilde{\lambda}_2^a \tilde{D}_\tilde{t}^{2a}\right)\tilde{S} = \mu \left(1 + \tilde{\lambda}_3^\beta \tilde{D}_\tilde{t}^\beta\right)\tilde{\gamma},$$  \hspace{1cm} (3)

Where $\tilde{S}$, $\tilde{\gamma}$ and $\mu$ are the shear stress, rate of shear strain and the viscosity, respectively. $\tilde{\lambda}_1$ and $\tilde{\lambda}_3$ ($<\tilde{\lambda}_1$) are the relaxation and the retardation times, respectively. $\tilde{\lambda}_2$ is the new material parameter of the Burgers’ fluid having the dimension of $t^2$, $\alpha$ and $\beta$ are the fractional time derivative parameters such that $0 \leq \alpha \leq \beta \leq 1$. $\tilde{D}_\tilde{t}^a$ is the upper convected fractional derivative defined:

$$\tilde{D}_\tilde{t}^a(\tilde{S}) = D^a(\tilde{S}) + (\tilde{V} \cdot \nabla)(\tilde{S}) - \tilde{L}(\tilde{S}) - (\tilde{S})\tilde{L}^T$$ \hspace{1cm} (4)

In which:

$$\tilde{\gamma} = (\nabla \tilde{V}) + (\nabla \tilde{V})^T$$ \hspace{1cm} (5)

Where $\tilde{L}$ is the velocity gradient and $\tilde{V}$ is the velocity vector, and $D^a = \partial_t^a$ is the fractional differentiation operator of order $\alpha > 0$ with respect to $t$ and may be defined as fractional complex transform:

$$D_t^\alpha \mathcal{C}(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_{t_0}^{t} (s-t)^{n-\alpha-1}[\mathcal{C}(s) - \mathcal{C}(t)] ds$$ \hspace{1cm} (6)

Or it can convert a fractional differential equation to a partial differential equation.

Here $\Gamma(.)$ denotes the Gamma function and

$$\tilde{D}_\tilde{t}^{2a}(\tilde{S}) = \tilde{D}_\tilde{t}^a \left(\tilde{D}_\tilde{t}^a(\tilde{S})\right)$$ \hspace{1cm} (7)
We choose a cylindrical coordinate system \((\tilde{r}, \tilde{z})\) where the \(\tilde{z}\)-axis is the longitudinal direction and the \(\tilde{r}\)-axis is transverse to it.

The governing equations of the motion for the fractional Burger fluids are given by

\[
\rho \left[ \tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{r}} + \tilde{w} \frac{\partial \tilde{u}}{\partial \tilde{z}} \right] = -\frac{\partial \tilde{p}}{\partial \tilde{r}} + \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{S}_{\tilde{r}\tilde{r}}) + \frac{\partial}{\partial \tilde{z}} (\tilde{S}_{\tilde{r}\tilde{z}}) - \frac{\tilde{S}_{\tilde{r}\tilde{z}}}{\tilde{r}} - \frac{\mu}{K} \tilde{u} - \rho g \cos(\varphi) - \tilde{\eta} \nabla^4 \tilde{u} \quad (8)
\]

\[
\rho \left[ \tilde{w} \frac{\partial \tilde{w}}{\partial \tilde{r}} + \tilde{w} \frac{\partial \tilde{w}}{\partial \tilde{z}} \right] = -\frac{\partial \tilde{p}}{\partial \tilde{z}} + \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{S}_{\tilde{r}\tilde{z}}) + \frac{\partial}{\partial \tilde{z}} (\tilde{S}_{\tilde{r}\tilde{z}})
- \frac{\mu}{K} \tilde{w} + \rho g \sin(\varphi) - \tilde{\eta} \nabla^4 \tilde{w} \quad (9)
\]

where \(\rho, \tilde{u}, \tilde{w}, \tilde{\eta}, \tilde{p}, \mu, K, \varphi\) are the fluid density, axial velocity, transverse velocity, material constant associated with couple-stress, pressure, viscosity, permeability parameter, inclination angle, respectively, and

\[
\nabla^2 = \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial}{\partial \tilde{r}} \right), \quad \nabla^4 = \nabla^2 \nabla^2
\]

