On α^{s*}-Regular, α^{s*} −Normal and α^{s*} −C-Compact spaces

A.Punitha tharani[1] And G.Hari Siva Annam[2]

[1] Department of Mathematics,
St.Mary's College (Autonomous)
Tuticorin 628 001,India.
[2] Research Scholar
Manonmaniam Sundaranar University,
Abisekapatti,Tirunelveli 627 002, India

Abstract

The purpose of this paper is to establish and project the theorems which exhibit the characterization of α^{s*}–Regular, α^{s*}–Normal and C-α^{s*}-compact space and obtain some of interesting properties of α^{s*}-Regular, α^{s*}–Normal and C-α^{s*}-Compact space.

Keywords: α^{s*}-Regular, α^{s*}–Normal, C-α^{s*}-Compact space, α^{s*}-Hausdorff, α^{s*}-continuous, M- α^{s*}continuous, α^{s*}- open, α^{s*}- closed.

1. INTRODUCTION

Viglino[11] introduced the family of C-Compact spaces, showing that every continuous function from a C-Compact space into a Hausdorff space is a closed function and that this class of spaces properly contains the class of compact spaces. In[2],Devi et al. introduced the concept of α -regular space study their properties as well as the relation among themselves. In [1], Alias et al. introduced α–Normal and discussed their properties.

Recently, the authors [3] introduced some new concepts namely α^{s*}-closed sets and α^{s*}-open sets in topological spaces. In this paper we define α^{s*}-Regular, α^{s*}-Normal and venture to generalize C-Compact space using α^{s*}-open sets and shall term them as C- α^{s*}-Compact space.

2. PRELIMINARIES

Throughout this paper X,Y, and Z will always denote topological spaces on which no separation axioms are assumed unless explicitly stated. If A is a subset of a space X, cl(A) and int(A) denote the closure and the interior of A in X respectively.

We recall the following definitions and results that will be useful in this paper. A subset A of a topological space(X,τ) is called generalized closed (briefly g-closed)if cl(A)⊆U whenever A⊆U and U is open in X, and generalized open(briefly g-open) if it complement. X\A is g-closed in X. The generalized closure [5] of A is defined as the intersection of all g-closed sets containing A and is denoted by cl*(A) and the generalized interior of A is defined as the union of all g-closed subsets of A and is denoted by int*(A).

A subset B of a topological space(X,τ) is called α-open[7] if B⊆int(cl(intB),α-closed if cl(int(cl(B))⊆B, α^{s^*}-open[3] if B⊆int*(cl(intB), α^{s^*}-closed if cl*(int(cl(B))⊆B. The α-closure of a subset A of X is the intersection of all α-closed sets containing A and is denoted by αcl(A). The α^{s^*}-closure of A is analogously defined is denoted by α^{s^*}cl(A)[3]. The α^{s^*}-interior of a subset A of X is the union of all α^{s^*}-open sets contained in A and is denoted by α^{s^*}int(A). The α^{s^*}-closure of A is analogously defined and that is denoted by α^{s^*}Int(A).

The collection of all α^{s^*}open (respectively α^{s^*}-closed) sets is denoted by $\alpha^{s^*}O(X,\tau)$ and $\alpha^{s^*}C(X,\tau)$.

Definition 2.1[9]

A function f: X→Y is called α^{s^*}-continuous if inverse image of each open set in Y is α^{s^*}-open in X.

Definition 2. 2[9]

A function f: X→Y is called M- α^{s^*}open if image of each α^{s^*}open set in X is α^{s^*}-open in Y.

Definition 2.3[9]

A function f:X→Y is called M- α^{s^*}closed if image of each α^{s^*}closed set in X is α^{s^*}closed in Y.

Definition 2.4[12]

A space (X,τ) is said to be regular if for every closed set F and a point x∉F, there exist disjoint open sets U and V such that x∈U and F⊆V

Definition 2.5

(i) A space (X,τ) is said to be α-regular **[2]** if for every closed set F and a point x∉F, there exist disjoint α-open sets U and V such that x∈U and F⊆V.

(ii) A space (X, τ) is said to be pre*-regular [10]if for each pre*-closed set A and a point x∉A, there exist disjoint open sets U and V such that A⊆U, x∈V.

Definition 2.6

(i) A space (X,τ) is said to be α-Normal[1] if for every α -closed set F and a point x∉F, there exist disjoint α-open sets U and V such that x∈U and F⊆V

(ii) A topological space (X, τ) is said to be pre*-normal [10] if for any two disjoint pre*-closed sets A and B, there exist disjoint open sets U and V such that A⊆U and B⊆V.

Definition 2.7[6]

A space (X,τ) is said to be g-regular if for every g-closed set F and a point $x \notin F$, there exist disjoint open sets U and V such that $x \in U$ and $F \subseteq V$.

