

A Single Server Markovian Feedback Queueing System with Discouraged Arrivals and Retention of Reneged Customers with controllable Arrival Rates.

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Abstract

In this paper, an $M/M/1/k$ queueing model with feedback, discouraged arrivals, Retention of Reneged customers with controllable arrival rate is considered. The steady state solutions of system size are derived explicitly. The analytical results are numerically illustrated and relevant conclusions are presented.

Keywords

Probability of Customers retention, renegeing, feedback discouraged Arrivals steady state solution, single server, controllable arrival rates, Bivariate Poisson Process, Finite Capacity.

1. Introduction

Feedback in queueing literature represents customer dissatisfaction because of in appropriate quality of service. In case of feedback after getting partial or incomplete service customer retries for service. An impatient customer (due to renegeing) may be convinced to stay in the system for his service by utilizing certain convincing mechanism. Such customers are termed as retained customers. When a customer gets impatient, he may leave the queue with some probability say p , and may remain in the queue for service with some complementary probability $q = (1 - p)$. In computer communication, the transmission of protocol data unit is sometimes repeated due to occurrence of an error. This usually happens

because of non-satisfactory quality of service. Rework in Industrial operations is also an example of queues with feedback. We assume that after the completion of service each customer may rejoin the system as a feedback customer for receiving another regular service with probability p and may not join with complementary probability $(1 - p)$. Queues with discouraged arrivals have applications in computers with batch job processing where job submissions are discouraged when the system is used frequently and arrivals are modeled as a Poisson process with λ_0, λ_1 as a faster and slower rate of arrivals which control the arrivals. The discouragement affect the arrival rate to the queuing system. Customer arrive in a Poisson fashion with rate that depends on the number of customers present in the system at that time. That is, the customers enter the system with faster arrival rates $\frac{\lambda_0}{n+1}$ and with the slower arrival rate $\frac{\lambda_1}{n+1}$. The reneging times follow exponential distribution with parameter ξ . The service times are exponentially distributed with rate μ

In this paper, an $M/M/1/k$ queueing model with feedback, discouraged arrivals, Retention of Reneged customers with controllable arrival rate is considered with the assumption that the arrival and service processes are correlated and follows a Bivariate Poisson process. It is assumed that, whenever the queue size reaches a prescribed number R , the arrival rate reduces from λ_0 to λ_1 and it continues with that rate as long as the content in the queue is greater than some prescribed integer r ($0 \leq r < R$). When the System size reaches r , the arrival rate changes back to λ_0 and the same process is repeated.

Much work has been reported in the literature regarding interdependent queueing model with controllable arrival rates. Ayyappan. G, Muthu Ganapathy Subramanian. A and Sekar.G[1] study $M/M/1$ retrial queueing system with loss and feedback under non-preemptive priority service by matrix geometric method. Srinivasa Rao. K, Shobha. T and Srinivasa Rao. P[2], The $M/M/1/\infty$ interdependent queueing model with controllable arrival rates have analyzed. Srinivasan. A and Thiagarajan. M [3, 4] have analysed $M/M/1/k$ interdependent queueing model with controllable arrival rates and $M/MC/K/N$ interdependent queueing model with controllable arrival rates balking reneging and spares. Kumar. R and Sharma S.K[5] have studied $M/M/1/N$ queueing system with retention of

reneged customers. An attempt is made in this paper to obtain relevant results for the $M/M/1/K$ interdependent queueing model with feedback, discouraged arrivals, Retention of Reneged customers with controllable arrival rate is considered. In section 2, the description of the model is given stating the relevant postulates. In section 3, the steady state equations are obtained. In section 4, the characteristics of the models are derived. In section 5, numerical results are calculated..