Introducing the following dimensionless parameters

\[
\begin{align*}
    z &= \frac{\tilde{z}}{\lambda}; \quad r &= \frac{\tilde{r}}{a_2}; \quad t &= \frac{c \tilde{t}}{\lambda}; \quad u &= \frac{\lambda \tilde{u}}{a_2 c}; \quad w &= \frac{\tilde{w}}{c}; \quad \lambda_1 = \frac{c \tilde{\lambda}_1}{\lambda}; \\
    \lambda_2 &= \frac{c \tilde{\lambda}_2}{\lambda}; \quad \lambda_3 = \frac{c \tilde{\lambda}_3}{\lambda}; \quad p &= \frac{\tilde{p} a_2^2}{\mu \lambda}; \quad r_1 = \frac{\tilde{r}_1}{a_2} = \frac{a_1}{a_2} = \epsilon < 1; \\
    r_2 &= \frac{\tilde{r}_2}{a_2} = 1 + \phi \sin 2\pi(z - t); \quad Q = \frac{\tilde{Q}}{\pi a_2^2 c}; \quad R_e = \frac{\rho c a_2}{\mu}; \quad \phi = \frac{b}{a_2}; \\
    F_r &= \frac{c^2}{g a}; \quad \Theta = \frac{T - T_0}{T_1 - T_0}; \quad \Omega = \frac{a_2^2 Q}{k(T_1 - T_0)}; \\
    \tilde{\varrho} &= \frac{\rho a_2}{\tilde{\eta}}; \quad \tilde{\varphi} = \frac{a_2}{e}; \quad \delta = \frac{a_2}{\lambda}. \quad (12)
\end{align*}
\]
\(\delta, \phi, R_e, F_r, \Omega, q\) are the wave number, amplitude ratio, Reynold’s number, the source/sink parameter, Froud number, couple stress parameter, respectively. Applying the long wavelength and low Reynolds number approximations in Eqs. (8-11), we get

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0
\]

\[
\frac{\partial p}{\partial r} = 0
\]

\[
\frac{\partial p}{\partial z} - \frac{R_e}{F_r} \sin(\varphi) = \frac{1}{r} \frac{\partial}{\partial r}(rS_{rz}) - \frac{1}{K} w - \frac{1}{q^2} \nabla^4 w
\]

\[
\frac{\partial^2 \Theta}{\partial r^2} + \frac{1}{r} \frac{\partial \Theta}{\partial r} + \Omega = 0
\]

Where \(S_{rr}, S_{\Theta r}, S_{zz} = 0\), and

\[
S_{rz} = \frac{(1 + \lambda_3^b \frac{\partial \mu}{\partial t})}{(1 + \lambda_1^{a} \frac{\partial a}{\partial t} + \lambda_2^{a} \frac{\partial^2 a}{\partial t^2})} \frac{\partial w}{\partial r}
\]

boundary conditions are given by :

\[
w = -1, \quad \frac{\partial^2 w}{\partial r^2} - \frac{\eta}{r} \frac{\partial w}{\partial r} = 0, u = 0, \Theta = 1 \quad \text{at} \quad r = r_1 = \epsilon
\]

\[
w = -1, \quad \frac{\partial^2 w}{\partial r^2} - \frac{\eta}{r} \frac{\partial w}{\partial r} = 0, u = 0, \Theta = 0 \quad \text{at} \quad r = r_2 = 1 + \phi \sin(2\pi z)
\]

Solving Eq.(16) using the boundary conditions (17), we get

\[
\Theta = \frac{4 \ln(r) + \epsilon^2 \Omega \ln(r) - r^2 \Omega \ln(\epsilon) - 4 \ln(r_2) + r^2 \Omega \ln(r_2) - \epsilon^2 \Omega \ln(r)}{4(\ln(\epsilon) - \ln(r_2))}
\times \frac{-\Omega \ln(r) r_2^2 + \Omega \ln(\epsilon) r_2^2}{4(\ln(\epsilon) - \ln(r_2))}
\]

The equation (15), can be written as
\[ \nabla^4 w = \frac{\alpha^2 \left( 1 + \frac{\lambda^\alpha}{\lambda^1 D^\alpha} D^\beta \right)}{\left( 1 + \lambda^\alpha D^\alpha + \lambda^\beta D^2 \right)} \mu \left( \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} \right) w + \frac{\alpha^2}{K} w \]

Therefore the general solution of equation (19) is

\[ w = B_1 I_0 \left( \xi \sqrt{|S_1|} \right) + B_2 K_0 \left( \xi \sqrt{|S_2|} \right) + B_3 I_0 \left( \xi \sqrt{|S_2|} \right) + B_4 K_0 \left( \xi \sqrt{|S_2|} \right) \]