Definition 2.8[4]

A collection ß of α^{s^*}–open sets in X is called α^{s^*}–open cover or cover by α^{s^*}–open sets of a subset B of X if $B \subseteq \cup\{U_\alpha : U_\alpha \in ß\}$holds.

Definition 2.9[4]

A topological space X is said to be α^{s^*}-compact if every α^{s^*}-open cover of X has a finite subcover.

Definition 2.10[8]

A subset A of a space (X, τ) is said to be quasi H-closed relative to X if for every cover $\{V\alpha: \alpha \in \nabla\}$ of A by open sets of X, there exists a finite subset ∇_0 of ∇ such that $A \subseteq \cup\{cl(V\alpha): \alpha \in \nabla_0\}$

Theorem 2.11[3]

(i) Every α-open set is α^{s*} open and every α-closed set is α^{s*}closed.
(ii) Every α^{s*}-open is pre*open and every α^{s*}-closed set is pre*-closed.
(iii)Every open set is α^{s*} open and every closed set is α^{s*}closed

3. α^{s^*}-Regular

In this section , we introduce α^{s^*}–Regular spaces using α^{s^*}–closed and α^{s^*}-open sets and find their relations themselves and with already existing spaces. Also , we find some characterizations of α^{s^*}–regular spaces.

Definition 3.1

A space (X, τ) is said to α^{s^*}–regular if for each α^{s^*}–closed set A and a point $x \notin A$, there exist disjoint open sets U and V such that $A \subseteq U$, $x \in V$

Theorem 3. 2

A space (X, τ) is said to α^{s^*}–regular if and only if (X, τ) is regular and every α^{s^*}-closed is closed.

Proof:

Suppose that (X,τ) is α^{s^*}– regular. Then clearly (X,τ) is regular. Now let $A \subseteq X$ be α^{s^*}–closed. For each $x \notin A$, there exists open sets V_x containing x such that $V_x \cap A = \emptyset$. If $V = \cup\{V_x : x \notin A\}$ then V is open and $V = X \setminus A$, hence A is closed.The converse is obvious.

Theorem 3.3

Every pre* regular is α^{s^*}-regular

Proof:

Let F be a α^{s^*}-closed set and $x \notin F$. Then by Theorem 2.11, F is pre*–closed. Since X is pre* –regular there exists disjoint open sets U and V such that $F \subseteq U$ and $x \in V$. Therefore X is α^{s^*}-regular.

However the converse of the above theorem is not true in the following example.

Example 3.4

Let X= {a,b,c} with $\tau = \{\emptyset,\{a,b\},\{c\},X\}$.Clearly (X,τ) is α^{s^*}–regular but not pre*-regular

Theorem 3.5

In a topological space X, the following are equivalent.

(i) X is α^{s^*}–regular
(ii) For every $x \in X$ and every α^{s^*}–open set G containing x, there exists an open set U such that $x \in U \subseteq cl(U) \subseteq G$.
(iii) For every α^{s^*}–closed set F, the intersection of all closed neighborhood of F is exactly F
(iv) For any set A and a α^{s^*}–open set B such that $A \cap B \neq \emptyset$, there exists an open set U such that $A \cap U \neq \emptyset$ and $cl(U) \subseteq B$.
(v) For every non-empty set A and a α^{s^*}–closed set B such that $A \cap B = \emptyset$, there exists disjoint open sets U and V such that $A \cap U \neq \emptyset$ and $B \subseteq V$

Proof:

(i)$\Rightarrow$(ii)

Suppose X is α^{s^*}–regular . Let $x \in X$ and let G be a α^{s^*}–open set containing x. then $x \notin X \setminus G$ and $X \setminus G$ is α^{s^*}–closed . since X is α^{s^*}–regular, there exist open sets U and V such that $U \cap V = \emptyset$ and $x \in U, X \setminus G \subseteq V$. It follows that $U \subseteq X \setminus V \subseteq G$ and hence $cl(U) \subseteq cl(X \setminus V) = X \setminus V \subseteq G$. That is $x \in U \subseteq cl(U) \subseteq G$.

(ii)$\Rightarrow$(iii)

Let F be any α^{s^*}–closed set and $x \notin F$.Then $X \setminus F$ is α^{s^*}–open and $x \in X \setminus F$. By assumption, there exists an open sets U such that $x \in U \subseteq cl(U) \subseteq X \setminus F$,Thus $F \subseteq X \setminus cl(U) \subseteq X \setminus U$. Now $X \setminus U$ is closed neighborhood of F which does not contains x. so we get the intersection of all closed neighborhoods of F is exactly F.

