TOTAL EDGE IRREGULARITY STRENGTH OF THE GENARALIZED WEB GRAPH $W_0(t, n)$

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Abstract

An edge irregular total k-labeling of a graph G is such a labeling of the vertices and edges with integers 1,2,...,k that the weights of any two different edges are distinct, where the weight of an edge is the sum of the label of the edge itself and the labels of its incident vertices. The minimum k, for which the graph G has an edge irregular total k-labeling, is called the total edge irregularity strength of the graph G and is denoted by tes(G).

In this paper, the upper bound of the total edge irregularity strength of the generalized web graph $W_0(t, n)$, $t \geq 2$ and n odd, is determined.

Keywords: Graphs, *an edge irregular total k-labeling, the total edge irregularity strength, the web graph $W_0(2, n)$, the generalized web graph $W_0(t, n)$,* $t \geq 2$ and n odd.

Subject Classification: *05C99*

1. INTRODUCTION

We consider only finite, simple and undirected graphs, without self-loops and multiple edges. Let G = (V, E) be a graph with the vertex set V and the edge set E.

A **walk** of a graph G is an alternating sequence of vertices and edges $v_0, e_1, v_1, e_2, v_3, \ldots, v_{n-1}, e_n, v_n$, beginning and ending with vertices, in which each edge is incident with two vertices immediately preceding and succeeding it. It is called a **v_0v_n -path** if all the vertices and edges are distinct. If the two end vertices are same, i.e. if $v_0=v_n$ in a path, then it is called a **cycle.** We henceforth denote the cycle consisting of n vertices and n edges by C_n. The **degree (**or **valency)** of a vertex v in a graph G, denoted by deg(v), is the number of edges incident with v. A vertex v is called a**pendent vertex**or an **end vertex** if deg (v) = 1. An edge incident to a pendent vertex is called a **pendentedge** or an **end edge**.

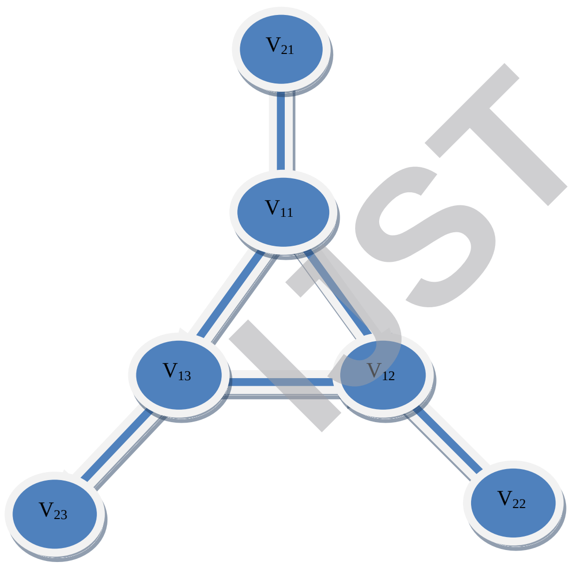

Fig 1: The Crown $C_3 \odot K_1$

For odd n, the **Crown $C_n \odot K_1$** is the graph obtained from a cycle C_n by attaching a pendent edge at each vertex of the cycle. The **web graph W_0 (2, n)** without center is the graph obtained from $C_n \odot K_1$ by joining the pendent vertices to form the cycle and then adding a single pendent edge at each vertex of the outer cycle. **The generalized web graph W_0(t, n)** without center is the graph obtained by iterating the process of constructing $W_0(2, n)$ from $C_n \odot K_1$ so that the web graph has exactly t cycles, $t \geq 2$ and n odd.

Graph Labeling is a mathematical discipline of Graph Theory, closely related to the field of Computer Science. It concerns the assignment of values, usually represented by positive integers, to the edges and vertices of a graph. Many of the graph labeling methods were motivated by application to Technology and Sports tournament Scheduling. By a labeling, we mean any mapping that carries a set of graph elements to set of numbers(usually positive integers) called labels.

The notions of the total edge irregularity strength and the total vertex irregularity strength were first introduced by Bača et al [1] in a recent paper. They are invariants analogous to irregularity strength of a graph G [3, 4, 6, 8, 9].