2. Description of the Model

It is assumed that the arrival process $[X_1(t)]$ and the service process $[X_2(t)]$ of the systems are correlated and follows a bivariate Poisson process given by

$$P(X_1 = x_1, X_2 = x_2; t) = e^{-(\lambda_i + \mu - \epsilon)t} \sum_{j=0}^{\min(x_1, x_2)} \frac{(\epsilon t)^d [(\lambda_i - \epsilon)t]^{x_1-d} [(\mu - \epsilon)t]^{x_2-d}}{j!(x_1 - j)!(x_2 - j)!}$$

Where $x_1, x_2 = 0, 1, 2, \dots$

$$0 < \lambda_i, \mu$$

$$0 < \epsilon < \min(\lambda_i, \mu) \quad i = 0, 1$$

with parameters $\lambda_0, \lambda_1, \mu$ and ϵ as mean faster rate of arrivals, mean slower rate of arrivals, mean services rate and mean dependence rate (covariance between the arrival and services processes) respectively.

3. Steady state equation:

We observe that only $P_n(0)$ exists when $n=0, 1, 2, \dots, c-1, c, \dots, r-1, r$ $P_n(0)$ and $P_n(1)$ exist when $n=r+1, r+2, \dots, R-2, R-1$ only $P_n(1)$ exist $n=R, R+1, \dots, K$. Further,

$$P_n(0) = P_n(1) = 0 \text{ if } n > K$$

The steady state equations are

$$-(\lambda_0 - \epsilon)P_0(0) + (\mu - \epsilon)q_1P_1(0) = 0 \tag{3.1}$$

$$-\left[\frac{(\lambda_0 - \epsilon)}{(n+1)} + (\mu - \epsilon)q_1 + (n-1)\xi p\right]P_n(0) + [(\mu - \epsilon)q_1 + n\xi p]P_{n+1}(0) + \frac{(\lambda_0 - \epsilon)}{n}P_{n-1}(0) = 0, \quad (3.2)$$

$$1 \leq n \leq r-1$$

$$-\left[\frac{(\lambda_0 - \epsilon)}{(r+1)} + (\mu - \epsilon)q_1 + (r-1)\xi p\right]P_r(0) + [(\mu - \epsilon)q_1 + r\xi p]P_{r+1}(0) + [(\mu - \epsilon)q_1 + r\xi p]P_{r+1}(1) + \frac{(\lambda_0 - \epsilon)}{r}P_{r-1}(0) = 0 \quad (3.3)$$

$$-\left[\frac{(\lambda_0 - \epsilon)}{(n+1)} + (\mu - \epsilon)q_1 + (n-1)\xi p\right]P_n(0) + [(\mu - \epsilon)q_1 + n\xi p]P_{n+1}(0) + \frac{(\lambda_0 - \epsilon)}{n}P_{n-1}(0) = 0 \quad 1 \leq n \leq r-1 \quad (3.4)$$

$$-\left[\frac{(\lambda_0 - \epsilon)}{R} + (\mu - \epsilon)q_1 + (R-2)\xi p\right]P_{R-1}(0) + \frac{(\lambda_0 - \epsilon)}{(R-1)}P_{R-2}(0) = 0 \quad (3.5)$$

$$-\left[\frac{(\lambda_1 - \epsilon)}{(r+2)} + (\mu - \epsilon)q_1 + r\xi p\right]P_{r+1}(1) + [(\mu - \epsilon)q_1 + (r+1)\xi p]P_{r+2}(0) = 0 \quad (3.6)$$

$$-\left[\frac{(\lambda_1 - \epsilon)}{(n+1)} + (\mu - \epsilon)q_1 + (n-1)\xi p\right]P_n(1) + [(\mu - \epsilon)q_1 + n\xi p]P_{n+1}(1) + \frac{(\lambda_1 - \epsilon)}{n}P_{n-1}(1) = 0 \quad r+2 \leq n \leq R-1 \quad (3.7)$$

$$-\left[\frac{(\lambda_1 - \epsilon)}{(R+1)} + (\mu - \epsilon)q_1 + (R-1)\xi p\right]P_R(1) + [(\mu - \epsilon)q_1 + R\xi p]P_{R+1}(1) + \frac{(\lambda_1 - \epsilon)}{R}P_{R-1}(1) + \frac{(\lambda_0 - \epsilon)}{R}P_{R-1}(0) = 0 \quad (3.8)$$

$$-\left[\frac{(\lambda_1 - \epsilon)}{(n+1)} + (\mu - \epsilon)q_1 + (n-1)\xi p\right]P_R(1) + [(\mu - \epsilon)q_1 + n\xi p]P_{n+1}(1) + \frac{(\lambda_1 - \epsilon)}{n}P_{n-1}(1) = 0 \quad (3.9)$$

$$-[(\mu - \epsilon)q_1 + (K-1)\xi p]P_K(1) + \frac{(\lambda_1 - \epsilon)}{K}P_{K-1}(1) = 0 \quad (3.10)$$