(iii)$\Rightarrow$(iv)

Suppose $A \cap B \neq \emptyset$ and B is α^{s^*}–open .Let $x \in A \cap B$. Since B is α^{s^*}–open,$X \setminus B$ is α^{s^*}–closed and $x \notin X \setminus B$.By using (iii), there exists a closed neighborhood V of $X \setminus B$ such that $x \notin V$. Now for the neighborhood V of $X \setminus B$ there exists an open set G such that $X \setminus B \subseteq G \subseteq V$. Take $U = X \setminus V$. Thus U is an open set containing x. Also $A \cap U \neq \emptyset$ and $cl(U) \subseteq X \setminus G \subseteq B$.

(iv)⇒(v)

Suppose A is a non-empty set and B is α^{s^*}–closed such that $A\cap B=\emptyset$.Then $X\setminus B$ is α^{s^*}–open and $A\cap(X-B)\neq\emptyset$. By our assumption, there exists an open set U such that $A\cap U\neq\emptyset$ and $cl(U)\subseteq X\setminus B$. Take $V=X\setminus cl(U)$. since cl(U) is closed , we have V is open. Also $B\subseteq V$ and $U\cap V\subseteq cl(U)\cap(X\setminus cl(U))=\emptyset$.

(v)⇒(i)

Let S be α^{s^*}–closed and $x\notin S$. Then $S\cap\{x\}=\emptyset$. By (v), there exists disjoint open sets U and V such that $U\cap\{x\}\neq\emptyset$ and $S\subseteq V$.That is U and V are disjoint open sets containing x and S respectively. This proves that (X,τ) is α^{s^*}-regular

Theorem 3.6

Every quasi H-closed subset relative to a α^{s^*}–regular space is g-closed.

Proof:

Suppose X is α^{s^*}–regular and a subset A of X is quasi H-closed relative to X. Let U be a open set in X containing A. Then by Theorem 2.11, U is a α^{s^*}–open in X containing A. since X is α^{s^*}–regular, by using Theorem3.5 (ii) for each $x\in A$, there exists an open set V_x such that $x\in V_x\subseteq cl(V_x)\subseteq U$. clearly $\{V_x:x\in A\}$ is an open cover for A. since A is quasi H-closed relative to X, there exists a finite subset A_0 of A such that $A\subseteq\cup\{cl(V_x):x\in A_0\}$. Since finite union of closed set is closed, we have $A\subseteq cl(A)\subseteq\cup\{cl(V_x):x\in A_0\}\subseteq U$. that is $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open. This shows that A is g-closed.

Theorem 3.7

A topological space X is α^{s^*}–regular if and only if for each α^{s^*}-closed F of X and each $x\in X\setminus F$, there exists open sets U and V such that $x\in U$ and $F\subseteq V$ and $cl(U)\cap cl(V)=\emptyset$

Proof:

Suppose X is α^{s^*}–regular. Let F be a α^{s^*}–closed set in X and $x\notin F$. Then there exists an open sets U_x and V such that $x\in U_x$, $F\subseteq V$ and $U_x\cap V=\emptyset$. This implies that $U_x\cap cl((V)=\emptyset$. also cl(V) is a closed set and hence cl(V) is α^{s^*}–closed set and $x\notin cl(V)$. since X is α^{s^*}–regular, there exist open sets G and H of X such that $x\in G$ and $cl(V)\subseteq H$ and $G\cap H=\emptyset$. This implies $cl(G)\cap H\subseteq cl(X\setminus H)\cap H=(X\setminus H)\cap H=\emptyset$. Take U=G.Now U and V are open sets in X such that $x\in U$ and $F\subseteq V$ .Also $cl(U)\cap cl(V)\subseteq cl(G)\cap H=\emptyset$.

Conversely , suppose for each α^{s^*}–closed set F of X and $x\in X\setminus F$, there exists open sets U and V of X such that $x\in U$ and $F\subseteq V$ and $cl(U)\cap cl(V)=\emptyset$.Now $U\cap V\subseteq cl(U)\cap cl(V)=\emptyset$.Therefore $U\cap V=\emptyset$.Hence X is α^{s^*}–regular.

Theorem 3.8

Let $f:X\rightarrow Y$ be a bijective function

(i) If f is M- α^{s^*}-continuous, open and X is α^{s^*}–regular. Then Y is α^{s^*}–regular.

(ii) If f is continuous,M- α^{s^*}closed and Y is α^{s^*}–regular, then X is α^{s^*}–regular

Proof:

(i) Suppose X is α^{s^*}–regular. Let S be any α^{s^*}–closed subset in Y such that $y\notin S$. since f is M- α^{s^*}-Continuous,$f^{-1}(S)$ is α^{s^*}–closed in X. since f is onto, there exists $x\in X$ such that $y=f(x)$. Now $f(x)=y\notin S \Rightarrow x\notin f^{-1}(S)$. since X is α^{s^*}–regular, there exists open sets U and V in X such that $x\in U, f^{-1}(S)\subseteq V$ and $U\cap V=\emptyset$. Now $x\in U\Rightarrow f(x)\in f(U)$ and $f^{-1}(S)\subseteq V\Rightarrow S\subseteq f(V)$. Also $U\cap V=\emptyset \Rightarrow f(U\cap V)=\emptyset \Rightarrow f(U)\cap f(V)=\emptyset$. since f is open map, f(U) and f(V) are disjoint open sets in Y containing y and S respectively. Thus Y is α^{s^*}–regular.

