

Inverse Weibull Scale Parameter Bayesian Estimation for Doubly Type II Censored Samples

Assistant prof.
Dr.Nada S.Karam & Dr.Awatif R.Mezaal
Department of Mathematics
Al- Mustansiryah University

الخلاصة

في هذا البحث تم ايجاد مقدر بيز لمعلمة الشكل λ للتوزيع معكوس وبيل وعلى افتراض وجود توزيعين مسبقة عن المعلمة باستخدام دالة Quasi ، والدالة الثانية متمثلة بتوزيع Gamma لايجاد الدالة اللاحقة وقد تم التقدير تحت عينات المراقبة المضاعفة من النوع الثاني وباستخدام ثلاث دوال خسارة (مربع الخطأ ، دي غروت والدالة الوقائية)، وقد تم اجراء المحاكاة لتوليد المتغيرات العشوائية استنادا الى دراسة موئن للمحاكاة وقد فورنت هذه المقدرات بالاعتماد على متوسط مربعات الخطأ (MSE) .

Abstract

In this paper we obtain Bayesian estimator of the scale parameter for the Inverse Weibull distribution under two priors distributions using Quasi prior and Gamma distribution have be assumed for posterior analysis .The estimation has been made under double type II censored samples and three different loss functions (squared error, De-Groot, and Percantionary loss function),we used the simulation be generate random variables baised on Monte – Carlo simulation study, these estimators compared depending on MSE.

Introduction

The Inverse Weibull distribution (IWD) plays an important role in many applications including the dynamic components of diesel engines, and the time to break down of an insulating fluid subject to the action of a constant tension.[8]

It is an important life time model in reliability and survival analysis which can be used to model a variety of failure characteristics such as infant mortality, useful life, wear out period, electron tubes. It can also be used to determine the coast effectiveness and maintenance periods of reliability centered maintenance activetise[7].

Additional results on the inverse Weibull distribution including work on reliability and tolerance limits, Bayes 2-sample prediction, and maximum likelihood and least squares estimation are given by: [see(1,3,4,)]

A random variable x , is said to have Inverse Weibull distribution with two parameters if its pdf is given by:

$$f(x; \alpha, \lambda) = \alpha \lambda e^{-\lambda x^{-\alpha}} x^{-(\alpha+1)} \quad x > 0, \alpha, \lambda > 0 \quad \dots \dots \dots (1)$$

Where λ and α are the scale and shape parameters respectively.
The cumulative and Reliability distribution function are given by:

$$F(x) = e^{-\lambda x^{-\alpha}} \quad \dots \dots \dots (2)$$

$$R(x) = 1 - e^{-\lambda x^{-\alpha}} \quad \dots \dots \dots (3)$$

Bayes method

The Bayesian statistics began in 1763[10], the model that the unknown parameter(λ) as random variable, and prior distribution said $p(\lambda)$.
Let $x_1, x_2, \dots, x_n \sim i.i.d.$ with $f(x/\lambda)$, the Bayesian views the density of each data observation as conditional density, which is conditional on realization of the random variable λ , then the posterior density given by:

$$p(\lambda|x) = \frac{f(x|\lambda)p(\lambda)}{\int_{\lambda} f(x|\lambda)p(\lambda)d\lambda}$$

Is the likelihood function, then the posterior density: $f(x/\lambda)$ Note that

$$p(\lambda/x) = \frac{L(x/\lambda).p(\lambda)}{\int_{\lambda} f(x/\lambda).p(\lambda)dx} \quad \dots \dots \dots (4)$$

Here used the Bayesian estimation under two prior functions for the scale parameter(λ), Quasi and Gamma functions with double type II concord samples as:[2]

$$L(\lambda / x) = \left(\frac{n!}{(r-1)!(n-s)!} \right) (F(x_r))^{r-1} (1-F(x_s))^{n-s} \prod_{i=r}^s f(x_i, \lambda) \quad \dots \dots \dots (5)$$

$$x_1 < x_2 < \dots < x_{r-1} < x_r < x_{r+1} < \dots < x_s < x_{s+1} < \dots < x_n$$

where a loss function $L(\hat{\lambda}, \lambda)$ represents losses incurred when we estimate λ , there is always some difference observed between the estimate and the parameter [9]. If we fix λ , we might get different values of the estimator, if $\lambda = \hat{\lambda}$ there is no loss, if it is less than ,we call it under estimation ,on the other hand if it is more , we say it is over estimation.