From (3.1) we get

$$P_1(0) = \frac{(\lambda_0 - \epsilon)}{(\mu - \epsilon)q_1} P_0(0)$$

Using the above result in (3.2) we get

$$P_2(0) = \frac{(\lambda_0 - \epsilon)^2}{2!(\mu - \epsilon)q_1[(\mu - \epsilon)q_1 + \xi p]} P_0(0)$$

And hence we recursively derive

$$P_n(0) = \frac{1}{n!} \prod_{k=1}^n \frac{(\lambda_0 - \epsilon)}{[(\mu - \epsilon)q_1 + (k-1)\xi p]} P_0(0) \quad (3.11)$$

Using (3.11) in (3.3) we get

$$P_{r+1}(0) = \frac{1}{(r+1)!} \prod_{k=1}^{r+1} \frac{(\lambda_0 - \epsilon)}{[(\mu - \epsilon)q_1 + (k-1)\xi p]} P_0(0) - P_{r+1}(1)$$

Using the above result and (3.11) in (3.4) we get

$$P_n(0) = \frac{1}{n!} \prod_{k=1}^n \frac{(\lambda_0 - \epsilon)}{[(\mu - \epsilon)q_1 + (k-1)\xi p]} P_0(0) - \frac{P_{r+1}(1)}{\prod_{l=r+1}^{n-1} [(\mu - \epsilon)q_1 + l\xi p]} \left[\frac{(\lambda_0 - \epsilon)^{n-r-1}}{nP_{n-r-1}} + \frac{(\lambda_0 - \epsilon)^{n-r-2}}{nP_{n-r-2}} [(\mu - \epsilon)q_1 + r\xi p] \right] \quad r+1 \leq n \leq R-1 \quad (3.12)$$

Using (3.12) in (3.5) we get

$$P_{r+1}(1) = \frac{\frac{1}{R!}(\lambda_0 - \epsilon)^R \prod_{k=1}^{r+1} \frac{1}{[(\mu - \epsilon)q_1 + (k-1)\xi p]} P_0(0)}{\frac{(\lambda_0 - \epsilon)^{R-r-1}}{RP_{R-r-1}} + \frac{(\lambda_0 - \epsilon)^{R-r-2}}{RP_{R-r-2}} [(\mu - \epsilon)q_1 + r\xi p] + \dots + [(\mu - \epsilon)q_1 + r\xi p] \dots [(\mu - \epsilon)q_1 + (R-2)\xi p]} \quad (3.13)$$

Using this result in (3.7), we recursively derive

$$P_n(1) = \frac{P_{r+1}(1)}{\prod_{l=r+1}^{n-1} [(\mu - \epsilon)q_1 + l\xi p]} \left[\frac{(\lambda_1 - \epsilon)^{n-r-1}}{nP_{n-r-1}} + \frac{(\lambda_1 - \epsilon)^{n-r-2}}{nP_{n-r-2}} [(\mu - \epsilon)q_1 + r\xi p] + \dots \right] \quad (3.14)$$

where $P_{r+1}(1)$ is given by (3.13)

Using (3.12) and (3.14) in (3.8) we get

$$P_{R+1}(1) = \frac{P_{r+1}(1)}{\prod_{l=r+1}^R [(\mu - \epsilon)q_1 + l\xi p]} \left[\frac{(\lambda_1 - \epsilon)^{R-r}}{(R+1)P_{R-r}} + \frac{(\lambda_1 - \epsilon)^{R-r-1}}{(R+1)P_{R-r-1}} [(\mu - \epsilon)q_1 + r\xi p] + \dots + \frac{(\lambda_1 - \epsilon)}{(R+1)P_1} \right]$$