(ii) Suppose Y is α^{s^*}–regular.Let F be any α^{s^*}–closed subset in X such that $x\notin F$. since f is M- α^{s^*}-Closed function,f(F) is α^{s^*}–closed in Y and $f(x)\notin f(F)$. Since Y is α^{s^*}–regular, there exists disjoint open sets U and V in Y such that $f(x)\in U, f(F)\subseteq V$ .Clearly $x\in f^{-1}(U)$ and $F\subseteq f^{-1}(V)$.since f is continuous,$f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open sets in X containing x and F respectively. Thus X is α^{s^*}–regular.

Theorem 3.9

Every α^{s^*}–compact subset of a space X is quasi H- closed relative to X.

Proof:

Let A be a α^{s^*}–compact relative to X. Let $\{V_\alpha:\alpha\in \nabla\}$ be an opencover for A in X. since every open set is α^{s^*} -open, $\{V_\alpha:\alpha\in \nabla\}$ is a α^{s^*}–open cover for A in X. since A is α^{s^*}–compact in X, there exists a finite subset ∇_o of ∇ such that $A\subseteq\cup\{V_\alpha:\alpha\in \nabla_o\}$.clearly $A\subseteq\cup\{V_\alpha:\alpha\in \nabla_o\}\subseteq\cup\{cl(V_\alpha):\alpha\in \nabla_o\}$.By definition A is quasi H-closed relative to X.

Theorem 3.10

If X is a α^{s^*}–regular space and a subset A of X is quasi H-closed relative to X, then A is α^{s^*}–compact in X.

Proof:

Suppose X is a α^{s^*}–regular space and a subset A of X is quasi H-closed relative to X. Let $\{V_\alpha:\alpha\in \nabla\}$ be a α^{s^*}–open cover or A. Then $A\subseteq\cup\{V_\alpha:\alpha\in \nabla\}$.Let $x\in A$. Then $x\in V_\alpha$ for some α. For each $x\in A$,take $V_x=V_\alpha$, where V_α is any one of the α^{s^*}–open sets in X containing x. since X is α^{s^*}–regular space and V_x is α^{s^*}–open. By Theorem 3.5 (ii), for each $x\in A$, there exists an open set U_x such that $x\in U_x\subseteq cl(U_x)\subseteq V_x$. Clearly $\{U_x:x\in A\}$ is an open cover of A. since A is quasi H-closed relative to X. there exists a finite subset A_o of A such that $A\subseteq\cup\{cl(Ux):x\in A_o\}\subseteq\cup\{V_x:x\in A_o\}$. That is $\{V_x:x\in A_o\}$ is a finite sub cover for the α^{s^*}–open cover $\{V_\alpha: \alpha\in \nabla\}$of A. This shows that A is α^{s^*}–compact in X.

Theorem 3.11

A topological space X is α^{s^*}–regular if and only if every pair consisting of a compact set and a disjoint α^{s^*}–closed set can be separated by open set.

Proof:

Let X be α^{s^*}–regular and A be a compact set ,B be α^{s^*}–closed set with $A \cap B = \emptyset$. Since X is α^{s^*}-regular , for each $x \in A$, there exist disjoint open set U_x and V_x such that $x \in U_x$, $B \subseteq Vx$. Clearly $\{U_x : x \in A\}$ is an open covering of A. since A is compact, there exists a finite subfamily $\{U_{xi} : 1 \leq i \leq n\}$ which covers A. It follows that $A \subseteq \cup\{U_{xi} : 1 \leq i \leq n\}$ and $B \subseteq \cap\{V_{xi} : 1 \leq i \leq n\}$. Put $U = \cup\{U_{xi} : 1 \leq i \leq n\}$ and $V = \cap\{V_{xi} : 1 \leq i \leq n\}$.Then U and V are open in X. Also $U \cap V = \emptyset$. Otherwise if $x \in U \cap V \Rightarrow x \in U_{xj}$ for some j and $x \in V_{xi}$ for every i. This implies that $x \in U_{xj} \cap V_{xj}$, which is a contradiction to $U_{xj} \cap V_{xj} = \emptyset$. Thus U and V are disjoint open sets containing A and B respectively.

Conversely, suppose every pair consisting of a compact set and a disjoint α^{s^*}–closed set can be separated by open sets. Let F be a α^{s^*}–closed set and $x \notin F$. Then $\{x\}$ is compact subset of x and $\{x\} \cap F = \emptyset$.By our assumption there exist disjoint open sets U and V such that $x \in U$ and $F \subseteq V$ .This proves that X is α^{s^*}–regular.