By using the definition of the fractional differential operator (6), putting

\[ 2 b_1 = \frac{\alpha^2 \left( 1 + \frac{\lambda^\alpha}{\lambda^1 D^\alpha} D^\beta \right)}{\left( 1 + \lambda^\alpha D^\alpha + \lambda^\beta D^2 \right)} \], \quad \lambda^4 = \frac{\alpha^2}{K} \]

Therefore the general solution of equation (19) is

\[ w = B_1 I_0 \left( \xi \sqrt{|S_1|} \right) + B_2 K_0 \left( \xi \sqrt{|S_2|} \right) + B_3 I_0 \left( \xi \sqrt{|S_2|} \right) + B_4 K_0 \left( \xi \sqrt{|S_2|} \right) \]

Where \( \xi = \omega r \) and

\[ S_1 = -\frac{b_1}{\omega^2} - \sqrt{\left( \frac{b_1}{\omega^2} \right)^2 - 1} \], \quad S_2 = -\frac{b_3}{\omega^2} + \sqrt{\left( \frac{b_3}{\omega^2} \right)^2 - 1} \]

Also, we know the corresponding stream functions are

\[ u = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad w = \frac{1}{r} \frac{\partial \psi}{\partial r} \]

For \( I_0 \) and \( K_0 \) are the modified Bessel functions of the first and second kind of order zero. By using the MATHEMATICA program and the boundary conditions given in equation (17) we have a constants \( B_1, B_2, B_3 \) and \( B_4 \).

The volume rate of flow in the fixed coordinate system \((R,Z)\) is given as

\[ Q(Z, t) = 2 \int_{\bar{R}_1}^{\bar{R}_2} \bar{w}(R, Z, \bar{t}) R d\bar{R}. \]
where \( q \) is the volume flow rate in moving coordinate system, and is given by:

\[
q = 2 \int_{r_1}^{r_2}Wr \, dr.
\]  

The time-mean flow over a period \( T = \lambda/c \) and a fixed position is defined as:

\[
\bar{Q} = \frac{1}{T} \int_{0}^{T} Q \, d\bar{t}
\]  

Substituting Eq.(22) into Eq.(24) and integrating we get

\[
\bar{Q} = q + \pi(a_2^2 - a_1^2 + \frac{b^2}{2})
\]  

Which may be written as:

\[
\frac{\bar{Q}}{2a_2^2c} = \frac{q}{2a_2^2c} + \frac{1}{2} \left(1 - \epsilon^2 + \frac{\phi^2}{2}\right)
\]  

With help of Eq.(23) and (26), we have

\[
\bar{Q} = Q + \frac{1}{2} \left(1 - \epsilon^2 + \frac{\phi^2}{2}\right)
\]  

The pressure difference and friction force (on the inner and outer tubes) across the one wavelength are given by

\[
\Delta p = \int_{0}^{1} \frac{dp(z)}{dz} \, dz
\]  

\[
F_{in} = \int_{0}^{1} r_1^2 \left(- \frac{dp(z)}{dz}\right) \, dz
\]  

\[
F_{out} = \int_{0}^{1} r_2^2 \left(- \frac{dp(z)}{dz}\right) \, dz
\]  

3. Results and discussions

This section represents the graphical results in order to be able to discuss the quantitative effects of the sundry parameters involved in the analysis.

The variation of the axial pressure gradient \( dp/dz \) with \( z \) for various values of \( \lambda_1, \lambda_2, \lambda_3 \) and \( K \) are shown in Fig.2(a-d). Figs.(2a,2b,2d) studies the effects of material constants \( (\lambda_1, \lambda_2, \lambda_3) \). These figures show that the axial pressure gradient decreases by
increasing $\lambda_1, \lambda_2$ and $K$. Fig.(2c) shows the variation of the axial pressure gradient $dp/dz$ with material constant $\lambda_3$.

In Fig.3 we studied the variation of time average flux $Q$ with $\Delta p$ for different values of material constant $(\lambda_1, \lambda_2, \lambda_3)$, fractional parameters $(\alpha, \beta)$. Fig.(3a,3b,3e) declares that the time average flux $Q$ decreases in the pumping region ($\Delta p > 0$), while in the co-pumping region ($\Delta p < 0$). $Q$ increases with an increases in $\lambda_1, \lambda_2$ and $K$. The situation is reversed in Fig.(3c,3d), the time average flux $Q$ increases in the pumping region ($\Delta p > 0$), while in the co-pumping region ($\Delta p < 0$). $Q$ decreases with an increase in $\lambda_3$ and $\alpha$.

