

## On Random Fixed Points of Uniformly Lipschitz in Random Operators

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### Abstract:

The purpose of this paper is to prove two random fixed point theorems for uniformly 1-lipschitz in random operator on non-star-shaped domain in Banach spaces. Our results is modified to results in [35].

**Keywords:** random operators, random fixed points, asymptotically non-expansive random operator, uniformly 1 –Lipschitz in random operator, Banach spaces.

### 1. Introduction

The random approximations and random fixed point theorems are stochastic generalizations of usual approximations and fixed points theorems. Recently, many researchers are interested in this subject such as Beg and Shahzad [5], [6] who proved some theorems about the best approximations as applications for some stochastic fixed point theorems. And, Khan, Thaheem and Hussain [14] are proved random fixed point theorems for nonexpansive random operators defined on a class of nonexpansive sets containing the subclass of starshaped sets. They establish generalizations of some fixed point theorems on a class of non-convex sets in a locally bounded topological vector space, and then, obtain Brosowski – Memardus type theorems about random invariant of approximation [9]. Nashine [17] establishes the existence of random fixed point as random best approximations with respect to compact and weakly compact domain. Alsaidy and el at [2] proved coincidence point results for pair of commuting mapping defined on weakly compact separable subset of complete p-normed space. And then, use them to study the random best approximation in p-normed space with separability condition [1].

In [fixed point and random best approximations results are proved for random Banach operator which is defined on separable closed subset of a complete p-normed space. Kirk and Ray [15] have shown that if  $X$  is uniformly convex Banach space,  $A$  is an unbounded closed convex subset of  $A$  and

$H : A \rightarrow A$  is a Lipschitzian pseudo contractive mapping for which the set  $G(x, Hx; x) = \{z \in X : \|z - Hx\| \leq \|z - x\|\}$  is bounded for some  $x \in A$  then  $H$  has a fixed point in  $A$ . Afterward Carbone and Marino [10] employed the structure of some geometric sets in Banach spaces with this property Penot [18] and Beg, and Abbas [7] imposing the condition of asymptotic contractivity on nonexpansive mappings defined on unbounded closed and convex

subset of a Banachspace, established some fixed point theorems. Here, we prove some random fixed point theorems for asymptotically nonexpansive random operators defined on an unbounded closed and non-star-shaped subset of a Banach space, where. employ the simple strandom iterative process to obtain the existence of random fixed points of such operators. As a consequence, a stochastic generalization and improvements of the comparable results valid for bounded convex sets in the literature ([12]and [18]) are obtained.

The following classes are needed

$2^X$  is the classes of all subsets of  $X$ ,

$CB(X)$  is the classes of all bounded closed subsets of  $X$ ,

$wk(X)$  is the classes of all weakly compact subsets of  $X$ ,

$\overline{\{coA\}}_w$  is the weak closure of convex hull of  $A$ .

## 2. Preliminaries

Let the pair  $(\Omega, \Sigma)$  denote to the measurable space with sigma algebra  $\Sigma$  of subsets of  $\Omega$  and  $X$  is a metric space.

**Definition (2.1):[22]**

A mapping  $F: \Omega \rightarrow 2^X$  is called measurable (respectively, weakly measurable) if, for any closed (respectively, open) subset  $B$  of  $X$ ,  $F^{-1}(B) = \{\omega \in \Omega: F(\omega) \cap B \neq \emptyset\} \in \Sigma$ .

**Definition (2.2): [20]**

A mapping  $\delta: \Omega \rightarrow X$  is called a measurable selector of a measurable mapping  $F: \Omega \rightarrow 2^X$ , if  $\delta$  measurable and  $\delta(\omega) \in F(\omega)$  for each  $\omega \in \Omega$ .

**Definition (2.3): [13]**

A mapping  $h: \Omega \times X \rightarrow X$  (or  $G: \Omega \times X \rightarrow CB(X)$ ) is called a random operator if for any  $x \in X$ ,  $h(\cdot, x)$  (respectively  $G(\cdot, x)$ ) is measurable.

**Definition (2.4):[21]**

A measurable mapping  $\delta: \Omega \rightarrow A$  is called random fixed point of a random operator  $h: \Omega \times X \rightarrow X$  (or  $G: \Omega \times X \rightarrow CB(X)$ ), if for every  $\omega \in \Omega$ ,  $\delta(\omega) = h(\omega, \delta(\omega))$  (respectively  $\delta(\omega) \in G(\omega, \delta(\omega))$ ).