4. α^{s^*}-Normal

A topological (X,τ) is said to be α^{s^*}–Normal if for any two disjoint α^{s^*}–closed set A and B, there exists disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 4.1

Every pre* Normal space is α^{s^*} –Normal.

Proof:

Suppose X is pre* - Normal. Let A and B be two disjoint α^{s^*}–closed sets in X . since every α^{s^*}–closed set is pre*-closed, A and B are pre*-closed in X. By Definition 2.6 there exists disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$. This proves that X is α^{s^*}–normal.

Remark 4. 2

But the converse of the above theorem is not true as shown by the following example.

Example 4.3

Let $X=\{a,b,c\}$ with topology $\tau = \{\emptyset,\{b\},\{a,c\},X\}$. Clearly (X,τ) is α^{s^*}–Normal but not pre*-Normal.

Theorem 4.4

For a topological space X. the following are equivalent.

(i) X is α^{s^*}–Normal
(ii) For every pair of α^{s^*}–open sets U and V whose union in X, there exists closed sets E and F such that $E \subseteq U$ and $F \subseteq V$ and $E \cup F = X$.

(iii) For every α^{s^*}–closed set F and every α^{s^*}–open set G containing F, there exists an open set U such that $F\subseteq U\subseteq cl(U)\subseteq G$.

Proof:

(i)⇒(ii)

Let U and V be pair of α^{s^*}–open sets in a α^{s^*}–Normal space X such that $X=U\cup V$.Then $(X\setminus U)\cap(X\setminus V)= X\setminus(U\cup V)= X\setminus X= \emptyset$ and $X\setminus U$ and $X\setminus V$ are disjoint α^{s^*}–closet sets. Since X is α^{s^*}–Normal, there exists disjoint open sets G and H such that $X\setminus U\subseteq G$ and $X\setminus V\subseteq H$.Let $E=X\setminus G$ and $F=X\setminus H$, then E and F are closed sets such that $E\subseteq U,F\subseteq V$. Also $E\cup F=(X\setminus G)\cup(X\setminus H)=X\setminus(G\cap H)=X\setminus\emptyset =X$.

(ii)⇒(iii)

Let F be a α^{s^*}–closed and Let G be a α^{s^*}–open set containing F.Then $X\setminus F$ and G are α^{s^*}–open sets whose union is X. Then by (ii) , there exists closed sets V_1 and V_2 such that $V_1\subseteq X\setminus F$ and $V_2\subseteq G$ and $V_1\cup V_2=X$. Then $F\subseteq X\setminus V_1$, $X\setminus G\subseteq X\setminus V_2$ and $(X\setminus V1)\cap(X\setminus V_2)=X\setminus(V_1\cup V_2)=X\setminus X= \emptyset$. Let $U=X\setminus V_1$ and $V=X\setminus V_2$ Then U and V are disjoint open sets such that $F\subseteq U\subseteq X\setminus V=V_2\subseteq G$.As $X\setminus V$ is closed, we have $cl(U) \subseteq X\setminus V$ and $F\subseteq U\subseteq cl(U)\subseteq G$.

(iii)⇒(i)

Let F_1 and F_2 be any two disjoint α^{s^*}–closed sets in X. Put $G=X\setminus F_2$, then $F_1\subseteq G$ and G is a α^{s^*}–open set in X. Then by (iii), there exists an open set U of X such that $F_1\subseteq U\subseteq cl(U)\subseteq G$. This implies that $F_2\subseteq X\setminus cl(U)$. Take $V=X\setminus cl(U)$. Then V is an open set containing F_2 and $U\cap V\subseteq cl(U)\cap(X\setminus cl(U))= \emptyset$. That is F_1 and F_2 are separated by open set U and V. It follows that X is α^{s^*}–Normal

Theorem 4.5

If X is α^{s^*}–Normal, then every pair of disjoint α^{s^*}–closed sets have open neighborhoods whose closures are disjoint.

Proof:

Assume that X is α^{s^*}–Normal.Let A and B be disjoint α^{s^*}–closed sets in X. since X is α^{s^*}–Normal , there exists open sets U_1 and U_2 such that $A\subseteq U_1$ and $B\subseteq U_2$ and $U_1\cap U_2= \emptyset$.By the above theorem 4.4, there exists open sets V_1 and V_2 such that $A\subseteq V_1\subseteq cl(V_1)\subseteq U_1$ and $B\subseteq V_2 \subseteq cl(V_2)\subseteq U_2$. Moreover, $cl(V_1)\cap cl(V_2)\subseteq U_1\cap U_2 = \emptyset$.