They are many types of loss function, in this paper we used three types (squared error, De-Groot, and Precautionary loss function).

I-Bayesian estimator for scale parameter (λ) under doubly type II cencored data:

By eq.[1,2 and 5] :

$$L(\lambda / x) = \frac{n!}{(r-1)!(n-s)!} \prod_{i=r}^s (\alpha \lambda e^{-\lambda x_i^{-\alpha}} x_i^{-(\alpha+1)}) (e^{-\lambda x_r^\alpha})^{r-1} (1 - e^{-\lambda x_s^{-\alpha}})^{n-s}$$

Now :

$$\prod_{i=r}^s (\alpha \lambda e^{-\lambda x_i^{-\alpha}} x_i^{-(\alpha+1)}) = \alpha^{s-r+1} \lambda^{s-r+1} e^{-\lambda \sum_{i=r}^s x_i^{-\alpha}} \prod_{i=r}^s x_i^{-(\alpha+1)}$$

let $m = s - r + 1$, and rewrite $\prod_{i=r}^s x_i^{-(\alpha+1)}$ as :

$$\exp\left(\sum_{i=r}^s \ln x_i^{-(\alpha+1)}\right) = \exp[-(\alpha+1) \sum_{i=r}^s \ln x_i]$$

and let this amount = Q_1

then:

$$L(\lambda / x) = \frac{n!}{(r-1)!(n-s)!} Q_1 \alpha^m \lambda^m e^{-\lambda \left(\sum_{i=r}^s x_i^{-\alpha} + (r-1)x_s^{-\alpha} \right)} (1 - e^{-\lambda x_s^{-\alpha}})^{n-s}$$

$$(1-x)^a = \sum_{j=0}^a C_j^a (-1)^j x^j \text{ For binomial property : ,then:}$$

$$(1 - e^{-\lambda x_s^{-\alpha}})^{n-s} = \sum_{j=0}^{n-s} C_j^{n-s} (-1)^j e^{-\lambda_j x_s^{-\alpha}}, \quad we \ get:$$

$$\begin{aligned} L(\lambda / x) &= \frac{n!}{(r-1)!(n-s)!} Q_1 \alpha^m \lambda^m e^{-\lambda (\sum_{i=r}^s x_i^{-\alpha} + (r-1)x_r^{-\alpha})} \sum_{j=0}^{n-s} C_j^{n-s} (-1)^j e^{-\lambda_j x_s^{-\alpha}} \\ L(\lambda / x) &= \frac{n!}{(r-1)!(n-s)!} Q_1 \alpha^m \sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \lambda^m e^{-\lambda Q} \end{aligned} \dots\dots\dots (6)$$

$$where: \quad Q = \sum_{i=r}^s x_i^{-\alpha} + (r-1)x_r^{-\alpha} + jx_s^{-\alpha}$$

The posterior distribution under Quasi prior : I)

The Quasi prior is define as [7]:

$$p(\lambda) = \frac{1}{\lambda^k} \quad \lambda, k > 0 \dots\dots\dots (7)$$

then by equation (4)(5)and(7) :

$$\begin{aligned} \int_0^\infty \frac{n!}{(r-1)!(n-s)!} Q_1 \alpha^m \sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \lambda^k \lambda^m e^{-\lambda Q_j} d\lambda = \\ \frac{n!}{(r-1)!(n-s)!} Q_1 \alpha^m \sum_{j=0}^{n-s} (-1)^j \int_0^\infty \lambda^{m+k} e^{-\lambda Q_j} d\lambda \end{aligned}$$

and the posterior distribution under this prior is given by :

$$p(\lambda / x) = \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \lambda^{m+k} e^{-\lambda Q_j}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m+k+1}{Q_j^{m+k+1}}} \dots\dots\dots (8)$$

$$by \quad \int_0^\infty x^{\alpha-1} e^{-\beta x} dx = \frac{\Gamma \alpha}{\beta^\alpha}$$