Using the above result in (3.9) and (3.10) we recursively derive

$$P_K(1) = \frac{P_{r+1}(1)}{\prod_{l=r+1}^{K-1} [(\mu - \epsilon)q_1 + l\xi p]} \left[\frac{(\lambda_1 - \epsilon)^{K-r-1}}{nP_{K-r-1}} + \frac{(\lambda_1 - \epsilon)^{K-r-2}}{KP_{K-r-2}} [(\mu - \epsilon)q_1 + r\xi p] + \dots + \frac{(\lambda_1 - \epsilon)^{K-R}}{KP_{n-R}} [(\mu - \epsilon)q_1 + r\xi p] \dots [(\mu - \epsilon)q_1 + (R-2)\xi p] \right] \quad (3.15)$$

$$R \leq n \leq K$$

where $P_{r+1}(1)$ is given by (3.13). Thus from (3.11) to (3.15), we find that all the steady state probabilities are expressed in terms of $P_0(0)$

4. Characteristics of the Model

The following system characteristics are considered and this analytical result are derived in this section

- I. The probability $P(0)$ that the system is in faster rate of arrivals.
- II. The probability $P(1)$ that the system is in slower rate of arrivals.
- III. The probability $P(0)$ that the system is in empty.
- IV. The expected number of customers in the system L_{s0} , when the system is in the faster rate of arrivals.
- V. The expected number of customers in the system L_{s1} , when the system is in the slower rate of arrivals.
- VI. The expected waiting time of the customer in the system W_s
- VII. The probability that the system is in faster rate of arrivals is

$$P(0) = \sum_{n=0}^K P_n(0) \\ = \sum_{n=0}^r P_n(0) + \sum_{n=r+1}^{R-1} P_n(0) + \sum_{n=R}^K P_n(0)$$

Since $P_n(0)$ exists only when $n = 0, 1, 2 \dots r-1, r, r+1 \dots R-2, R-1$ we get

$$P(0) = \sum_{n=0}^r P_n(0) + \sum_{n=r+1}^{R-1} P_n(0) \quad (4.1)$$

From (3.11) (3.12) (3.13) and (4.1) we get

$$P(0) = \sum_{n=0}^{R-1} \frac{1}{n!} \prod_{k=1}^n \frac{(\lambda_0 - \epsilon)}{[(\mu - \epsilon)q_1 + (K-1)\xi p]} P_0(0) - \frac{\sum_{n=r+1}^{R-1} A}{B} \frac{(\lambda_0 - \epsilon)^R}{R!} \prod_{K=1}^{R-1} \frac{1}{[(\mu - \epsilon)q_1 + (K-1)\xi p]} P_0(0), \quad (4.2)$$

where

$$A = \frac{(\lambda_0 - \epsilon)^{n-r-1}}{nP_{n-r-1}} + \frac{(\lambda_0 - \epsilon)^{n-r-2}}{nP_{n-r-2}} [(\mu - \epsilon)q_1 + r\xi p] + \dots + [(\mu - \epsilon)q_1 + r\xi p] \dots [(\mu - \epsilon)q_1 + (n-2)\xi p]$$

$$\mathbf{B} = \frac{(\lambda_0 - \epsilon)^{R-r-1}}{RP_{R-r-1}} + \frac{(\lambda_0 - \epsilon)^{R-r-2}}{RP_{R-r-2}} [(\mu - \epsilon)q_1 + r\xi p] + \dots + [(\mu - \epsilon)q_1 + r\xi p] \dots [(\mu - \epsilon)q_1 + (R-2)\xi p]$$

The Probability that the system is in slower rate of arrival is

$$\begin{aligned} P(1) &= \sum_{n=0}^K P_n(1) \\ &= \sum_{n=0}^r P_n(1) + \sum_{n=r+1}^{R-1} P_n(1) + \sum_{n=R}^K P_n(1) \end{aligned}$$

