SOLUTION OF THE EQUATION $y^2=6x^2+1$

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ABSTRACT

A binary quadratic equation $y^2=6x^2+1$ representing hyperbola is considered for finding its infinitely many non zero integral solutions.

KEY WORDS: Diophantine, quadratic, Binary

INTRODUCTION:

The binary quadratic equation is of the form $y^2 = Dx^2 + 1$, where D is a non-square positive integer. A binary Diophantine equation is an equation relating integer quantities. The binary quadratic Diophantine equations gives us an unlimited field for research because of their variety This communication concerns with interesting binary quadratic equation $\mathbf{y^2=6x^2+1}$ which is Representing a hyperbola, for finding out its various non zero integral solutions. Also, following are the few interesting relations among the solution are given.

METHOD OF ANALYSIS:

Consider the equation,

$$y^2=6x^2+1$$

Which is a Pellian equation, whose general solution (x_n, y_n)is given by

$$x_n = \frac{1}{2\sqrt{6}} g_n \qquad y_n = \frac{1}{2} f_n$$

Where $f_n = \{[5+2\sqrt{6}]^{n+1} + [5-2\sqrt{6}]^{n+1}\}$

$g_n = \{[5+2\sqrt{6}]^{n+1} - [5-2\sqrt{6}]^{n+1}\}$ n =0,1,2,3,……

The relations satisfied with x_n and y_n are given by

$y_{n+2}-10y_{n+1}+y_n=0$ with $y_0=5$ & $y_1= 49$

similarly,

$x_{n+2}-10x_{n+1}+x_n=0$ with $x_0=2$ & $x_1=20$

n	x_n	y_n
0	2	5
1	20	49
2	198	485
3	1960	4801
4	19402	9505
5	192060	470449

A few intereting solutions among the solutions are as follows

1. x_n is always even.
2. y_n is always odd.
3. $y_{2n} \equiv 0 \pmod 5$.
4. $x_{2n+1} \equiv 0 \pmod{20}$.
5. $2y_{2n+1}+2$ is a perfect square.
6. $x_{n+1}=5x_n+2y_n$
7. $x_{n+2}=49x_n+20y_n$
8. $y_{n+1}=5y_n+12x_n$
9. $5xn+1-2y_{n+1}=x_n$
10. $5y_{n+1}-2xn+1=yn$
11. $5x_n+2-20y_{n+1}=5x_n$
12. $49y_{n+1}-12x_{n+2}=5y_n$
13. $49x_n+2-20y_{n+2}=x_n$

14.$20x_{n+1}-2x_{n+1}=2x_n$

15.$5y_ny_{n+1}-12x_nx_{n+2}=5x_ny_n$

16.$49x_ny_{n+1}-12x_nx_{n+2}=5x_ny_n$

17.$x_ny_{n+1}-y_nx_{n+1}=12x_n^2-2y_n^2$

18..$5y_{n+2}-49y_{n+1}=12x_n$

19.$y_{n+2}=49y_n+120x_n$

20.$x_ny_{n+2}-y_nx_{n+2}=10x_n^2-20y_n^2$

21.$120x_ny_{n+1}-12x_ny_{n+2}=12x_ny_n$

22.$y_{3n+2}+3y_n=2y_n(y_{2n+1}+1)$

23.$y_nx_{n+2}-4y_ny_{n+1}=x_ny_n$

CONCLUSION:

In the paper, I have presented infinitely many integer solutions for the equation $y^2=6x^2+1$.As the Binary quadratic Diophantine equation have variety of choices, one may search for the other Diophantine equations and find out their integer solutions along with suitable properties.

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