**Definition (2.5):[7]**

A mapping  $h: A \rightarrow X$  is called demi-closed with respect to  $y \in X$  if for each sequence  $\{x_n\}$  in  $A$  such that  $\{x_n\}$  converges weakly to  $x \in X$  and  $\{h(x_n)\}$  converges strongly to  $y$  imply that  $x \in A$  and  $h(x) = y$ .

**Definition (2.6):[7]**

Let  $X$  be a normed space. A mapping  $G: \Omega \times X \times X \rightarrow \mathbb{R}$  which satisfies the following conditions

i.  $G(\omega, \lambda x, y) = \lambda G(\omega, x, y)$

ii.  $G(\omega, x + y, z) = G(\omega, x, z) + G(\omega, y, z)$

iii.  $\|x\| \leq G(\omega, x, x)$

iv. There is  $M > 0$  such that  $|G(\omega, x, y)| \leq M\|x\| \cdot \|y\|$

For any  $x, y, z \in X$ ,  $\lambda > 0$  and  $\omega \in \Omega$ .

The mapping  $G$  is used to obtain a random fixed point of an uniformly 1-lipschitz random operators defined on unbounded domain.

**Definition (2.7):[3],[19]**

Let  $X$  be a normed space,  $\emptyset \neq A \subseteq X$  and  $h: \Omega \times A \rightarrow A$  be a random operator, then  $h$  is said to be

- i- asymptotically nonexpansive, if there exists a sequence of measurable mappings  $k_n: \Omega \rightarrow [1, \infty)$  with  $\lim_{n \rightarrow \infty} k_n(\omega) = 1$  for each  $\omega \in \Omega$ , such that for arbitrary  $x, y \in A$  we have

$$\|h^n(\omega, x) - h^n(\omega, y)\| \leq k_n(\omega) \|x - y\|, \text{ for each } \omega \in \Omega.$$

ii- uniformly  $L$  –Lipschitzain, if for any  $x, y \in A$  ,  
 $\|h^n(\omega, x) - h^n(\omega, y)\| \leq L \|x - y\|$  , where  $L > 0$  ,  $n = 1, 2, \dots$  .

**Remark (2.1):**[ 19]

Every asymptotically nonexpansive mapping is uniformly  $L$  –Lipschitzain .The random operator  $h: \Omega \times A \rightarrow A$  is said to be uniformly 1 –Lipschitzain iff  $L = 1$  in definition above.

**Definition (2.8):**[7]

A random operator  $h: \Omega \times A \rightarrow A$  is said to be satisfy property (q), if for any bounded sequence  $\langle x_n \rangle$  in  $A$  with  $\lim_{n \rightarrow \infty} \|h^n(\omega, x_n) - x_n\| = 0$  for all  $\omega \in \Omega$  implies that  $\lim_{n \rightarrow \infty} \|x_{n+1} - x_n\| = 0$  .

Note that, throughout this paper we assume that  $h$  satisfies the property (q) .

**Definition (2.9):**

A set  $A$  is said to have property (p) if

- (i)  $h: \Omega \times A \rightarrow A$  ,
- (ii)  $(1 - k_n)q + k_n h^n(\omega, x) \in A$  , for some  $q \in A$  and sequence of real numbers  $k_n$  ( $0 < k_n < 1$ ) converging to 1 and for each  $x \in A$  .

**Definition (2.10):**[7]

Let  $X$  be a normed space,  $\emptyset \neq A \subseteq X$  and  $h: \Omega \times A \rightarrow A$  be a random operator,  $h$  is said to satisfy condition (B) if for fixed  $x_0 \in A$  , we have

$$\lim_{\substack{\|x\| \rightarrow \infty \\ x \in A}} \sup \frac{\|h^n(\omega, x) - h(\omega, x_0)\|}{\|x - x_0\|} < 1$$

For each  $\omega \in \Omega$  and  $n \in \mathbb{N}$  .

**Definition (2.11):** [4]

Let  $X$  be a normed space,  $\emptyset \neq A \subseteq X$  and  $h: \Omega \times A \rightarrow A$  be a random operator,  $h$  is said to be asymptotically contractive, if for each  $x \in A$  there exists  $x_0 \in A$  with

$$\lim_{\substack{\|x\| \rightarrow \infty \\ x \in A}} \sup \frac{\|h(\omega, x) - h(\omega, x_0)\|}{\|x - x_0\|} < 1$$

For each  $\omega \in \Omega$  .

