ALMOST CONTRA *vg*-OPEN AND ALMOST CONTRA *vg*-CLOSED MAPPINGS

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Abstract: The aim of this paper is to introduce and study the concepts of almost contra *vg*-open and almost contra *vg*-closed mappings and the interrelationship between other contra-closed maps.
Keywords: *vg*-open set, *vg*-open map, *vg*-closed map, contra-closed map, contra-pre closed map, contra *vg*-open map, contra *vg*-closed map, almost contra *vg*-open map and almost contra *vg*-closed map.
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§1. INTRODUCTION:

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1969, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced α-open and α-closed mappings in the year in 1982, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-closed mappings in the year 1989 and G.B.Navalagi further verified few results about semipro-closed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced β-open mappings in the year 1983. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. During the years 2010 to 2014, S. Balasubramanian together with his research scholars defined and studied a variety of open, closed, almost open and almost closed mappings for *v*-open, rp-open *gpr*-closed and *vg*–closed sets as well contra-open and contra-closed mappings for semi-open, pre-open, rp-open, β-open and gpr-closed sets. Inspired with these concepts and its interesting properties the author of this paper tried to study a new variety of open and closed maps called almost contra *vg*-open and almost contra *vg*-closed maps. Throughout the paper *X*, *Y* means topological spaces (*X*, τ) and (*Y*,σ) on which no separation axioms are assured.

§2. Preliminaries:

Definition 2.1: A⊆ X is said to be

a) regular open[pre-open; semi-open; α-open; β-open] if A = int(cl(A)) [A ⊆int(cl(A)); A⊆ cl(int(A)); A⊆ int(cl(int(A))); A⊆ cl(int(cl(A)))] and regular closed[pre-closed; semi-closed; α-closed; β-closed] if A = cl(int(A))[cl(int(A)) ⊆ A; int(cl(A))⊆ A; cl(int(cl(A)))⊆ A; int(cl(int(A)))⊆ A]

b) *v*-open if there exists regular-open set U such that U⊆A⊆cl(U).

c) *v*-dense in X if *v*cl(A) = X.

d) θ-closed if A = Cl_θ(A). The complement of a θ-closed set is said to be θ-open.

e) g-closed[rg-closed] if cl(A)⊂U[rcl(A)⊂U] whenever A⊂U and U is open[r-open] in X.

f) g-open[rg-open] if its complement X - A is g-closed[rg-closed].

g) Zero[semi-zero] set of X if there exists a continuous [semi-continuous] function *f:* X → R such that A = {x∈X : *f*(x) = 0}. Its complement is called co-zero[co-semi-zero] set of X.

Remark 1: We have the following implication diagrams for closed sets.

g.closed $\downarrow$ gs.closed $\downarrow$

rαg.closed $\rightarrow$ rgα.closed $\rightarrow$ rg.closed $\rightarrow$ ***vg*.closed** $\leftarrow$ sg.closed $\leftarrow$ βg.closed

$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$

$\rightarrow\rightarrow$ rα.closed $\rightarrow$ $\rightarrow$ *v*.closed $\rightarrow$

$\uparrow$ $\downarrow$ $\uparrow$ $\uparrow$

r.closed $\rightarrow$ π.closed $\rightarrow$ closed $\rightarrow$ α.closed $\rightarrow$ s.closed $\rightarrow$ β.closed

$\downarrow$ $\downarrow$ $\downarrow\rightarrow\rightarrow\rightarrow$ $\downarrow$

πg.closed p.closed $\rightarrow$ ω.closed gα.closed

$\downarrow$ $\downarrow$

gp.closed $\leftarrow$ pg.closed rω.closed

The same relation is true for open sets also.

Definition 2.2: A function *f*:*X*→*Y* is said to be

a) continuous[resp:semi-continuous, r-continuous, *v*-continuous] if the inverse image of every open set is open [resp: semi open, regular open, *v*--open].

b) irresolute [resp: r-irresolute, *v*-irresolute] if the inverse image of every semi open [resp: regular open, *v*-open] set is semi open [resp: regular open, *v*-open].

c) closed[resp: semi-closed, r-closed] if the image of every closed set is closed [resp: semi closed, regular closed].

d) g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed].