An edge irregular total k-labeling of a graph G is such a labeling of the vertices and edges with integers 1, 2,..., k that the weights of any two different edges are distinct, where the weight of an edge is the sum of the label of the edge itself and the labels of the two end vertices. The minimum k for which the graph G has an edge irregular total k-labeling, is called the total edge irregularity strength of the graph G and in denoted by tes(G).

In [1], Bača et al put forward the lower bound of tes(G) and tvs(G), in terms of the maximum degree Δ, minimum degree δ, the size $|E(G)|$ of the edge set and the size $|V(G)|$ of the vertex set, which may be stated as in Theorem 1.1 and 1.2 respectively.

Theorem 1.1: tes(G) $\geq$ max $\left\{\left\lceil \frac{\Delta+1}{2} \right\rceil, \left\lceil \frac{|E(G)|+2}{3} \right\rceil\right\}$

Based on this theorem, Bača et al[1] determined the exact values of the total edge irregularity strength of path P_n, star S_n, wheel W_n and friendship graph F_n.Ivančo and Jendrol′ [5] determined the total edge irregularity strength of any tree. Jendrol′ et al [7] proved the exact values of the total edge irregularity strength of complete graphs and complete bipartite graphs. Miskuf and Jendrol′ [10] determined the exact values of the total edge irregularity strength of grids. Brandt et al [2] proved a conjecture about edge irregular total labeling. Tong Chunling et al[11] obtained the exact values of the total edge irregularity strength of some families of graphs including the generalized Petersen graph, Ladder, Mobius band etc.

Note: The symbol $\lfloor x \rfloor$ means the greatest integer not greater than x and

$\lceil x \rceil$ means the smallest integer not smaller than x .

2 MAIN RESULTS

In this paper,the upper bound of the total edge irregularity strength of the generalized web graph $W_0(t, n)$, $t \geq 2$ and n odd, is determined.

Theorem 2.1: For $t \geq 2$ and n odd,

$$tes(W_0(t, n)) = 2n - 1, \quad \text{if } t = 2$$

$$= (n - 1)(t - 1) + 3, \quad \text{if } t \geq 3.$$

Proof: For $t \geq 2$ and n odd, let $W_0(t, n) = (V, E)$ be the generalized web graph without center on n(t+1) vertices and 2nt edges, defined on $V = V_1 \cup V_2$,

where $V_1 = \{ v_{ij} : i = 1,2,\ldots,t ; j = 1,2,\ldots, n \}$

$V_2 = \{v_{(t+1)j} : j = 1,2,\ldots, n\}$

with v_{ij} ($i = 1, 2,\ldots, t ; j = 1, 2,\ldots n$) as j^{th} vertex in i^{th} cycle

and $E = E_1 \cup E_2 \cup E_3$,

where $E_1 = \{v_{ij}v_{i(j+1)}: i = 1,2,\ldots,t ; j= 1,2,\ldots, n \text{ modulo } n\}$

$E_2 = \{v_{ij}v_{(i+1)j}: i = 1, 2,\ldots, t ; j = 1, 2, \ldots, n\}$

& $E_3 = \{v_{tj}v_{(t+1)j}: j = 1, 2,\ldots, n\}$

Now let us construct a mapping f for $W_0(t, n)$, $t \geq 2$ and n odd, as follows:

For i = 1, $f(v_{1j}) = 1$, $j = 1, 2,\ldots, n$

$f(v_{1j}v_{1(j+1)}) = j$, $j = 1, 2,\ldots,n$ modulo n

$f(v_{1j}v_{2j}) = j$, $j = 1, 2,\ldots, n$

For $i = 2, 3,\ldots, t$, $f(v_{ij}) = (n - 1)(i - 1) + 2$, $j = 1, 2,\ldots, n$

$f(v_{ij}v_{i(j+1)}) = 2i+j- 4$, $j = 1, 2,\ldots, n$ modulo n

$f(v_{ij}v_{(i+1)j}) =2i+j-3$, $j = 1, 2,\ldots, n$

and $f(v_{(t+1)j}) = (n - 1)(i - 1) + 3$, $j = 1, 2, \ldots, n$

$f(v_{tj}v_{(t+1)j}) = n + 2t + j - 5$, $j = 1, 2, \ldots, n$

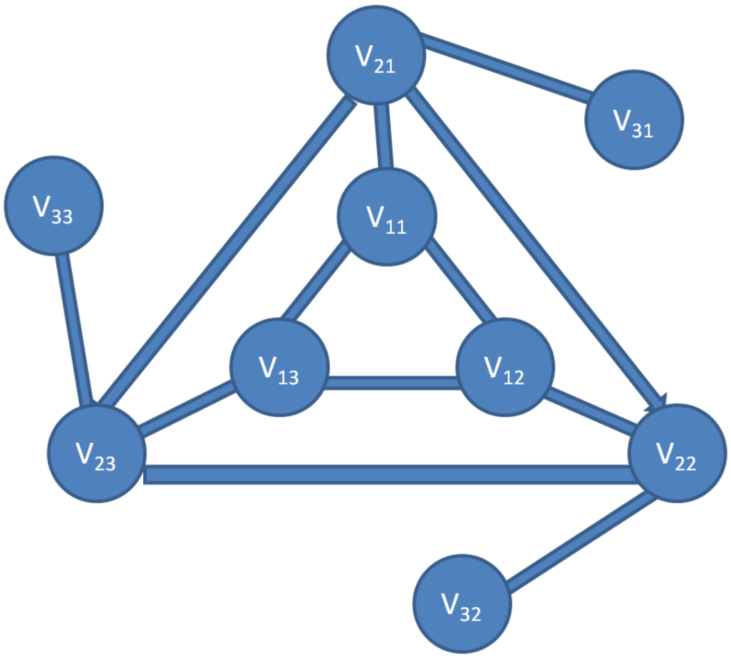

Fig 2: The web graph $W_0(2, 3)$without center

Since wt $(v_{ij}v_{i(j+1)}) = f(v_{ij}) + f(v_{ij}v_{i(j+1)}) + f(v_{i(j+1)})$

$= (n - 1)(i - 1) + 2 + 2i + j - 4 + (n - 1)(t - 1) + 2$

$= 2 (n - 1)(i - 1) + 4+2i +j-4,$

$= 2(n - 1)(i - 1) + 2i + j \quad 2\leq i \leq t, j =1,2,\ldots, n$modulo n

$wt(v_{ij}v_{(i+1)j}) = f(v_{ij}) + f(v_{ij}v_{(i+1)j}) + f(v_{(i+1)j})$

$= (n - 1)(i - 1) + 2 + 2i + j – 3 + (n - 1) i + 2$

$= (n - 1) (2i – 1)+ 2i +j+1, \qquad 2 \leq i \leq t-1, j =1,2,\ldots, n$

wt $(v_{tj}v_{(t+1)j}) = f(v_{tj}) + f(v_{tj}v_{(t+1)j}) + f(v_{(t+1)j})$

$= (n - 1)(t - 1) + 2 + n + 2t + j – 5 + (n - 1)(t - 1) + 3$

$= 2 (n - 1)(t - 1) + n + 2t + j, \qquad j = 1, 2,\ldots, n$

the weights of the edges of the generalized web graph $W_0(t, n)$ without center, under the labeling f, constitute the set {3, 4,5,...,2nt+2} and the mapping f is a mapping from $V \cup E$ into {1,2,...,(n-1)(t-1)+3}. Clearly f is a labeling of the vertices and edges with integers 1, 2, ,...,(n-1)(t-1)+3such that the weights of two different edges are distinct, for $t \geq 2$.Therefore f is an edge irregular total k labeling and tes $(W_0(t, n)) \leq (n-1)(t-1)+3$, $t \geq 3$.

But $tes(W_0(t, n)) \geq \text{Max}\left\{\left\lceil \frac{\Delta+1}{2} \right\rceil, \left\lceil \frac{|E|+2}{3} \right\rceil\right\}$

$$\geq \text{Max}\left\{\left\lceil \frac{n+1}{2} \right\rceil, \left\lceil \frac{2nt+2}{3} \right\rceil\right\}$$

$$\geq \left\lceil nt + \tfrac{2}{3} \right\rceil$$

$$= nt+1$$

$$= (n-1)(t-1) + n + t$$

$$\geq (n-1)(t-1) + 3 \quad \text{for } n \geq 3,\ t \geq 2$$

Hence tes $(W_0(t, n)) = (n-1)(t-1) + 3$, for $n \geq 3$, $t \geq 2$

Conjecture: *Is the Generalized Web Graph $W_0(t, n)$ Vertex Irregular Total K- labeling?*

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