### 3. Main results

We begin with

**Theorem ( 3.1):**

Let  $X$  be a separable reflexive Banach space,  $\emptyset \neq A \subseteq X$  unbounded closed subset of  $X$  has property (p) w.r.t.  $h$  ,  $h: \Omega \times A \rightarrow X$  be a uniformly 1 –Lipschitzain random operator and  $I - h(\omega, \cdot)$  be demi-closed for all  $\omega \in \Omega$ . If  $\lim_{\|x\| \rightarrow \infty} \sup \frac{G(\omega, h^n(\omega, x) - q, x)}{\|x\|} < 1$  , then  $h$  has a random fixed point.

**Proof:**

Since  $A$  has property (p) w.r.t.  $h$  , then

$$(1 - k_n)q + k_n h^n(\omega, x) \in A$$

for some  $q \in A$  and a fixed real sequence  $\langle k_n \rangle$  converging to 1 ( $0 < k_n < 1$ ) , for all  $x \in A$  and for all  $\omega \in \Omega$  .now, for each  $n \geq 1$  , defined the random operators  $h_n$  by

$$h_n(\omega, x) = (1 - k_n)q + k_n h^n(\omega, x) , \text{ for all } x \in A \text{ and all } \omega \in \Omega .$$

so,  $h_n: \Omega \times A \rightarrow A$  . Since  $h$  is uniformly 1 –Lipschitzain random operator , then

$$\|h_n(\omega, x) - h_n(\omega, y)\| = |k_n| \|x - y\|$$

for all  $x \in A$ , and all  $\omega \in \Omega$ . Therefore each  $h_n$  is contractive random operators.

Hence by [8] there is a measurable mapping  $\delta_n: \Omega \rightarrow X$  such that

$$\delta_n(\omega) = h_n(\omega, \delta_n(\omega)) \text{ for all } \omega \in \Omega \dots (3.1.1)$$

Next, we show that  $\{\delta_n(\omega)\}$  is bounded sequence for all  $\omega \in \Omega$ .

Suppose that  $\|\delta_n(\omega)\| \rightarrow \infty$ , for some  $\omega \in \Omega$ .

Let  $\alpha \in (0,1)$  and  $\beta > 0$  such that

$$G(\omega, h^n(\omega, x) - q, x) \leq \alpha \|x\|^2 \text{ for all } \omega \in \Omega \text{ and } x \in A \text{ with } \|x\| \geq \beta \dots (3.1.2)$$

From properties of  $G$ , using (3.1.1) and (3.1.2) we have

$$\begin{aligned} \|\delta_n(\omega)\|^2 &\leq G(\omega, \delta_n(\omega), \delta_n(\omega)) \\ &= G(\omega, h_n(\omega, \delta_n(\omega)), \delta_n(\omega)) \\ &= G(\omega, (1 - k_n)q + k_n h^n(\omega, \delta_n(\omega)), \delta_n(\omega)) \\ &= G(\omega, k_n(h^n(\omega, \delta_n(\omega)) - q) + q, \delta_n(\omega)) \\ &= k_n G(\omega, h^n(\omega, \delta_n(\omega)) - q, \delta_n(\omega)) + G(\omega, q, \delta_n(\omega)) \end{aligned}$$

$$\|\delta_n(\omega)\|^2 \leq k_n \alpha \|\delta_n(\omega)\|^2 + M \|q\| \|\delta_n(\omega)\|$$

Dividing by  $\|\delta_n(\omega)\|^2$  and taking limit as  $n \rightarrow \infty$ , we have  $1 \leq \alpha$  which is contraction. Therefore,  $\{\delta_n(\omega)\}$  is bounded sequence for all  $\omega \in \Omega$ .

