

NEW TYPE OF C-COMPACT SPACE BY T-PRE OPERATOR

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Abstract:

In this paper ,we introduce and study new types of C-compact space ,namely T-pre-c-compact space. And several properties of these space are proved.

Keywords: T-pre-open set, T-pre-closed set, T-pre-open cover , T-pre c-compact space.

1. Introduction

In 1969 G.Viglione [2]introduced the concept of c-compact space and investigated its properties. Since then ,a tremendous number of papers such as Sakai[8], Herringaton et al[6], Viglino[3], Goss. And Viglino[1], and Kim[5] have a peared on c-compact space. In 2013 N.G.Mansur. And H.K.Mousa[7] introduced the concept of T-pre-operator topological space. Also introduced the notion of T-pre open set.

In this paper ,we present c-compact space by using T-pre-open set and shall term them as T-pre c-compact space. Several properties of these space are proved.

2. Preliminaries

Through this paper (X,Γ) will always denoted a topological space. And Γ_{pre} denoted the set of all pre open set

Definition(2-1):

Let (X, Γ) be a topological space and T be an operator from Γ_{pre} to $P(X)$, i.e., $T: \Gamma_{\text{pre}} \rightarrow P(X)$. We say that T is a pre-operator associated with Γ_{pre} if $U \subseteq T(U)$ for every $U \in \Gamma_{\text{pre}}$. And the triple (X, Γ, T) is called T -pre-operator topological space.

Definition(2-2):

Let (X, Γ) be a topological space and T be a pre-operator on Γ . A subset A of X is said to be T -pre open set if for each $x \in A$, there exist a pre open set U containing x such that $T(U) \subseteq A$. We denote the set of all T -pre open sets by $T_{\Gamma_{\text{pre}}}$.

A subset B of X is said to be T -pre closed set if its complement is T -pre open set.

Definition(2-3):

Let (X, Γ) and (Y, σ) be two topological spaces and T, L be an operator on Γ and σ , respectively. A function $f: (X, \Gamma, T) \rightarrow (Y, \sigma, L)$ is (T, L) pre-continuous if and only if for every L -open set U in Y , $f^{-1}(U)$ is T -pre open set in X .

Definition(2-4):

Let (X, Γ) and (Y, σ) be two topological spaces and T, L be an operators on Γ and σ , respectively. A function $f: (X, \Gamma, T) \rightarrow (Y, \sigma, L)$ is (T, L) pre-irresolute continuous if and only if for every L -pre open set U in Y , $f^{-1}(U)$ is T -pre open set in X .

Definition(2-5):

Let (X, Γ) and (Y, σ) be two topological spaces and T, L be an operators on Γ and σ , respectively. A function $f: (X, \Gamma, T) \rightarrow (Y, \sigma, L)$ is said to be (T, L) strongly pre-continuous if and only if for every L -pre open set U in Y , $f^{-1}(U)$ is T -open set in X .

Theorems(2-6):

1. Every (T, L) pre-irresolute continuous function is (T, L) pre –continuous function.
But the converse is not true.

2. Every (T,L) strongly pre-continuous function is (T,L) pre–irresolute continuous function. But the converse is not true.
3. Every (T,L) strongly pre- continuous function is (T,L) pre –continuous function. But the converse is not true.

Definition(2-7):

Let (X,Γ,T) be a pre-operator topological space. A subset A of X is said to be T -pre compact if for any T -pre open cover $\{U_\alpha : \alpha \in \Omega\}$ of A , has a finite collection that covers A and $A \subseteq \bigcup_{i=1}^n T(U_{\alpha_i})$.

3. T-pre c-compact space

Definition(3-1):

Let (X,Γ,T) be an operator topological space. A T -closed subset A of X is called T -c-compact if for each T -open cover Ω of A there exist a finite subcover $\beta = \{c_1, c_2, \dots, c_n\}$ of Ω such that $A \subseteq \bigcup_{i=1}^n cl(ci)$

Definition(3-2):

Let (X,Γ,T) be a pre-operator topological space. A T -pre closed subset A of X is called T -pre c-compact if for each T -pre open cover Ω of A there exist a finite subcover $\beta = \{c_1, c_2, \dots, c_n\}$ of Ω such that $A \subseteq \bigcup_{i=1}^n cl(ci)$

Theorem(3-3):

Every T -pre compact space is T -pre c-compact.

Proof:

Suppose that (X,Γ,T) be a T -pre compact space

To prove that X is T -pre c-compact space , Assume that Ω be a T -pre open cover of X such that $X \subseteq \bigcup_{i \in \Lambda} ci$, where $ci \in \Omega$.

Since X is T -pre compact space therefore there exists a finite subcover $\beta = \{c_1, c_2, \dots, c_n\}$

Hence $X \subseteq \bigcup_{i=1}^n cl(ci)$ of Ω such that $X \subseteq \bigcup_{i=1}^n ci$

Thus X is T -pre c-compact space.