Fig.2: Pressure gradient $dp/dz$ versus axial distance $z$ when $Q = 0.2$ and $t = 1$ corresponding to (a) different values of $\lambda_1$ with $\epsilon = 0.1, \phi = 0.8, \beta = 0.4, \alpha = 0.3, \lambda_2 = 2, \lambda_3 = 3, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.7, \rho = 2.5, \eta = 0.01$. (b) different values of $\lambda_2$ with $\epsilon = 0.1, \phi = 0.8, \beta = 0.4, \alpha = 0.3, \lambda_2 = 2, \lambda_3 = 3, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.7, \rho = 2.5, \eta = 0.01$. (c) different values of $\lambda_3$ with $\epsilon = 0.1, \phi = 0.8, \beta = 0.4, \alpha = 0.3, \lambda_2 = 2, \lambda_3 = 3, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.7, \rho = 2.5, \eta = 0.01$. (d) different values of $K$ with $\epsilon = 0.1, \phi = 0.8, \beta = 0.4, \alpha = 0.3, \lambda_2 = 2, \lambda_3 = 3, R_e = 0.008, F_r = 0.005, \varphi = 0.7, \rho = 2.5, \eta = 0.01$.

Fig.4(a-j) present the variation of friction force with averaged flow rate for different values of pertinent parameters. It can be seen that the effect of increasing the flow rate is to enhance the frictional force. As expected, Fig.4(a-e) shows that friction force reduces in magnitude with rise $\lambda_1, \lambda_2, \alpha$ and $\beta$. The influence of the third material constant $\lambda_3$ on friction force is shown in Fig.(4c). It is found that the effect of material constant $\lambda_3$ on friction force is opposite to that of the first and second fractional parameter $(\alpha, \beta)$ and material constants $(\lambda_1, \lambda_2)$. Fig.4(f-j) shows that friction force reduces in magnitude with rise $\lambda_2$ and $\beta$. The influence of $\lambda_1, \lambda_3, \alpha$ on friction force is shown in Fig.(4f),(4h) and (4i). It is found that the effect of second material constant $\lambda_2$ and second fractional parameter $\beta$ on friction force is opposite $\lambda_1, \lambda_3$ and $\alpha$. 
In Figs.(5a,5e) we represent the axial velocity $w$ for different values of same physical parameters. These Figures indicate that the velocity $w$ increases with increasing $\lambda_2, \alpha$, while it decreases with an increase in $\lambda_1, \lambda_3, \beta$.

Fig.6(a-c) indicates the behavior of parameters appearing in the temperature distribution. Figs.(6a,6b) shows that the magnitude of temperature decreases near the lower wall of the channel and increases in the rest part of the channel when there is an increase in $\phi$ and $\Omega$. Fig.6(c) illustrates that the temperature decreases when $\epsilon$ increases.

Fig.3: Pressure rise $\Delta p$ versus time-averaged flow rate when $t = 1$ corresponding to (a) different values of $\lambda_1$ with $\epsilon = 0.3, \phi = 0.3, \beta = 0.4, \alpha = 0.3, \lambda_2 = 2, \lambda_3 = 3, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.5, \varphi = 0.5, q = 2.5, \eta = 0.1$. (b) different values of $\lambda_3$ with $\epsilon = 0.3, \phi = 0.3, \beta = 0.4, \alpha = 0.3, \lambda_2 = 1, \lambda_3 = 3, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.5, q = 2.5, \eta = 0.1$. (c) different values of $\lambda_2$ with $\epsilon = 0.3, \phi = 0.3, \beta = 0.4, \alpha = 0.3, \lambda_1 = 2, \lambda_2 = 4, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.5, q = 2.5, \eta = 0.1$. (d) different values of $\alpha$ with $\epsilon = 0.1, \phi = 0.8, \beta = 0.4, K = 0.9, \lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 3, R_e = 0.008, F_r = 0.005, \varphi = 0.5, q = 2.5, \eta = 0.1$. (e) different values of $\beta$ with $\epsilon = 0.3, \phi = 0.3, \alpha = 0.3, \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.5, q = 2.5, \eta = 0.1$. 
Figs. 7(a-c) depict the behavior of heat transform coefficient at the wall. Heat transform coefficient has an oscillatory behavior due to peristalsis. Figs. 7(a-c) shows the variation of heat transfer coefficient $Z$ with $\epsilon, \phi$ and $\Omega$. Absolute value of heat transfer coefficient increases with an increase in $\epsilon, \phi$ and $\Omega$ respectively.