Now the bayes estimator for λ and risk function under Quasi prior use the three loss functions as:

a. Squared error loss function:

The bayes estimator for λ and risk function using squared error loss function are[2]:

$$\hat{\lambda}_s = E(\lambda / x)$$

So:

$$E(\lambda / x) = \int_{\lambda} \lambda.p(\lambda / x)d\lambda$$

$$\hat{\lambda}_{SQ} = \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \int_0^{\infty} \lambda^{m-k+1} e^{-\lambda Q_j} d\lambda}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma{m-k+1}}{Q_j^{m-k+1}}}$$

then the risk function will be:

$$\begin{aligned}
R_s &= E(\lambda^2 / x) - [E(\lambda / x)]^2 \\
E(\lambda^2 / x) &= \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 3}{Q_j^{m-k+3}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 1}{Q_j^{m-k+1}}} \\
\therefore R_{SQ} &= \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 3}{Q_j^{m-s+3}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 1}{Q_j^{m-a+1}}} - \left[\frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 2}{Q_j^{m-k+2}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 1}{Q_j^{m-k+1}}} \right]^2
\end{aligned} \quad \dots\dots\dots(10)$$

b. De Groot loss function:

The bayes estimator for λ and risk function using De Groot loss function are:

$$\hat{\lambda}_D = \frac{E(\hat{\lambda}^2 / x)}{E(\hat{\lambda} / x)}$$

then:

$$\begin{aligned}\hat{\lambda}_{DQ} &= \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 3}{Q_j^{m-k+3}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 1}{Q_j^{m-k+1}}} \cdot \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 1}{Q_j^{m-k+1}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 2}{Q_j^{m-k+2}}} \\ \hat{\lambda}_{DQ} &= \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 3}{Q_j^{m-k+3}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 2}{Q_j^{m-k+2}}} . \quad \dots \dots \dots (11)\end{aligned}$$

and the risk function will be:

$$\begin{aligned}R_{DQ} &= \hat{\lambda}_{DG}^{-2} E(\hat{\lambda} / x) - 2 \hat{\lambda}_{DG}^{-1} E(\hat{\lambda} / x) + 1 \\ &= \frac{E(\hat{\lambda} / x)^2}{E(\hat{\lambda}^2 / x)^2} \cdot E(\hat{\lambda}^2 / x) - 2 \frac{E(\hat{\lambda} / x)}{E(\hat{\lambda}^2 / x)} \cdot E(\hat{\lambda} / x) + 1 \\ &= \frac{E(\hat{\lambda} / x)^2}{E(\hat{\lambda}^2 / x)} - 2 \frac{E(\hat{\lambda} / x)^2}{E(\hat{\lambda}^2 / x)} + 1 \\ &= 1 - \frac{E(\hat{\lambda} / x)^2}{E(\hat{\lambda} / x)} \\ &= 1 - \left[\frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 2}{Q_j^{m-k+2}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 1}{Q_j^{m-k+1}}} \right]^2 \left[\frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 1}{Q_j^{m-k+1}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 3}{Q_j^{m-k+3}}} \right] \\ \therefore R_{DQ} &= 1 - \frac{\left[\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 2}{Q_j^{m-k+2}} \right]^2}{\left[\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 1}{Q_j^{m-k+1}} \right] \left[\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 3}{Q_j^{m-k+3}} \right]} \quad \dots \dots \dots (12)\end{aligned}$$

c. precautionary loss function:

The bayes estimator for λ and risk function using precautionary loss function are:

$$\therefore \hat{\lambda}_{PQ} = \left[\frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 3}{Q_j^{m-k+3}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m - k + 1}{Q_j^{m-k+1}}} \right]^{\frac{1}{2}} \dots \dots \dots (13)$$

and the risk function will be:

$$R_P = [E(\lambda^2/x)]^{1/2} - 2 E(\lambda/x) + [E(\lambda^2/x)]^{-1/2} [E(\lambda^2/x)]$$

II) the posterior distribution under Gamma prior:

The Gamma prior distribution define as :[1]

$$f(\lambda) = \frac{b^a}{\Gamma a} \lambda^{a-1} e^{-b\lambda} \quad \lambda > 0 \quad \dots \dots \dots \quad (15)$$

then by eq.(6),(15):

$$\int_{\mathbb{R}} L(\lambda/x) f(\lambda) d\lambda = \frac{n! Q_1 \alpha^m}{(r-1)! (n-s)!} \sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \lambda^m e^{-\lambda Q_j} \frac{b^a}{\Gamma a} \lambda^{a-1} e^{-b\lambda} d\lambda$$

and the posterior distribution by equation(4) given as:

$$P(\lambda / x) = \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \lambda^{m+a-1} e^{-(b+Q_j)\lambda}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a}{(b + Q_j)^{m+a}}} \quad \dots \dots \dots \quad (16)$$

Now the bayes estimator for λ and risk function under Gamma prior use the three loss functions as:

a. Squared error loss function: the bayes estimator for λ and risk function using squared error loss function are[2]:

where the risk function given by :

$$\therefore R_{SG} = \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 2}{(b + Q_j)^{m+a+2}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a}{(b + Q_j)^{m+a}}} - \left[\frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 1}{(b + Q_j)^{m+a+1}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a}{(b + Q_j)^{m+a}}} \right]^2 \quad(18)$$

b. De Groot loss function:

The bayes estimator for λ and risk function using De Groot loss function are:

$$\hat{\lambda}_D = \frac{E(\lambda^2 / x)}{E(\lambda / x)}$$

then:

$$\begin{aligned} \hat{\lambda}_{DG} &= \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 2}{(b + Q_j)^{m+a+2}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a}{(b + Q_j)^{m+a}}} \cdot \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a}{(b + Q_j)^{m+a}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 1}{(b + Q_j)^{m+a+1}}} \\ \therefore \hat{\lambda}_{DG} &= \frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 2}{(b + Q_j)^{m+a+2}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 1}{(b + Q_j)^{m+a+1}}} \quad \dots \dots \dots \quad (19) \end{aligned}$$

Where the risk function given by :

$$R_D = \hat{\lambda}_{DG}^{-2} E(\lambda^2 / x) - 2 \hat{\lambda}_{DG}^{-1} E(\lambda / x) + 1$$

$$= 1 - \frac{E(\lambda / x)^2}{E(\lambda^2 / x)}$$

then:

$$\therefore R_{DG} = 1 - \frac{\left[\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 1}{(b + Q_j)^{m+a+1}} \right]^2}{\left[\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a}{(b + Q_j)^{m+a}} \right] \left[\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 2}{(b + Q_j)^{m+a+2}} \right]} \quad \dots \dots \dots (20)$$

c. Precautionary loss function:

bayes estimator for λ and risk function using precautionary loss function are:

$$\therefore \hat{\lambda}_{PG} = \left[\frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 2}{(b + Q_j)^{m+a+2}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a}{(b + Q_j)^{m+a}}} \right]^{\frac{1}{2}} \dots \dots \dots \quad (21)$$

And the risk function given by:

$$\begin{aligned} R_p &= \hat{\lambda}_{PG} - 2E(\lambda/x) + \hat{\lambda}_{PG}^{-1} E(\lambda^2/x) \\ &= 2[E(\lambda^2/x)]^{\frac{1}{2}} - 2E(\lambda/x) \\ \therefore R_{PG} &= 2 \left[\frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 2}{(b + Q_j)^{m+a+2}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a}{(b + Q_j)^{m+a}}} \right]^{\frac{1}{2}} - 2 \left[\frac{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a + 1}{(b + Q_j)^{m+a+1}}}{\sum_{j=0}^{n-s} C_j^{n-s} (-1)^j \frac{\Gamma m + a}{(b + Q_j)^{m+a}}} \right] \quad(22) \end{aligned}$$

Simulation Results

In this simulation study , we generated values for r.v.'s of size ($n = 10,20,30,50$) for (IWD) to represent small and large sample size with the scale parameter ($\lambda = 1.5,3,5$) and [$\alpha = 0.9$, $k = 2$, $a = 3$, $b = 0.8$] the parameters values of prior distributions.