Since $P_n(1)$ exists only when $n = r+1, r, r+2, \dots, R-2, R-1, \dots, K$ we get

$$\begin{aligned} P(1) &= \sum_{n=r+1}^R P_n(1) + \sum_{n=R+1}^K P_n(1) \\ P(1) &= \frac{\left(\sum_{n=r+1}^R C \right)}{B} \left[\frac{(\lambda_0 - \epsilon)^R}{R!} \right] \prod_{l=1}^{R-1} \frac{1}{[(\mu - \epsilon)q_1 + (K-1)\xi p]} P_0(0) \\ &\quad + \frac{\left(\sum_{n=R+1}^K D \right)}{B} \left[\frac{(\lambda_0 - \epsilon)^R}{R!} \right] \prod_{l=1}^{K-1} \frac{1}{[(\mu - \epsilon)q_1 + l\xi p]} P_0(0) \end{aligned} \quad (4.3)$$

where

$$\begin{aligned} C &= \frac{(\lambda_1 - \epsilon)^{n-r-1}}{nP_{n-r-1}} + \frac{(\lambda_1 - \epsilon)^{n-r-2}}{nP_{n-r-2}} [(\mu - \epsilon)q_1 + r\xi p] + \dots + \frac{(\lambda_1 - \epsilon)^{n-R}}{nP_{n-R}} [(\mu - \epsilon)q_1 + r\xi p] \dots [(\mu - \epsilon)q_1 + (n-2)\xi p] \\ D &= \frac{(\lambda_1 - \epsilon)^{K-r-1}}{KP_{K-r-1}} + \frac{(\lambda_1 - \epsilon)^{K-r-2}}{KP_{K-r-2}} [(\mu - \epsilon)q_1 + r\xi p] + \dots + \frac{(\lambda_1 - \epsilon)^{K-R}}{KP_{K-R}} [(\mu - \epsilon)q_1 + r\xi p] \dots [(\mu - \epsilon)q_1 + (R-2)\xi p] \end{aligned}$$

The probability $[P_0(0)]$ that the system is empty can be calculated from the normalizing condition

$$P(0) + P(1) = 1$$

$$P_0(0) = \frac{1}{1 + \sum_{n=0}^{R-1} \frac{1}{n!} \prod_{k=1}^n \frac{(\lambda_0 - \epsilon)}{[(\mu - \epsilon)q_1 + (k-1)\xi p]} - \frac{\sum_{n=r+1}^{R-1} A}{B} \frac{(\lambda_0 - \epsilon)^R}{R!} \prod_{k=1}^{R-1} \frac{1}{[(\mu - \epsilon)q_1 + (k-1)\xi p]} + \frac{\left(\sum_{n=r+1}^R C\right)}{B} \left[\frac{(\lambda_0 - \epsilon)^R}{R!} \prod_{l=1}^{R-1} \frac{1}{[(\mu - \epsilon)q_1 + l\xi p]} + \frac{\left(\sum_{n=R+1}^K D\right)}{B} \left[\frac{(\lambda_0 - \epsilon)^R}{R!} \prod_{l=1}^{K-1} \frac{1}{[(\mu - \epsilon)q_1 + l\xi p]} \right]} \right] \quad (4.4)$$

Now, we calculated the expected number of customers in the system. Let L_s denote the average number of customers in the system then we have

$$L_s = L_{s0} + L_{s1}$$

$$L_{s0} = \sum_{n=0}^r nP_n(0) + \sum_{n=r+1}^{R-1} nP_n(0)$$

$$L_{s1} = \sum_{n=r+1}^{R-1} nP_n(1) + \sum_{n=R}^K nP_n(1)$$

$$L_{s0} = \left[n \sum_{n=0}^{R-1} \frac{1}{n!} \prod_{k=1}^n \frac{(\lambda_0 - \epsilon)}{[(\mu - \epsilon)q_1 + (k-1)\xi p]} - \frac{\sum_{n=r+1}^{R-1} A}{B} n \left[\frac{(\lambda_0 - \epsilon)^R}{R!} \right] + \prod_{k=1}^{R-1} \frac{1}{[(\mu - \epsilon)q_1 + (k-1)\xi p]} \right] P_0(0)$$

where $P_0(0)$ is given by (4.4)

$$L_{s1} = \left[\frac{\sum_{n=r+1}^R C}{B} n \left[\frac{(\lambda_0 - \epsilon)^R}{R!} \prod_{l=1}^{R-1} \frac{1}{[(\mu - \epsilon)q_1 + l\xi p]} + \frac{\sum_{n=R+1}^K D}{B} n \left[\frac{(\lambda_0 - \epsilon)^R}{R!} \prod_{l=1}^{K-1} \frac{1}{[(\mu - \epsilon)q_1 + l\xi p]} \right] \right] P_0(0)$$

where $P_0(0)$ is given by (4.4)