The formation of an internally circulating bolus of the fluid by closed streamline is called trapping and this trapped bolus pulled ahead along with the peristaltic wave.

Fig. 4(a-e): Friction force on outer tube vs. averaged flow rate for (a) various values of $\lambda_1$ with $\epsilon = 0.1, \phi = 0.3, t = 1, \beta = 0.4, \alpha = 0.3, \lambda_2 = 2, \lambda_3 = 3, K = 0.9, R_e = 0.008, F_r = 0.005, \phi = 0.5, q = 2.5, \eta = 0.1$. (b) various values of $\lambda_2$ with $\epsilon = 0.3, t = 1, \phi = 0.3, \beta = 0.4, \alpha = 0.3, \lambda_1 = 1, \lambda_3 = 3, K = 0.9, R_e = 0.008, F_r = 0.005, \phi = 0.5, q = 2.5, \eta = 0.1$. (c) various values of $\lambda_3$ with $\epsilon = 0.3, t = 1, \phi = 0.3, \beta = 0.4, \alpha = 0.3, \lambda_1 = 1, \lambda_2 = 2, K = 0.9, R_e = 0.008, F_r = 0.005, \phi = 0.5, q = 2.5, \eta = 0.1$. (d) various values of $\alpha$ with $\epsilon = 0.1, t = 1, \phi = 0.8, \beta = 0.4, K = 0.9, \lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 3, R_e = 0.008, F_r = 0.005, \phi = 0.5, q = 2.5, \eta = 0.1$. (e) various values of $\beta$ with $\epsilon = 0.3, t = 1, \phi = 0.3, \alpha = 0.3, \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3, K = 0.9, R_e = 0.008, F_r = 0.005, \phi = 0.5, q = 2.5, \eta = 0.1$. 

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Since this bolus appears to be trapped by the wave, the bolus moves with the same speed as that of the wave. The streamlines for different values of $\lambda_1$ are shown in Fig.(8a,8b). It is at evident that the trapped bolus which are moving as whole increases in size with the increase in $\lambda_1$. The effects of second material constant $\lambda_2$ can be seen through Fig.(8c,8d). It is observed that when we increase $\lambda_2$, the size of trapped bolus decreases. Fig.(8e,8f) is plotted to see the effects of $K$, it observed that trapped bolus increasing with the increase of $K$ and in Fig.(8g,8h) trapped bolus are going to be vanish as we increase $\beta$.