Definition 2.3: A function *f*: *X*→*Y* is said to be

a) contra closed[resp: contra semi-closed; contra pre-closed; contra α-closed; contra rα-closed; contra β-closed; contra g-closed; contra r*g*-closed; contra sg-closed; contra gs-closed; contra pg-closed; contra gp-closed;] if the image of every closed set in *X* is open[resp: semi-open; pre-open; α-open; rα-open; β-open; g-open; r*g*-open; sg-open; gs-open; pg-open; gp-open] in *Y*.

b) contra open[resp: contra semi-open; contra pre-open; contra α-open; contra rα-open; contra β-open; contra g-open; contra r*g*-open; contra sg-open; contra gs-open; contra pg-open; contra gp-open;] if the image of every open set in *X* is closed[resp: semi-closed; pre-closed; α-closed; rα-closed; β-closed; g-closed; r*g*-closed; sg-closed; gs-closed; pg-closed; gp-closed] in *Y*.

c) almost contra closed[resp: almost contra semi-closed; almost contra pre-closed; almost contra α-closed; almost contra rα-closed; almost contra β-closed; almost contra g-closed; almost contra rg-closed; almost contra sg-closed; almost contra gs-closed; almost contra pg-closed; almost contra gp-closed;] if the image of every closed set in *X* is open[resp: semi-open; pre-open; α-open; rα-open; β-open; g-open; r*g*-open; sg-open; gs-open; pg-open; gp-open] in *Y*.

d) almost contra open[resp: almost contra semi-open; almost contra pre-open; almost contra α-open; almost contra rα-open; almost contra β-open; almost contra g-open; almost contra r*g*-open; almost contra sg-open; almost contra gs-open; almost contra pg-open; almost contra gp-open;] if the image of every open set in *X* is closed[resp: semi-closed; pre-closed; α-closed; rα-closed; β-closed; g-closed; r*g*-closed; sg-closed; gs-closed; pg-closed; gp-closed] in *Y*.

Definition 2.4: *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

§3. ALMOST CONTRA *vg*-OPEN MAPPINGS:

Definition 3.1: A function $f: X \to Y$ is said to be almost contra *vg*-open if the image of every *r*-open set in *X* is *vg*-closed in *Y*.

Theorem 3.1: Every contra *vg*-open map is almost contra *vg*-open map, but not conversely.

Theorem 3.2: We have the following interrelation among the following almost contra open mappings

(i)

al.c.g.open al.c.gs.open
↓ ↓
al.c.rαg.open → al.c.rgα.open → al.c.rg.open → al.c.***vg*.open** ← al.c.sg.open ← al.c.βg.open
↑ ↑ ↑ ↑ ↑
→ → al.c.rα.open → → al.c.*v*.open → → →
↑ ↓ ↑ ↑
al.c.r.open → al.c.π.open → al.c.open → al.c.α.open → al.c.s.open → al.c.β.open
↓ ↓ ↓
al.c.πg.open al.c.p.open → al.c.ω.open al.c.gα.open
↓ ↓
al.c.gp.open ← al.c.pg.open al.c.rω.open None is reversible

(ii)

c.g.open c.gs.open
↓ ↓
c.rαg.open → c.rgα.open → c.rg.open → al.c.***vg*.open** ← c.sg.open ← c.βg.open
↑ ↑ ↑ ↑ ↑
→ → c.rα.open → → c.*v*.open →
↑ ↓ ↑ ↑
c.r.open → c.π.open → c.open → c.α.open → c.s.open → c.β.open None is reversible

(iii) If *v*GC(Y) = RC(Y), then the reverse relations hold for all almost contra open maps.

al.c.g.open al.c.gs.open
↑↓ ↑↓
al.c.rαg.open ↔ al.c.rgα.open ↔ al.c.rg.open ↔ al.c.***vg*.open** ↔ al.c.sg.open ↔ al.c.βg.open
↑↓ ↑↓ ↑↓ ↑↓ ↑↓
↔ ↔ al.c.rα.open ↔ ↔ ↔ al.c.*v*.open ↔
↑↓ ↑↓ ↑↓ ↑↓
al.c.r.open ↔ al.c.π.open ↔ al.c.open ↔ al.c.α.open ↔ al.c.s.open ↔ al.c.β.open

Example 1: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f:X \to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is almost contra *vg*-open, almost contra rg-open and almost contra rgα-open but not almost contra open, almost contra semi-open, almost contra pre-open, almost contra α-open, almost contra rα-open, almost contra *v*-open, almost contra π-open, almost contra β-open, almost contra g-open, almost contra sg-open, almost contra gs-open, almost contra pg-open, almost contra gp-open and almost contra βg-open .