The Monte – Carlo simulation study used to compare the best Bayes estimator for scale parameter under two prior functions by using the Mean Square Error (MSE) as an index for precision to compare the capacity of each estimator , where :

$$MSE(\hat{\lambda}) = \frac{\sum_{i=1}^L (\hat{\lambda}_i - \lambda)^2}{L}$$

The results were summarized and tabulated in the following tables: Tables(1,2,3) represent the values of the scale parameter λ we assumed and the values we estimate for parameter and the risk function to show whose of these estimators closer to the truth , where in tables (4,5,6) we used MSE to show which estimator of the scale parameter is the best under which loss and prior function for all sample sizes.

Table (1) the estimation value of λ and risk function(R) with
 $\lambda = 1.5; \alpha = 0.9; k = 2; a = 3; b = 0.8$

n	SQ	DQ	PQ	SG	DG	PG
10	λ	1.3357	1.5027	1.4168	1.7676	1.9149
	R	0.2231	0.1111	0.1620	0.2604	0.0769
20	λ	1.3115	1.3938	1.3520	1.5395	1.6168
	R	0.1080	0.0591	0.0811	0.1191	0.0478
30	λ	1.3356	1.3914	1.3632	1.4920	1.5454
	R	0.0744	0.0401	0.0552	0.0796	0.0345
50	λ	1.4440	1.4802	1.4620	1.5441	1.5792
	R	0.0523	0.0244	0.0360	0.0543	0.0223

Table(2)
 $\lambda = 3$

n		SQ	DQ	PQ	SG	DG	PG
10	λ	2.9518	3.3212	3.1311	3.4189	3.7040	3.5586
	R	1.0902	0.1112	0.3584	0.9748	0.0770	0.2794
20	λ	2.9352	3.1191	3.0257	3.1999	3.3603	3.2791
	R	0.5396	0.0589	0.1811	0.5130	0.0477	0.1584
30	λ	2.9207	3.0429	2.9812	3.1058	3.2171	3.1610
	R	0.3567	0.0401	0.1209	0.3457	0.0346	0.1103
50	λ	3.0856	3.1629	3.1240	3.1970	3.2698	3.2332
	R	0.2383	0.0244	0.0768	0.2325	0.0222	0.0723

Table(3)
 $\lambda = 5$

n		SQ	DQ	PQ	SG	DG	PG
10	λ	4.7077	5.2969	4.9937	4.8012	5.2016	4.9974
	R	2.7739	0.1112	0.5719	1.9226	0.0770	0.3924
20	λ	4.7223	5.0181	4.8680	4.7754	5.0145	4.8935
	R	1.3966	0.0589	0.2912	1.1421	0.0477	0.2362
30	λ	5.0496	5.2601	5.1538	5.0424	5.2226	5.1317
	R	1.0627	0.0400	0.2083	0.9083	0.0345	0.1785
50	λ	4.9899	5.1148	5.0520	4.9908	5.1044	5.0473
	R	0.6233	0.0244	0.1241	0.5668	0.0222	0.1129

Table (4)
 The Mean and (MSE) with $\lambda = 1.5$

n		SQ	DQ	PQ	SG	DG	PG
10	Mean	1.4982	1.6859	1.5893	1.9268	2.0878	2.0057
	MSE	0.3101	0.4272	0.3569	0.5330	0.7573	0.6358
20	Mean	1.5032	1.5973	1.5495	1.7412	1.8284	1.7843
	MSE	0.1371	0.1642	0.1481	0.2144	0.2801	0.2448
30	Mean	1.4943	1.5566	1.5251	1.6580	1.7173	1.6874
	MSE	0.0848	0.0952	0.0890	0.1187	0.1478	0.1322
50	Mean	1.5045	1.5422	1.5232	1.6055	1.6420	1.6236
	MSE	0.0632	0.0681	0.0653	0.0786	0.0908	0.0843

Table (5)
 The Mean and (MSE) with $\lambda = 3$

n	SQ		DQ	PQ	SG	DG	PG
10	Mean	2.9642	3.3356	3.1445	3.3603	3.6409	3.4978
	MSE	1.1138	1.5213	1.2727	0.8863	1.2986	1.0673
20	Mean	3.0253	3.2147	3.1186	3.2607	3.4240	3.3413
	MSE	0.6115	0.7358	0.6632	0.5868	0.7519	0.6614
30	Mean	3.0189	3.1448	3.0812	3.1890	3.3031	3.2456
	MSE	0.3889	0.4426	0.4114	0.3874	0.4692	0.4246
50	Mean	2.9815	3.0561	3.0186	3.0907	3.1611	3.1257
	MSE	0.2423	0.2547	0.2484	0.2371	0.2659	0.2504