$n \rightarrow \infty$

$$\begin{aligned} \|\delta_n(\omega) - h^n(\omega, \delta_n(\omega))\| &= \|h_n(\omega, \delta_n(\omega)) - h^n(\omega, \delta_n(\omega))\| \\ &= \|(1 - k_n)q + k_n h^n(\omega, x) - h^n(\omega, \delta_n(\omega))\| \\ &= \|(1 - k_n)q - h^n(\omega, \delta_n(\omega))(1 - k_n)\| \\ &= \|(1 - k_n)(q - h^n(\omega, \delta_n(\omega)))\| \\ &= (1 - k_n) \|h^n(\omega, \delta_n(\omega)) - q\| \rightarrow 0 \end{aligned}$$

Moreover,

$$\begin{aligned} \|\delta_n(\omega) - h(\omega, \delta_n(\omega))\| &= \|\delta_n(\omega) - h^n(\omega, \delta_n(\omega)) + h^n(\omega, \delta_n(\omega)) - h(\omega, \delta_n(\omega))\| \\ &\leq \|\delta_n(\omega) - h^n(\omega, \delta_n(\omega))\| + \|h^n(\omega, \delta_n(\omega)) - h(\omega, \delta_n(\omega))\| \\ &\leq (1 - k_n) \|h^n(\omega, \delta_n(\omega)) - q\| + \|h(\omega, h^{n-1}(\omega, \delta_n(\omega))) - h(\omega, \delta_n(\omega))\| \\ &\leq (1 - k_n) \|h^n(\omega, \delta_n(\omega)) - q\| + \|h^{n-1}(\omega, \delta_n(\omega)) - \delta_n(\omega)\| \\ &\leq (1 - k_n) \|h^n(\omega, \delta_n(\omega)) - q\| + \|h^{n-1}(\omega, \delta_n(\omega)) - h^{n-1}(\omega, \delta_{n-1}(\omega))\| \\ &\quad + \|h^{n-1}(\omega, \delta_{n-1}(\omega)) - \delta_{n-1}(\omega)\| + \|\delta_{n-1}(\omega) - \delta_n(\omega)\| \\ &\leq (1 - k_n) \|h^n(\omega, \delta_n(\omega)) - q\| + \|\delta_n(\omega) - \delta_{n-1}(\omega)\| + \|h^{n-1}(\omega, \delta_{n-1}(\omega)) - h_{n-1}(\omega, \delta_{n-1}(\omega))\| \\ &\quad + \|\delta_{n-1}(\omega) - \delta_n(\omega)\| \\ &\leq (1 - k_n) \|h^n(\omega, \delta_n(\omega)) - q\| + \|\delta_n(\omega) - \delta_{n-1}(\omega)\| \\ &\quad + (1 - k_{n-1}) \|h^{n-1}(\omega, \delta_{n-1}(\omega)) - q\| + \|\delta_{n-1}(\omega) - \delta_n(\omega)\| \end{aligned}$$

Since  $\{\delta_n(\omega)\}$  is a bounded sequence in a reflexive Banach space for all  $\omega \in \Omega$ , then for each  $n$ , we can define  $Q_n: \Omega \rightarrow \text{wk}(X)$  by  $Q_n(\omega) = \overline{\text{co}\{\delta_i(\omega): i \geq n\}}_w$ . Define  $Q: \Omega \rightarrow \text{wk}(X)$  by  $Q(\omega) = \bigcap_{n=1}^{\infty} Q_n(\omega)$ .

Since the weakly topology on  $X$  is metric topology [Dunford and Schwartz 1958, p.434] and the mapping  $Q$  is weakly measurable so  $Q$  has measurable selector  $\delta$  [Kuratowski and Ryll 1965]. For any fixed  $\omega$  in  $\Omega$ , we may assume that there exists a subsequence  $\{\delta_{m_i}(\omega)\}$  of  $\{\delta_n(\omega)\}$  converges weakly to  $\delta(\omega)$ .

Since  $\delta_{m_i}(\omega) - h(\omega, \delta_{m_i}(\omega))$  converges to zero and  $I - h(\omega, \cdot)$  is demiclosed for all  $\omega \in \Omega$ . Hence  $h$  has a random fixed point  $\delta: \Omega \rightarrow A$  such that  $\delta(\omega) = h(\omega, \delta(\omega))$ . ■

### Corollary (3.1) :

Let  $X$  be a separable reflexive Banach space,  $\emptyset \neq A \subseteq X$  unbounded closed and starshaped w.r.t.  $q \in X$  subset of  $X$ ,  $h: \Omega \times A \rightarrow X$  be an uniformly 1-Lipschitzain random operator with  $h(\omega, A) \subseteq A$  and  $I - h(\omega, \cdot)$  be demiclosed for all  $\omega \in \Omega$ . If  $\lim_{\|x\| \rightarrow \infty} \sup \frac{G(\omega, h^n(\omega, x) - q, x)}{\|x\|^2} < 1$ , then  $h$  has a random fixed point.

