New Results of New Type of Sets by S.M-Open Set

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Abstract:

The definitions of S.M-open sets, S.M-continuous function and S.M-Connected sets are introduced and some of their properties are studied.

Key words: S. M-open set , S. M-closed set, S. M-continuous function , S. M-connected space.

INTRODUCTION

The concept of θ-interior set, θ-closure set , δ-interior set and δ-closure set, were first introduced by Velicko[8]. A subset A of a topological space(X,T) is called regular open (resp. regular closed)[8] if A=int(cl(A)) (resp. A=cl(int(A))). A point $x\in X$ is said to be θ-adherent point of A[9] iff $cl(U)\cap A\neq\Phi$,For every open set U containing x. The set of all θ-adherent points of A is called θ-closure of A, and is denoted by $cl_\theta(A)$.The complement of θ-closed set is called θ-open set, a point $x\in X$ is called θ-interior of A [9] if there exist an open set U containing x such that $cl(U)\subseteq A$. The set of all θ-interior points of A is called θ-interior set and denoted by $int_\theta(A)$.A subset A is θ-open if $A=int_\theta(A)$.

The δ-interior point [6]of a subset A of topological space(X,T) is the union of all regular open sets of (X,T) contained A and is denoted by $int_\delta(A)$. A subset A of (X,T) is called δ-open[3] if $A=int_\delta(A)$ .The complement of δ-open set is called δ-closed , a point x of a topological space (X,T) is called δ-adherent point of A if and only if $int(cl(U))\cap A\neq\Phi$,for every open set U containing x .The set of all δ-adherent points of A is called δ-closure of A and denoted by $cl_\delta(A)$,and A is δ-closed set if $A=cl_\delta(A)$.

PRELIMINARIES

In this section we recall the following known definitions, notations , and some properties . A subset A of a topological space (X,T) is called pre-open[2] (resp. δ-pre open [10], semi open[7], δ-semi open[5], θ-semi open [4]) if $A\subseteq int(cl(A)$ (resp. $A\subseteq int(cl_\delta(A))$, $A\subseteq cl(int(A))$, $A\subseteq cl(int_\delta(A)$, $A\subseteq cl(int_\theta(A))$).

And A is M-open set[1] if $A\subseteq cl(int_\theta(A)\cup int(cl_\delta(A))$.

The union of all δ-semi open (resp. δ-preopen , semi open ,preopen , θ-semi open ,θ-preopen) set contained A is called the δ-semi interior (resp. δ-pre interior, semi interior, pre interior ,θ-semi interior ,θ-pre interior) of A and is denoted by $Sint_\delta(A)$ (resp. $Pint_\delta(A)$,Sint(A) ,Pint(A) ,$Sint_\theta(A)$,$Pint_\theta(A)$).

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The intersection of all δ-semi closed (resp. δ-pre closed, semi closed , pre closed, θ-semi closed , θ-pre closed)sets containing A is called the δ-semi closure (resp. δ-pre closur , semi closure , pre closure, θ-semi closure, θ-pre closure) of A and denoted by $Scl_\delta(A)$ (resp. $Pcl_\delta(A)$,Scl(A), Pcl(A), $Scl_\theta(A)$, $Pcl_\theta(A)$)

Proposition(1-2):

Let A,B be subsets of a topological space(X,T) ,The following statements hold:

1. $Scl_\theta(A)=A\cup int(cl_\theta(A))$

$Sint_\theta(A)=A\cap cl(int_\theta(A))$

2. $Pcl_\delta(A)=A\cup cl(int_\delta(A))$

$Pint_\delta(A)=A\cap int(cl_\delta(A))$

3. $\delta\text{-}Scl_\theta(X\backslash A)=X\backslash\ \delta\text{-}int_\theta(A)$

$\delta\text{-}Scl_\theta(A\cup B)\subseteq \delta\text{-}Scl_\theta(A)\cup \delta\text{-}Scl_\theta(B)$

4. $\delta\text{-}Pcl_\theta(X\backslash A)=X\backslash\delta\text{-}Pint_\theta$

$\delta\text{-}Pcl_\theta(A\cup B)\subseteq \delta\text{-}Pcl_\theta(A)\cup \delta\text{-}Pcl_\theta(B)$

5. $X\backslash(int_\delta(A))=cl_\delta(X\backslash A)$

$X\backslash(cl_\delta(A)=int_\delta(X\backslash A)$

2. S. M-OPEN SET

Definition(2-1):

Let (X,T) be a topological space , A subset A of X is said to be Semi M-open set if $A\subseteq cl[cl(int_\theta(A))\cup int(cl_\delta(A))]$

And A is Semi M- closed set if

$A\supseteq int[int(cl_\theta(A))\cap cl(int_\delta(A))$.