**Fig. 4(f-g):** Friction force on inner tube vs. averaged flow rate for (f) various values of $\lambda_1$ with $\epsilon = 0.1, \phi = 0.3, t = 1, \beta = 0.4, \alpha = 0.3, \lambda_2 = 2, \lambda_3 = 3, K = 0.9, R_s = 0.008, F_r = 0.005, \varphi = 0.5, q = 2.5, \eta = 0.1$. (g) various values of $\lambda_2$ with $\epsilon = 0.3, t = 1, \phi = 0.3, \beta = 0.4, \alpha = 0.3, \lambda_1 = 1, \lambda_3 = 3, K = 0.9, R_s = 0.008, F_r = 0.005, \varphi = 0.5, q = 2.5, \eta = 0.1$. (h) various values of $\lambda_3$ with $\epsilon = 0.3, t = 1, \phi = 0.3, \beta = 0.4, \alpha = 0.3, \lambda_1 = 1, \lambda_2 = 2, K = 0.9, R_s = 0.008, F_r = 0.005, \varphi = 0.5, q = 2.5, \eta = 0.1$. (i) various values of $\alpha$ with $\epsilon = 0.1, t = 1, \phi = 0.8, \beta = 0.4, K = 0.9, \lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 3, R_s = 0.008, F_r = 0.005, \varphi = 0.5, q = 2.5, \eta = 0.1$. (j) various values of $\beta$ with $\epsilon = 0.3, t = 1, \phi = 0.3, \alpha = 0.3, \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3, K = 0.9, R_s = 0.008, F_r = 0.005, \varphi = 0.5, q = 2.5, \eta = 0.1$. 
Fig. 5: Axial velocity $W$ versus radial direction $r$ when $t = 0.1$ corresponding to (a) different values of $\lambda_1$ with $\epsilon = 0.1, \phi = 0.2, \beta = 0.4, \alpha = 0.3, \lambda_2 = 0.2, \lambda_3 = 0.3, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.3, \rho = 2.5, \eta = 0.01, Q = 0.2, z = 0.5$. (b) different values of $\lambda_2$ with $\epsilon = 0.1, \phi = 0.2, \beta = 0.4, \alpha = 0.3, \lambda_1 = 0.1, \lambda_3 = 0.3, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.3, \rho = 2.5, \eta = 0.01, Q = 0.2, z = 0.5$. (c) different values of $\lambda_3$ with $\epsilon = 0.1, \phi = 0.2, \beta = 0.4, \alpha = 0.3, \lambda_1 = 0.1, \lambda_2 = 0.2, K = 0.9, R_e = 0.008, F_r = 0.005, \varphi = 0.3, \rho = 2.5, Q = 0.2, z = 0.5, \eta = 0.01$. (d) different values of $K$ with $\epsilon = 0.1, \phi = 0.2, \beta = 0.4, \alpha = 0.3, \lambda_1 = 0.1, \lambda_2 = 0.2, K = 0.9, \lambda_3 = 0.3, R_e = 0.008, F_r = 0.005, \varphi = 0.3, \rho = 2.5, Q = 0.2, z = 0.5, \eta = 0.01$. (e) different values of $\varphi$ with $\epsilon = 0.1, \phi = 0.2, \beta = 0.4, \alpha = 0.3, \lambda_1 = 0.1, \lambda_2 = 0.2, \lambda_3 = 0.3, K = 0.9, R_e = 0.008, F_r = 0.005, Q = 0.2, \rho = 2.5, z = 0.5, \eta = 0.01,$ $\varphi = 0.3, 0.5, 0.7, 0.9.$
Fig. 6: Effect of $\phi, \Omega, \epsilon$ on temperature $\theta$ for (a) $\epsilon = 0.3$, $\alpha = 0.1$, $\Omega = 4$
(b) $\epsilon = 0.3$, $\phi = 0.6$, $\alpha = 0.1$.
(c) $\phi = 0.6$, $\alpha = 0.1$, $\Omega = 5$.

Fig. 7: Effect of $\phi, \Omega, \epsilon$ on heat transfer coefficient $Z$ for
(a) $\phi = 0.2$, $\Omega = 1$.
(b) $\epsilon = 0.2$, $\Omega = 5$.
(c) $\phi = 0.6$, $\epsilon = 2$.

Fig. 8(a,b): Streamlines for $\epsilon = 0.1$, $\phi = 0.2$, $\beta = 0.4$, $\alpha = 0.3$, $\lambda_2 = 3$, $\lambda_3 = 3$, $K = 0.9$, $R_x = 0.008$, $F_t = 0.005$, $\varphi = 0.2$, $\varphi = 3$, $\eta = 0.01$, $Q = 0.1$, $\tau = 1$. and for different $\lambda_1$:
(a) $\lambda_1 = 1$.
(b) $\lambda_1 = 6$. 
Fig. 8(c,d). Streamlines for 
and for different 

Fig. 8(e,f,g,h). Streamlines for 
for different where (g), (h) and for different where (c), (d)
4. Conclusion

In this paper, viscoelastic fluid flow with fractional Burger’s model for heat effect with peristaltic transport through uniform inclined tubes with permeable walls. The effects of couple-stress parameters, permeability parameter, material constants and fractional parameter have been emphasized. It is concluded that

- The pressure gradient increases with increasing third material constant $\lambda_3$.
- The linear relation is found between pressure and flow.
- The linear relation is found between friction force and flow.
- The pressure rise $\Delta p$ increasing in the pumping region ($\Delta p > 0$) for an increase in first fractional parameter $\alpha$ and third material constant $\lambda_3$.
- The fractional force on inner tube decreasing in the pumping region for an increase in $\lambda_3$. While opposing behavior is seen for $\lambda_1$ and $\alpha$ for outer tube.
- The axial velocity $W$ increases with increasing $\lambda_2, \alpha$.
- Temperature increases with increasing value of amplitude ratio $\phi$ and source/sink parameter.
- The absolute value of heat coefficient increases with increase of $\epsilon, \phi$ and $\Omega$.
- The occurrence of trapping can be reduced by increasing the magnitude of $\lambda_1, K$.

References