### Corollary (3.2) Theorem (3.1)[7]

Let  $A$  be a nonempty unbounded closed and starshaped subset w.r.t some point  $q$  in a separable reflexive Banach space  $X$  and  $h: \Omega \times A \rightarrow X$  be an asymptotically nonexpansive random operator with  $h(\omega, A) \subseteq A$  and  $I - h(\omega, \cdot)$  be demiclosed for each  $\omega \in \Omega$ . If  $\lim_{\|x\| \rightarrow \infty} \sup \frac{G(\omega, h^n(\omega, x) - u, x)}{\|x\|^2} < 1$ , then  $h$  has a random fixed point.

### Theorem (3.2):

Let  $A$  be a nonempty unbounded closed subset of separable reflexive Banach space  $X$  and  $h: \Omega \times A \rightarrow X$  be an uniformly 1-Lipschitzain with  $A$  has property (p) w.r.t  $h$  and  $I - h(\omega, \cdot)$  be demiclosed for all  $\omega \in \Omega$ . If  $h$  satisfy condition (B), then  $h$  has a random fixed point.

### Proof :

By similar proof of theorem (3.1) there is measurable mapping  $\delta_n: \Omega \rightarrow A$  such that  $\delta_n(\omega) = h_n(\omega, \delta_n(\omega))$  for all  $\omega \in \Omega$ . Now we show that  $\{\delta_n(\omega)\}$  is bounded sequence for all  $\omega \in \Omega$ .

Suppose that  $\|\delta_n(\omega)\| \rightarrow \infty$ , for some  $\omega \in \Omega$ .

Let  $\alpha \in (0, 1)$  and  $\beta > 0$  such that  $\|h^n(\omega, x) - h(\omega, q)\| \leq \alpha\|x - q\|$  for all  $\omega \in \Omega$  and  $x \in A$  with  $\|x\| \geq \beta$ , consider

$$\begin{aligned} \|\delta_n(\omega)\| &= \|h_n(\omega, \delta_n(\omega))\| = \|(1 - k_n)q + k_n h^n(\omega, \delta_n(\omega))\| \\ &= \|(1 - k_n)q + k_n h(\omega, q) - k_n h(\omega, q) + k_n h^n(\omega, \delta_n(\omega))\| \\ &\leq k_n [\|h^n(\omega, \delta_n(\omega)) - h(\omega, q)\| + \|h(\omega, q)\|] + (1 - k_n)\|q\| \end{aligned}$$

Dividing by  $\|\delta_n(\omega)\|$  and taking limit as  $n \rightarrow \infty$  we have  $1 \leq \alpha$  which is contraction. therefore  $\{\delta_n(\omega)\}$  is a bounded sequence for each  $\omega \in \Omega$ . Hence by similarly proof of Theorem (3.1) there is a measurable mapping  $\delta: \Omega \rightarrow A$  such that  $\delta(\omega) = h(\omega, \delta(\omega))$ . ■

### Corollary (3.3)

Let  $A$  be a nonempty unbounded closed and starshaped w.r.t. some  $q$  in a separable reflexive Banach space  $X$  and  $h: \Omega \times A \rightarrow X$  be an uniformly 1-Lipschitzain with  $h(\omega, A) \subseteq A$  for all  $\omega$  in  $\Omega$  and  $I - h(\omega, \cdot)$  be demiclosed for all  $\omega \in \Omega$ . If  $h$  satisfy condition (B), then  $h$  has a random fixed point.

### Corollary (3.4) theorem (3.3) [7]

Let  $A$  be a nonempty unbounded closed starshaped subset w.r.t  $q$  in a separable reflexive Banach space  $X$  and  $h: \Omega \times A \rightarrow X$  be an asymptotically nonexpansive random operator satisfying condition (B) with  $h(\omega, A) \subseteq A$  for each  $\omega \in \Omega$ . If and  $I - h(\omega, \cdot)$  is demiclosed for each  $\omega$  in  $\Omega$ , then  $h$  has a random fixed point.

### Corollary (3.5) corollary (3.4)[7]

Let  $A$  be a nonempty unbounded closed starshaped subset w.r.t some  $q$  in a separable reflexive Banach space  $X$  and  $h: \Omega \times A \rightarrow X$  be a nonexpansive asymptotically contractive random operator with  $h(\omega, A) \subseteq A$  for all  $\omega$  in  $\Omega$ . Then  $h$  has a random fixed point.

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