Example 2: Let $X = Y = \{a, b, c, d\}$; $\tau = \{\varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let $f:X \to Y$ be defined $f(a) = b$, $f(b) = a$, $f(c) = d$ and $f(d) = c$. Then f is almost contra *vg*-open but not contra *vg*-open.

Theorem 3.3:
(i) If (Y, σ) is discrete, then f is almost contra open of all types.

(ii) If *f* is almost contra open and *g* is *vg*-closed then *gof* is almost contra *vg*-open.
(iii) If *f* is almost open and *g* is contra *vg*-open then *gof* is almost contra *vg*-open.

Corollary 3.1: If *f* is almost contra open and *g* is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα*-; *rα*-; *α-; s-; p*-; *β-; v-;* π-; *r*-] closed then *gof* is almost contra *vg*-open.

Corollary 3.2: If *f* is almost open[almost *r*-open] and *g* is c-*g*-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα*-; c-*rα*-; c-*α-; c-s-; c-p*-; c-*β-;c*-π-] open then *gof* is almost contra *vg*-open.

Theorem 3.4: If *f*: X→ Y is almost contra *vg*-open, then $vg(\mathrm{cl}(f(\mathrm{A}))) \subset f(\mathrm{cl}(\mathrm{A}))$

Proof: Let A⊂X be r-open and *f*: X→ Y is almost contra *vg*-open gives $f(\mathrm{cl}\{\mathrm{A}\})$ is *vg*-closed in Y and $f(\mathrm{A}) \subset f(\mathrm{cl}(\mathrm{A}))$ which in turn gives $vg(\mathrm{cl}(f(\mathrm{A}))) \subset vg\mathrm{cl}(f(\mathrm{cl}(\mathrm{A})))$- - - - - (1)
Since $f(\mathrm{cl}(\mathrm{A}))$ is *vg*-closed in Y, $vg\mathrm{cl}(f(\mathrm{cl}(\mathrm{A}))) = f(\mathrm{cl}(\mathrm{A}))$ - - - - - - (2)
From (1) and (2) we have $vg(\mathrm{cl}(f(\mathrm{A}))) \subset f(\mathrm{cl}(\mathrm{A}))$ for every subset A of X.

Remark 2: Converse is not true in general.

Corollary 3.3: If *f*: X→ Y is al-c-*g*-[al-c-*rg-;* al-c-*sg-;* al-c-*gs-;* al-c-*βg-;* al-c-*rαg-;* al-c-*rgα*-; al-c-*r*-; al-c-*rα*-; al-c-*α-;* al-c-*s-;* al-c-*p*-; al-c-*β-;* al-c-*v-;* al-c-π-] open, then $vg(\mathrm{cl}(f(\mathrm{A}))) \subset f(\mathrm{cl}(\mathrm{A}))$

Theorem 3.5: If *f*:*X*→*Y* is almost contra *vg*-open and A⊂*X* is *r*-open, *f*(A) is τ_{vg}-closed in *Y.*

Proof: Let A⊂X be r-open and *f*: X→ Y is almost contra *vg*-open implies $vg(\mathrm{cl}(f(\mathrm{A}))) \subset f(\mathrm{cl}(\mathrm{A}))$ which in turn implies $vg(\mathrm{cl}(f(\mathrm{A}))) \subset f(\mathrm{A})$, since $f(\mathrm{A}) = f(\mathrm{cl}(\mathrm{A}))$. But $f(\mathrm{A}) \subset vg(\mathrm{cl}(f(\mathrm{A})))$. Combining we get $f(\mathrm{A}) = vg(\mathrm{cl}(f(\mathrm{A})))$. Hence *f*(A) is τ_{vg}-closed in Y.

Corollary 3.4: If *f*:*X*→*Y* is al-c-*g*-[al-c-*rg-;* al-c-*sg-;* al-c-*gs-;* al-c-*βg-;* al-c-*rαg-;* al-c-*rgα*-; al-c-*r*-; al-c-*rα*-; al-c-*α-;* al-c-*s-;* al-c-*p*-; al-c-*β-;* al-c-*v-;* al-c-π-]open, then *f*(A) is τ_{vg} closed in *Y* if A is *r*-open in *X*.