Table (6)
 The Mean and (MSE) with $\lambda = 5$

n	SQ		DQ	PQ	SG	DG	PG
10	Mean	4.9720	5.5950	5.2743	4.8420	5.2461	5.0400
	MSE	3.6361	4.9574	4.1661	1.2787	1.5319	1.3598
20	Mean	5.0267	5.3414	5.1817	4.9669	5.2156	5.0897
	MSE	1.7940	2.1414	1.9386	1.0385	1.1903	1.0974
30	Mean	5.0013	5.2100	5.1046	4.9738	5.1517	5.0620
	MSE	1.1370	1.2770	1.1953	0.7997	0.8802	0.8314
50	Mean	5.0008	5.1267	5.0634	4.9901	5.1038	5.0467
	MSE	0.6533	0.7022	0.6735	0.5319	0.5671	0.5461

For table(1) show that when $\lambda=1.5$ and ($n=10,30$) the estimator λ with De Groot loss function under Quasi and Gamma priors is the best while in ($n=20,50$) the best one with square error loss function under Gamma prior .

For table (2) and when $\lambda=3$ for all sample size the best value represent λ with square error loss function under Qausi prior, as well as in table (3) when $\lambda=5$ the same loss function and the same prior is the best .

Now for the risk function whith assumption values of ($\lambda=1.5,3,5$) and for all sample size ($n=10,20,30,50$) the sumallest value of the risk function under Gamma prior.

In table (4) and for $\lambda=1.5$, shot heat the best estimator of the parameter λ whith sequare error loss function under Quasi prior and for all sample size , where in table (5) when $\lambda=3$ also the best estimator whith square error loss function but under Gamma prior for all sample size except in $n=10$ the best with precautionary under the same prior.

In the last table (6) in $\lambda=5$, the best estimator with square error loss function undere Gamma prior and for all sample size.

In general, we conclude that the best estimator of the scale parameter for (IWD) is with square error loss function under Gamma prior in large value of $\lambda=3$ and 5 , and the same loss function but under Qausi prior in small value of $\lambda=1.5$ and for all sample size.

References

-----:

1. Ahmed H. Abd Ellah ,/2012/ " Bayesian and non- Bayesian estimation of the inverse Weibull model based on generalized order statistics" / Intelligent information Management, v4, p 23-31/ Egyp.
2. Ahmed Kasim,/2014/" Bayes estimators of the shape parameter of Exponentiated Rayleigh distribution"/ thesis/ Al-Mustasirya University.
3. Calabria. R. and Pulcini G.,/1994/ " Bayes 2-sample prediction for the Inverse Weibull distribution "/Communications in Statistics/23(6), p 1811-1824.
4. Calabria. R. and Pulcini G.,/1990/" On the maximum likelihood and least squares estimation in Inverse Weibull distribution "/Statistics Applicata/2.p53-66.
5. Jink Xiong /2010/" Weighted Inverse Weibull and Beta – Inverse Weibull" Electronic theses / Georgia Southern university.
6. K.S. Sultan and N.H. Alsadat , D.K. /2014/" Bayesian and maximum likelihood estimations of the Inverse Weibull parameters under progressive Type –II Censoring "/
7. M.A.T. Elshahat & A. E. Ismail /2014/" Quasi- Bayesian Estimation Inverse Weibull distribution "/International Journal of Mathematics and Statistics Studies/Vol 2, No.2, p 64-75.

8. Mavis Pararai, G.W. /2014/ " A new class of Generalized IWD with applications" / Journal of applied Mathematics ,Vol.4, N0.2, p 17- 35.
9. Surles , J.G. and Padgett, W.J./2001/ " Inference for reliability and stress-strength for a scale Burr type X distribution "/Lifetime Data Analysis Vol.7, p187-200.
10. Zellner, A. /1971/ " An Introduction to Bayesian inference in Econometrics" /Inc. New York.

IJST