Theorem 4. 6

Let $f:X\rightarrow Y$ be a function

(i) If f is injective,M- α^{s^*}-continuous,open and X is α^{s^*}–Normal. Then Y is α^{s^*}–Normal.
(ii) If f is continuous,M- α^{s^*}closed and Y is α^{s^*}–Normal, then X is α^{s^*}–Normal

Proof:

(i) Suppose X is α^{s^*}–Normal. Let A and B be disjoint α^{s^*}–closed sets in Y. since f is M- α^{s^*}-Continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are α^{s^*}–closed in X. since X is α^{s^*}–Normal, there exists disjoint open sets U and V in X such that $f^{-1}(A)\subseteq U$ and $f^{-1}(B)\subseteq V$. Now $f^{-1}(A)\subseteq U\Rightarrow A\subseteq f(U)$ and $f^{-1}(B)\subseteq V\Rightarrow B\subseteq f(V)$.since f is an open map,f(U) and f(V) are open sets in Y. Also $U\cap V=\emptyset \Rightarrow f(U\cap V)=\emptyset \Rightarrow f(U)\cap f(V)=\emptyset$. Thus f(U) and f(V) are disjoint open sets in Y containing A and B respectively.Thus Y is α^{s^*}–Normal.

(ii) Suppose Y is α^{s^*}–Normal. Let A and B be disjoint α^{s^*}–closed sets in X.since f is M- α^{s^*}-Closed function, f(A) and f(B) are α^{s^*}–closed in Y. Since Y is α^{s^*}–normal, there exists disjoint open sets U and V in Y such that $f(A)\subseteq U, f(B)\subseteq V$. That is $A\subseteq f^{-1}(U)$ and $B\subseteq f^{-1}(V)$.since f is continuous,$f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open sets in X containing A and B respectively. Thus X is α^{s^*}–Normal.

5. C- α^{s^*} compact

In this section we introduce C - α^{s^*}-Compact space.

Definition 5.1 [**11**]

A topological space X is called C- Compact if for each closed subset $A\subseteq X$ and for each open cover $\mathcal{U}=\{U_\alpha / \alpha\in \nabla\}$of A, there exists a finite sub collection $\{U_{\alpha i} / 1\le i\le n\}$of $\mathcal{U}$, such that $A\subseteq\bigcup_{i=1}^{n} cl(U_{\alpha i})$

Definition 5.2

An α^{s^*}–open set U is said to be α^{s^*}–regular if $\alpha^{s^*}int(\alpha^{s^*}cl(U))=U$. Also α^{s^*}–closed set U is said to be α^{s^*}–regular if $\alpha^{s^*}cl(\alpha^{s^*}int(U))=U$.

Definition 5.3

X is said to be an α^{s^*}–Hausdorff space if for any pair of distinct points x and y in X, there exists an α^{s^*}–open sets U and V in X such that $x\in U$, $y\in V$ and $U\cap V=\emptyset$.

Definition 5.4

A set U in a topological space X is an α^{s^*}–neighborhood of a point x if U contains an α^{s^*}–open set V, such that $x\in V$.

Theorem 5.5

If A is α^{s^*}–closed then $\alpha^{s^*}intA$ is α^{s^*}–regular open

Proof:

Clearly $\alpha^{s^*}intA\subseteq \alpha^{s^*}cl(\alpha^{s^*}intA)$

$\Rightarrow$ α^{s^*} int(α^{s^*}intA)$\subseteq$ α^{s^*}int(α^{s^*}cl(α^{s^*}int(A)))

$\Rightarrow$ α^{s^*}intA$\subseteq$ α^{s^*}int(α^{s^*}cl(α^{s^*}int(A))).

Also α^{s^*}int(A)$\subseteq$A,

$\Rightarrow$ α^{s^*}cl(α^{s^*}int(A))$\subseteq$ α^{s^*}cl(A)=A (since A is α^{s^*}–closed)

$\Rightarrow$ α^{s^*}int(α^{s^*}cl(α^{s^*}int(A)))$\subseteq$ α^{s^*}int(A)

Theorefore α^{s^*}int(α^{s^*}cl(α^{s^*}int(A)))= α^{s^*}int(A).

Hence α^{s^*}int(A) is α^{s^*}–regular open.

Theorem 5. 6

If A is α^{s^*}–open then α^{s^*}cl(A) is α^{s^*}–regular closed

Proof:

Follows pre-Theorem 5.5

Note 5.7

By pre-Theorems we can say that α^{s^*}int(α^{s^*}cl(U)) is α^{s^*}–regular open and α^{s^*}cl(α^{s^*}int(U)) is α^{s^*}–regular closed.