Theorem(3-4):

A T-pre c-compact space is (T,L)pre-irresolute topological property .

Proof:

Suppose that $f:(X,\Gamma,T) \rightarrow (Y,\sigma,L)$ be a (T,L) pre-irresolute homeomorphism function from a T-pre c-compact space (X,Γ,T) onto L-pre operator topological space (Y,σ,L) .

To prove that Y is L-pre-c-compact space assume that Ω be L-pre open cover of Y such that $Y \subseteq \bigcup_{i \in \Lambda} c_i$ where $c_i \in \Omega$,for each $i \in \Lambda$

Since f is (T,L) pre-irresolute continuous function , therefore $f^{-1}(Y) \subseteq f^{-1}(\bigcup_{i \in \Lambda} c_i)$

$$X \subseteq \bigcup_{i \in \Lambda} f^{-1}(c_i)$$

Since X is T-pre c-compact space, then there exists a finite sub collection $\{f^{-1}(c_1), f^{-1}(c_2), \dots, f^{-1}(c_n)\}$ covers X.

Also f is onto then $f(X) \subseteq \bigcup_{i=1}^n cl(f^{-1}(c_i))$

Then $Y \subseteq \bigcup_{i=1}^n cl(c_i)$, Thus Y is L-pre c-compact space.

Theorem(3-5):

If (X,Γ,T) is T-pre c-compact space and $A \subseteq X$ be both T-pre open and T- pre closed set .Then a subspace (A,Γ_A,T) is T-pre c-compact space

Proof:

Suppose that $B \subseteq A$ is T-pre closed set in A and $k = \{c_\alpha : \alpha \in \Lambda\}$ be T-pre open cover of B where c_α is T-pre open set in A for all $\alpha \in \Lambda$.Since B is T-pre closed set in A and A is T-pre closed set in X then B is T-pre closed set in X.

Also c_α is T-pre open set in A and A is T-pre open in X ,then c_α is T-pre open set in X.

Since X is T-pre c-compact space , B is T-pre closed set in X and k is T-pre open cover of B then there exist $\{c_{\alpha_1}, c_{\alpha_2}, \dots, c_{\alpha_n}\}$ covers B such that $B \subseteq \bigcup_{i=1}^n cl(c_{\alpha_i})$.

Thus (A,Γ_A,T) is T-pre c-compact space.

Theorem(3-6):

Every (T,L) pre-continuous image of T-pre c-compact space is T-c-compact space.

proof :

suppose that $f: (X, \Gamma, T) \rightarrow (Y, \sigma, L)$ is (T, L) pre-continuous function from T -pre c -compact space (X, Γ, Y) onto L -operator topological space (Y, σ, L) .

To prove Y is T - c -compact then let Ω be T -open cover of Y that is $Y \subseteq \bigcup_{i \in \Lambda} c_i$ where $c_i \in \Omega$ for each $i \in \Lambda$.

Since f is (T, L) pre-continuous function therefore $X \subseteq \bigcup_{i \in \Lambda} f^{-1}(c_i)$, but X is T -pre c -compact space, then there exist $\{f^{-1}(c_1), f^{-1}(c_2), \dots, f^{-1}(c_n)\}$ of Ω such that

$$X \subseteq \bigcup_{i=1}^n f^{-1}(cl(c_i))$$

$$f(X) \subseteq f(\bigcup_{i=1}^n f^{-1}(cl(c_i)))$$

$$Y \subseteq \bigcup_{i=1}^n cl(c_i)$$

Hence Y is T - c -compact space.

Corollary(3-7):

Every (T, L) strongly pre-continuous image of T - c -compact space is T -pre c -compact space.

Proof:

suppose that $f: (X, \Gamma, T) \rightarrow (Y, \sigma, L)$ is (T, L) strongly pre-continuous function from T - c -compact space (X, Γ, Y) onto L -pre operator topological space (Y, σ, L) .

To prove Y is T -pre c -compact then let Ω be T -pre open cover of Y that is $Y \subseteq \bigcup_{i \in \Lambda} c_i$ where $c_i \in \Omega$.

Since f is (T, L) strongly pre-continuous function therefore $f^{-1}(Y) \subseteq \bigcup_{i \in \Lambda} f^{-1}(c_i)$, where $c_i \in \Omega$ is T -pre open cover of X such that $X \subseteq \bigcup_{i \in \Lambda} f^{-1}(c_i)$.

Since X is T -pre c -compact space therefore there exist $f^{-1}(c_1), f^{-1}(c_2), \dots, f^{-1}(c_n)$ of Ω such that

$$X \subseteq \bigcup_{i=1}^n f^{-1}(cl(c_i))$$

$$Y \subseteq \bigcup_{i=1}^n cl(c_i)$$

Thus Y is T -pre c -compact space.

Corollary(3-8):

Every (T,L)pre-irresolute continuous image of T-pre-c-compact space is T-pre-c-compact space.

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