The family of all Semi M-open (Semi M-closed) subset of (X,T) will be denoted by S.M-open (S.M-closed) set

Theorem(2-2):

Every M-open set is S. M-open set.

Proof: Let A be M-open set in topological space(X,T)

$A\subseteq cl(int_\theta(A))\cup int(cl_\delta(A))$

Since $A \subseteq cl(A)$

Then $A \subseteq cl[cl(int_\theta(A)) \cup int(cl_\delta(A))]$

$\therefore$ A is S.M-open set

Remark(2-3):

The converse of the above theorem is not true. As shown that by the following example

Example(2-4):- Let X={ 1,2,3,4} T={Φ,X,{1},{2},{1,2},{2,4},{1,2,3}}

Then {2,3} is S. M-open set but not M-open set.

Proposition(2-5):-

The union of Semi M-open sets is Semi M-open set

Proof:

Let {Ai, i∈I} be a family of S.M-open set. Then $A \subseteq cl[cl(int_\theta(A)) \cup int(cl_\delta(A))$

$\bigcup_i Ai \subseteq \bigcup_i[\ cl(cl(int_\theta(A)) \cup int(cl_\delta(A)))]$

$\subseteq cl[cl(int_\theta(\bigcup_i Ai)) \cup int(cl_\delta(\bigcup_i Ai))]\ ,\forall i \in I$

$\therefore \bigcup_i Ai$ is Semi M-open set

Proposition(2-6):

The intersection of Semi M-closed sets is Semi M-closed set

Remark(2-7):

The intersection of any two Semi M-open sets is not semi M-open set.

As shown that by the following example.

Example(2-8):

Let X={1,2,3} ,T={X,Φ,{2},{3},{2,3}}

Then A={1,3} and B={1,2} are Semi M-open sets but A∩B={1} is not Semi M-open set.

Definition(2-9):

Let A be a subset of topological space (X,T)then

1 . The union of all Semi M-open sets of X contained in A is called Semi M- Interior point of A . And is denoted by S.M-int(A)

2 . The intersection of all Semi M-closed sets of X containing A is called the Semi M-closure of A and is denoted by S.M-cl(A)

Theorem(2-10):

Let $A \subseteq X$,Then

1 . S.M-cl(A)=int $[Scl_\theta(A) \cap Pcl_\delta(A)]$

2 . S.M-int(A)= cl $[Sint_\theta(A) \cup Pint_\delta(A)]$

Proof:

1 . It easy to see that S.M-cl(A)$\subseteq$int$[Scl_\theta(A) \cap Pcl_\delta(A)]$

And

$int[Scl_\theta(A) \cap Pcl_\delta(A)]=int[(A \cup int(cl_\theta(A)))\cap(A \cup cl(int_\delta(A)))]$

$=int(A)\cup int(int(cl_\theta(A))) \cap int(A) \cup int(cl(int_\delta(A)))$

$=int(A) \cup[int(cl_\theta(A))\cap int(cl(int_\delta(A)))]$

But S.M-cl is S.M.closed set

$S.M.CL(A)\supset int(int(cl_\theta(S.M.CL(A))\cap int(cl(int_\delta(S.M.CL(A)))$

$\supset int[int(cl_\theta(A)\cap cl(int_\delta(A))]$

$\therefore int(A)\cup[int(cl_\theta(A))\cap cl(int_\delta(A))]=int(A)\cup S.M\text{-}CL(A)$

$=S.M\text{-}CL(A)$

2 . Abvious

Theorem(2-11):

Let A be a subset of topological space (X,T) then:

1. A is S.M-open set if and only if $A=cl(Sint_\theta(A)\cup Pint_\delta(A))$

2. A is S.M-closed set if and only if $A=int(Scl_\theta(A)\cap Pcl_\delta(A))$

Proof:

($\rightarrow$)Let A be A S.M-open set ,Then

$A\subseteq cl[cl(int_\theta(A)) \cup int(cl_\delta(A))]$

Hence by proposition(1-2)

$cl(Sint_\theta(A)\cup Pint_\delta(A)) =cl[(A\cap cl(int_\theta(A))) \cup(A\cap int(cl_\delta(A)))]$

$=cl[A\cap(cl(int_\theta(A)\cup int(cl_\delta(A))]$

$=cl(A)$

$\supset A$

$\therefore A=cl[Sint_\theta(A)\cup Pint_\delta(A)]$

$(\leftarrow)$ Let $A=cl\,[Sint_\theta(A)\cup Pint_\delta(A)]$

Then by proposition (1-2)