Definition 5.8

A topological space X is called C- α^{s^*}-Compact if for each α^{s^*}-closed subset A$\subseteq$X and for each α^{s^*}–open cover $\mathcal{U} = \{U_\alpha / \alpha \in \nabla\}$of A, there exists a finite sub collection $\{U_{\alpha i} / 1 \le i \le n\}$of $\mathcal{U}$, such that A$\subseteq \bigcup_{i=1}^{n} \alpha^{s*}$cl($U_{\alpha i}$)

Lemma 5.9

A topological space X is C- α^{s^*}-compact iff for each α^{s^*}-closed subset A$\subseteq$X and for each α^{s^*}–regular open cover $\{U_\alpha : \alpha \in \nabla\}$ of A, there exists a finite subcollectiion $\{U\alpha_i : 1 \le i \le n\}$ such that A$\subseteq \bigcup_{i=1}^{n} \alpha^{s*}$cl($U_{\alpha i}$)

Proof:

If X is C-α^{s^*}-compact, the condition follows from Definition.

Now suppose the condition holds and let $\{U_\alpha : \alpha \in \nabla\}$ be any cover of A by α^{s^*}–open sets. Then $\gamma = \alpha^{s^*}$int(α^{s^*}cl(U_α)) is a α^{s^*}–regular open cover of A and so there exists a finite sub collection {α^{s^*}int(α^{s^*}cl($U_{\alpha i}$)): $1 \le i \le n$} of γ such that A$\subseteq \bigcup_{i=1}^{n} \alpha^{s*}$cl($\alpha^{s^*}$int($\alpha^{s^*}$cl($U_{\alpha i}$))).But for each i , we have α^{s^*}cl(α^{s^*}int(α^{s^*}cl($U_{\alpha i}$))) = α^{s^*}cl($U_{\alpha i}$) Therefore A$\subseteq$ $\bigcup_{i=1}^{n} \alpha^{s*}$cl($U_{\alpha i}$) which shows that X is C- α^{s^*}-Compact

Theorem 5.10

A M-α^{s^*}–continuous image of a C- α^{s^*}-compact space is C- α^{s^*}-compact

Proof:

Let A be a α^{s^*}-closed subset of Y and γ be an α^{s^*}–open cover of A. By M-α^{s^*}–continuity of f, $f^{-1}(A)$ is an α^{s^*}–closed subset of X and P={$f^{-1}(V)$:V∈ γ} is a cover of $f^{-1}(A)$ by α^{s^*}–open sets. By C- α^{s^*}-compactness of X, there exists a finite collection say{Pi:1≤i≤n} of P such that $f^{-1}(A)\subseteq \bigcup_{i=1}^{n}\{\alpha^{s*}cl(f^{-1}(Vi)):1\leq i\leq n\}$.Now by M-$\alpha^{s^*}$–continuity of f, $A\subseteq \bigcup_{i=1}^{n}\{\alpha^{s*}cl((Vi):1\leq i\leq n\}$.Thus Y is C- α^{s^*}-compact.

Theorem 5.11

For any Topological space X, the following properties of X are equivalent

(i) X is C- α^{s^*}-compact
(ii) For each α^{s^*}-closed A⊆X and each α^{s^*}–regular open cover {U_α: α∈ ∇} of A there exists a finite sub collection {$U_{\alpha i}$: 1≤i≤n} such that $A\subseteq \bigcup_{i=1}^{n} \alpha^{s*}cl(U_{\alpha i})$
(iii) For each α^{s^*}-closed A⊆X and each collection of non-empty α^{s^*}–regular closed sets {F_α: α∈ ∇} such that $(\bigcap_\alpha F_\alpha)\cap A=\emptyset$, there exists a finite sub collection {$F_{\alpha i}$: 1≤i≤n} such that $(\bigcap_{i=1}^{n} \alpha^{s*}\mathrm{int}(F_{\alpha i})\cap A=\emptyset$
(iv) For each α^{s^*}-closed A⊆ X and each collection of non empty α^{s^*}–regular closed sets{F_α: α∈ ∇} , if each finite sub collection {$F_{\alpha i}$: 1≤i≤n}has the property that $(\bigcap_{i=1}^{n} \alpha^{s*}\mathrm{int}(F_{\alpha i})\cap A\neq\emptyset$, then $(\bigcap_\alpha F_\alpha)\cap A\neq\emptyset$.

Proof:

(i)if and only if (ii) has been shown in Lemma 5.9

(ii)⇒(iii)

Let A be a α^{s^*}-closed subset of a C- α^{s^*}- compact space X and $\mathcal{F}$ a family of α^{s^*}–regular closed sets of X with $\cap\mathcal{F}\cap A=\emptyset$. since $\mathcal{U}=\{X-F\ /\ F\in\mathcal{F}\}$is a family of α^{s^*}–regular open sets of X covering A, there is a finite number of elements of $\mathcal{U}$, say U_i= X-F_i,1≤i≤n, with $\bigcup_{i=1}^{n} \alpha^{s*}cl(Ui)\supseteq A$. Therefore, $\bigcap_{i=1}^{n} \alpha^{s*}\mathrm{int}(F_i)=X-\bigcup_{i=1}^{n} \alpha^{s*}cl(Ui)\subseteq X-A$.