Using Little's formula the expected waiting time of the customer in the system is calculated as

$$W_s = \frac{L_s}{\bar{\lambda}}$$

where $\bar{\lambda} = \lambda_0 P_0(0) + \lambda_1 P(1)$

This model includes the earlier model as particular cases. For example when $q_1=1$ this model reduces to a single server Markovian queueing system with discouraged arrivals and Retention of Reneged customers with controllable arrival rates[6].

5. Numerical Illustration:

Table 5.1

r	R	K	λ_0	λ_1	μ	ϵ	ξ	P	q_1	$P_0(0)$	P(0)	P(1)
4	6	10	4	3	5	1	1	1	1	.4958	.99992	.000060
4	6	10	4	3	5	0.5	1	1	1	.4818	.99817	.000103
4	6	10	4	3	5	0	1	1	1	.4707	.99842	.000160
4	6	10	4	3	5	0.5	1	1	1	.5443	.99943	.000061
4	6	10	4	3	5	0.5	1	1	1	.6347	.99886	.000038
4	6	10	4	4	5	0.5	1	1	1	.4818	.99817	.000103
4	6	10	4	4	5	0	1	1	1	.4707	.99843	.000163
4	6	10	3	2	5	0.5	1	1	1	.5884	.99926	.000016
4	6	10	2	1	5	0.5	1	1	1	.7233	.99988	.0000009
4	6	10	6	5	5	0.5	1	1	1	.3284	.99767	.001025
4	6	10	6	5	4	0.5	1	1	1	.2541	.99834	.001704
4	6	10	5	5	5	0	1	1	1	.3946	.99805	.000519
4	6	10	4	3	5	1	2	2	2	.7018	.99986	.0000004
4	6	10	4	3	5	1	1	2	1	.5095	.99910	.000010
4	6	10	4	3	5	0	2	1	1	.4841	.99979	.000035
4	6	10	4	3	5	2	2	2	2	.7308	.99998	.00000007
4	6	10	4	3	5	1	0.5	0.5	0.5	.2474	.99640	.003587

Table 5.2

r	R	K	λ_0	λ_1	μ	ϵ	ξ	P	q_1	L_s	W_s
4	6	10	4	3	5	1	1	1	1	.66062763	.16516263
4	6	10	4	3	5	0.5	1	1	1	.69044864	.172915237
4	6	10	4	3	5	0	1	1	1	.71579596	.17921035
4	6	10	4	3	5	0.5	1	1	1	.58414556	.146112046
4	6	10	4	3	5	0.5	1	1	1	.44381405	.111076129
4	6	10	4	4	5	0.5	1	1	1	.69047209	.172916303
4	6	10	4	4	5	0	1	1	1	.71583095	.17920981
4	6	10	3	2	5	0.5	1	1	1	.50779694	.06938841
4	6	10	2	1	5	0.5	1	1	1	.314987362	.10507193
4	6	10	6	5	5	0.5	1	1	1	1.22789807	.20502141
4	6	10	6	5	4	0.5	1	1	1	1.24535016	.20760813
4	6	10	5	5	5	0	1	1	1	.87898251	.17604842
4	6	10	4	3	5	1	2	2		.335462338	.08387696
4	6	10	4	3	5	1	1	2	1	.61302386	.15339276
4	6	10	4	3	5	0	2	1	1	.66485394	.16624403
4	6	10	4	3	5	2	2	2	2	.295931586	.07398385
4	6	10	4	3	5	1	0.5	0.5	0.5	1.36875042	.34249885

Conclusion:

It is observed from the table 5.1 and table 5.2, when the mean depends rate increases and the other parameters are kept fixed L_s and W_s decrease. When the mean dependence rate increases and the other parameters are kept fixed, L_s and W_s decrease. When the arrival rate decreases (the other parameters are kept fixed) L_s and W_s decrease.

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