$cl(Sint_\theta(A)\cup Pint_\delta(A)) =cl[(A\cap cl(int_\theta(A)))\cup(A\cap int(cl_\delta(A)))]$

$=cl[A\cap(cl(int_\theta(A)\cup int(cl_\delta(A))]$

Then we have $A\subseteq cl\,[cl(int_\theta(A))\cup int(cl_\delta(A))]$

Hence A is S.M-open set

Theorem(2-12):

Let A be a subset of topological space (X,T) ,Then

1 . A is S.M-open set iff A=S.M-int(A)
2 . A is S.M-closed set iff A= S.M-cl(A)

Proof:
Let A be a Semi M-open set ,Then by theorem(2-10)

$A=cl\,[Sint_\theta(A)\cup Pint_\delta(A)]$ and by theorem (2-9)

$Cl\,[Sint_\theta(A)\cup Pint_\delta(A)]$ =S.M-int(A)

Then we have A=S.M-int(A).

Conversely:

Let A=S.M-int(A) , Then by theorem(2-9)

$A=cl\,[Sint_\theta(A)\cup Pint_\delta(A)]$ and by theorem (2-10)

A is S.M-open set

Definition(2-13):

Let A be a subset of topological space (X,T), A point $x\in X$ is called S.M-limit point of A if for every S.M-open set U containing x contains a point of A other than x.

The set of all S.M-limit points of A is called S.M-Derived set of A and is denoted by S.M-D(A)

Definition(2-14):

Asubset A of a topological space(X,T)is called S.M-neighborhood of a point $p\in X$ if there exists a S.M-open set U such that $p\in U\subseteq A$ and denoted by S.M-nbd

The set of all S.M-nbd of $p\in X$ is called S.M-nbd system of p and denoted by S.M-N_P .

Theorem(2-15):

A subset A of a topological space(X,T) is S.M-open set iff it is S.M- neighborhood for every point $p\in A$.

Proof:-

Let A be a S.M-open set. Then A is S.M-nbd. for each point $p\in A$

Conversely:

Let A be S.M-nbd for each $p\in A$, Then there exists a S.M-open set U_P containing p such that $p\in U_p\subseteq A$ so $A=\bigcup_{p\in A} U_p$

Therefore A is S.M-open set.

Theorem(2-16):

Let A be a subset of topological space (X,T) .Then

1 . S.M-cl(S.M-cl(A))= S.M-cl(A) ,And

S.M-int(S.M-int(A))=S.M-int(A)

Proof:

S.M-cl(S.M-cl(A))= int[Scl_θ(S.M-cl(A))$\cap$ Pcl_δ(S.M-cl(A))]

$= int[Scl_\theta (int(Scl_\theta(A)\cap Pcl_\delta(A)) \cap(Pcl_\delta(int(Scl_\theta(A)\cap Pcl_\delta(A)))]$

$= int\ (int)[Scl_\theta (Scl_\theta(A)\cap Pcl_\delta(A)) \cap(Pcl_\delta(Scl_\theta(A)\cap Pcl_\delta(A)))]$

$= int[(Scl_\theta(A)\cap Scl_\theta(A)Pcl_\delta(A)) \cap(Pcl_\delta(Scl_\theta(A)\cap Pcl_\delta(A))]$

$= int[Scl_\theta(A)\cap Pcl_\delta(A))]$

=S.M-cl(A)

$\therefore$S.M-cl(S.M-cl(A)$\subseteq$S.M-cl(A)

But S.M-cl(A)$\subseteq$S.M-cl(S.M-cl(A))

Then we have S.M-cl(S.M-cl(A))=S.M-cl(A)

2 . S.M.cl(X\A)=X \ S.M-int(A) , And

S.M.int(X\A)=X \ S.M-cl(A)

Proof:

S.M-cl(X\A)= int[Scl_θ(X\A)∩Pcl_δ(X\A))]

=int[(X\$Sint_\theta$(A)) ∩ (X\$Pint_\delta$(A))]

Then by proposition (1-2)

=X \ [int($Sint_\theta$(A))∩$Pint_\delta$(A))]

=X \ S.M-int(A)

Theorem(2-17):

Let A and B be subsets of a topological space (X,T), Then the following are hold

1 . If A⊆B ,Then S.M-cl(A) ⊆ S.M-cl(B) , And

S.M-int(A) ⊆ S.M-int(B)

Proof:

S.M-cl(A)= int[Scl_θ(A) ∩ Pcl_δ(A)] , And A⊆B

∴ S.M-cl(A)=int[Scl_θ(A) ∩ Pcl_δ(A)]⊆ int[Scl_θ(B) ∩ Pcl_δ(B)] ⊆S.M-cl(B)

2 . S.M-int(A∩B) ⊆ S.M-int(A) ∩ S.M-int(B) , And

S.M-cl(A∩B)⊆ S.M-cl(A) ∩ S.M-cl(B)

Proof:

S.M-int(A∩B)=cl[$Sint_\theta$(A∩B) ⋃ $Pint_\delta$(A∩B)]

⊆ cl[($Sint_\theta$(A)∩$Sint_\theta$(B))⋃($Pint_\delta$(A)∩$Pint_\delta$(B))]

⊆ cl[($Sint_\theta$(A)⋃($Pint_\delta$(A))∩($Sint_\theta$(B)⋃($Pint_\delta$(B))]

⊆ cl[$Sint_\theta$(A)⋃ $Pint_\delta$(A)] ∩ cl[$Sint_\theta$(B)⋃ $Pint_\delta$(B)]

⊆S.M-int(A) ∩ S.M-cl(B)

∴S.M-int(A∩B)⊆ S.M-int(A) ∩ S.M-int(B)

3 . S.M-cl(A)⋃ S.M-cl(B)⊆ S.M-cl(A⋃ B) , And

S.M-int(A)⋃ S.M-int(B)⊆ S.M-int(A⋃ B)

Proof:

S.M-cl(A)$\cup$ S.M-cl(B)= int[Scl$_\theta$(A)$\cap$ Pcl$_\delta$(A)]$\cup$ int[Scl$_\theta$(B)$\cap$Pcl$_\delta$(B)]

$\subseteq$int [(Scl$_\theta$(A)$\cap$Pcl$_\delta$(A)) $\cup$(Scl$_\theta$(B)$\cap$Pcl$_\delta$(B))]

$\subseteq$int [(Scl$_\theta$(A) $\cup$(Scl$_\theta$(B))$\cap$(Pcl$_\delta$(A)$\cup$ Pcl$_\delta$(B))]

$\subseteq$int [(Scl$_\theta$(A$\cup$ B) $\cap$Pcl$_\delta$(A$\cup$ B))]

$\subseteq$S.M-cl(A$\cup$ B).

3. S. M -Continuous Function

Definition(3-1):

A function f: (X,T)$\rightarrow$(Y,σ) is called

1 . S. M- Continuse if f^{-1}(u) $\in$ S.M-open (X) for all u$\in$(Y,σ)

2 . S. M-irresolute if f^{-1}(u) $\in$ S.M-open (X) for all u$\in$S.M-open in(Y,σ)

3 . S. M-open if f(u) $\in$S.M-open(Y,σ) for all u$\in$(X,T)

4 . Pre -S. M-open if f(u)$\in$S .M-open(Y,σ) for all u$\in$ S.M-open(X,T)

5 . Supper S. M-open if f(u)$\in\sigma$,for all u$\in$ S.M-open(X,T)

6 . S. M- Homeomorphism if f is bijective , S. M- irresolute ,and pre-S. M-open function.

Definition(3-2):

A topological space (X,T) is said to be S. M-connected space if X cannot be expressed as the union of two disjoint non empty S. M-open sets in X.

Example(3-3):

Let X={1,2,3,4} ,T={Φ,X,{1},{2},{1,2},{2,4},{1,2,3}}

Let A={2} and B={3,4} are disjoint semi M-open sets and A$\cup$ B $\neq$X

Theorem(3-4):

If f:(X,T)$\rightarrow$(Y,σ) be S. M-continuous function and (X,T) is S. M-connected space ,Then (Y,σ) is connected space.

Proof:

Assume that Y is not connected space

Then Y=U$\cup$ V ,Where U,V are non empty disjoint open set in Y.

Since f is S.M-continuous function
Then f^{-1}(U), f^{-1}(V) are S.M-open sets in X and
f^{-1}(U) ∪ f^{-1}(V)=X

∴X is S.M-disconnected C!

Hence Y is connected.

Lemma(3-5):

If f:(X,T)→(Y,σ) be S.M-irresolute continuous function and (X,T) is S. M-connected space ,Then (Y,σ) is S. M-connected space.

Lemma(3-6):

If f:(X,T)→(Y,σ) be super S.M-continuous function and (X,T) be connected space ,Then (Y,σ) be S. M-connected space.

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