(iii)⇒(ii)

Let {U_α: α∈ ∇} be a α^{s^*}–regular open cover of A. Then $A\subseteq\bigcup_\alpha U_\alpha$ implies $(\bigcap_\alpha(X-U\alpha))\cap A=\emptyset$. since X-$U_\alpha$ is α^{s^*}–regular closed for each α∈∇, the hypothesis of (iii) implies that there is a finite subcollection {X-$U_{\alpha i}$ / 1≤ i ≤n}such that $(\bigcap_{i=1}^{n} \alpha^{s*}\mathrm{int}(X-U_{\alpha i}))\cap A=\emptyset$. It follows that $A\subseteq i=1n(X-\alpha^{s*}\mathrm{int}(X-U_{\alpha i}))$. However, $X-\alpha^{s*}\mathrm{int}(X-U_{\alpha i})=\alpha^{s*}cl(X-(X-U_{\alpha i}))=\alpha^{s*}cl(U_{\alpha i})$ for each i=1, 2,3,4………n. Therefore, $A\subseteq\bigcup_{i=1}^{n} \alpha^{s*}cl(U_{\alpha i})$ which is condition(ii).

(iii) If and only if (iv) is clear.

Theorem 5.12

Every M-α^{s^*}-Continuous function from a C- α^{s^*}-compact space to a α^{s^*}–Hausdorff space is M-α^{s^*}-closed.

Proof:

Let f be α^{s^*}–continuous function from a C- α^{s^*} -compact space X to a α^{s^*}–Hausdorff space Y. Let C be a α^{s^*}-closed set in X and let $p\notin f(C)$. Now for every $x\in f(C)$, $x\neq p$ and hence choose a open neighborhood N_x such that $p\notin \alpha^{s^*}cl(N_x)$. Clearly $\{f^{-1}(N_x) : x\in f(C)\}$ is a α^{s^*}–open cover of C. Let $\{x_i : 1\leq i\leq n\}$ be such that $C\subseteq\bigcup_{i=1}^{n} \alpha^{s*}cl(f^{-1}(N_{xi})) : 1\leq i\leq n\}$(since X is C-$\alpha^{s^*}$−Compact space). Thus by the M-α^{s^*}–continuity of f ,Y-$\bigcup_{i=1}^{n}\{\alpha^{s*}cl(N_{xi}): 1\leq i\leq n\}$ is a α^{s^*}–neighborhood of p disjoint from f(C). Hence f(C) is α^{s^*}-closed, hence M-α^{s^*}–continuous function f from a C- α^{s^*}--Compact space X to a α^{s^*}–Hausdorff space Y is M-α^{s^*}-closed.

Theorem 5.13

A space X is C-α^{s^*}-Compact iff for each α^{s^*}-closed subset C of X and α^{s^*}–open cover $\mathcal{G}$ of X-C and a α^{s^*}-open –neighborhood U of C, there exists a finite collection $\{G_i\in\mathcal{G} : 1\leq i\leq n \}$ such that $X= U\cup \bigcup_{i=1}^{n}\{\alpha^{s*}cl(G_i) : 1\leq i\leq n \}$

Proof:

Since U is an α^{s^*}-open neighborhood of C, therefore $C\subseteq U\subseteq cl(C)$, or $X-U\subseteq X-C$ where X-U is a α^{s^*}–closed set. Since $\mathcal{G}$ is a α^{s^*}–open cover of X-C, $\mathcal{G}$ is a α^{s^*}–open cover of the α^{s^*}–closed set X-U. Now by C- α^{s^*}-compactness of X, there exists a finite subfamily $\{G_i : 1\leq i\leq n \}$ of $\mathcal{G}$ such that $X-U\subseteq\bigcup_{i=1}^{n}\{\alpha^{s*}cl(G_i) : 1\leq i\leq n \}$ which implies $X=U\cup \bigcup_{i=1}^{n}\{\alpha^{s*}cl(Gi) : 1\leq i\leq n \}$.

Conversely, Let A be a α^{s^*}-closed subset of X and $\mathcal{G}$ be a α^{s^*}-open cover of A, Therefore $A\subseteq\cup\{G:G\in\mathcal{G}\}$ =H(say), obviously H is α^{s^*}-open . Therefore X-H is α^{s^*}-closed and $C\subseteq X-A$. since X-A is α^{s^*}-open. Therefore we can take X-A= U is an α^{s^*}-open neighborhood of C. Thus by the given statement $X=U\cup \bigcup_{i=1}^{n}\{\alpha^{s*}cl(Gi) : 1\leq i\leq n \}$ .Hence X is C- α^{s^*}- compact

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