Theorem 3.6: If $vg(\mathrm{cl}(f(\mathrm{A}))) = \mathrm{rcl}(\mathrm{A})$ for every A⊂Y and X is discrete space, then the following are equivalent:
a) *f*: X→ Y is almost contra *vg*-open map
b) $vg(\mathrm{cl}(f(\mathrm{A}))) \subset f(\mathrm{cl}(\mathrm{A}))$

Proof: (a) ⇒ (b) follows from theorem 3.4

(b) ⇒ (a) Let A be any *r*-open set in X, then $f(\mathrm{A}) = f(\mathrm{cl}(\mathrm{A})) \supset vg(\mathrm{cl}(f(\mathrm{A})))$ by hypothesis. We have $f(\mathrm{A}) \subset vg(\mathrm{cl}(f(\mathrm{A})))$. Combining we get $f(\mathrm{A}) = vg(\mathrm{cl}(f(\mathrm{A}))) = \mathrm{r}(\mathrm{cl}(f(\mathrm{A})))$[by given condition] which implies *f*(A) is *r*-closed and hence *vg*-closed. Thus *f* is almost contra *vg*-open.

Theorem 3.7: If $v(\mathrm{cl}(\mathrm{A})) = \mathrm{rcl}(\mathrm{A})$ for every A⊂Y and X is discrete space, then the following are equivalent:
a) *f*: X→ Y is almost contra *vg*-open map
b) $vg(\mathrm{cl}(f(\mathrm{A}))) \subset f(\mathrm{cl}(\mathrm{A}))$

Proof: (a) ⇒ (b) follows from theorem 3.4

(b) ⇒ (a) Let A be any *r*-open set in X, then $f(\mathrm{A}) = f(\mathrm{cl}(\mathrm{A})) \supset vg(\mathrm{cl}(f(\mathrm{A})))$ by hypothesis. We have $f(\mathrm{A}) \subset vg(\mathrm{cl}(f(\mathrm{A})))$. Combining we get $f(\mathrm{A}) = vg(\mathrm{cl}(f(\mathrm{A}))) = \mathrm{rcl}(f(\mathrm{A}))$[by given condition] which implies *f*(A) is *r*-closed and hence *vg*-closed. Thus *f* is almost contra *vg*-open.

Theorem 3.8: *f*:*X*→*Y* is almost contra *vg*-open iff for each subset S of *Y* and each $\mathrm{U} \in \mathrm{RC}(\mathrm{X}, f^{-1}(\mathrm{S}))$, there is an *vg*-open set V of *Y* such that S⊆V and $f^{1}(\mathrm{V}) \subseteq \mathrm{U}$.

Proof: Assume $f{:}X\to Y$ is almost contra *vg*-open. Let $S\subseteq Y$ and $U\in RC(X, f^{-1}(S))$. Then X-U is r-open in X and $f(X\text{-}U)$ is *vg*-closed in Y as f is almost contra *vg*-open and $V=Y\text{-}f(X\text{-}U)$ is *vg*-open in Y. $f^{-1}(S)\subseteq U \Rightarrow S\subseteq f(U) \Rightarrow S\subseteq V$ and $f^{-1}(V) = f^{-1}(Y\text{-}f(X\text{-}U)) = f^{-1}(Y)\text{-}f^{-1}(f(X\text{-}U)) = f^{-1}(Y)\text{-}(X\text{-}U) = X\text{-}(X\text{-}U) = U$

Conversely Let F be r-open in $X \Rightarrow F^c$ is r-closed. Then $f^{-1}(f(F^c))\subseteq F^c$. By hypothesis there exists an *vg*-open set V of Y, such that $f(F^c)\subseteq V$ and $f^{-1}(V)\supset F^c$ and so $F\subseteq[f^{-1}(V)]^c$. Hence $V^c\subseteq f(F)\subseteq f[f^{-1}(V)^c]\subseteq V^c \Rightarrow f(F)\subseteq V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is *vg*-closed in Y. Therefore f is almost contra *vg*-open.

Remark 3: Composition of two almost contra *vg*-open maps is not almost contra *vg*-open in general.

Theorem 3.9: Let X, Y, Z be topological spaces and every *vg*-closed set is *r*-open in Y. Then the composition of two almost contra *vg*-open maps is almost contra *vg*-open.
Proof: (a) Let f and g be almost contra *vg*-open maps. Let A be any *r*-open set in $X \Rightarrow f(A)$ is *r*-open in Y (by assumption) $\Rightarrow g(f(A)) = g\,o\,f(A)$ is *vg*-closed in Z. Therefore gof is almost contra *vg*-open.

Theorem 3.10: Let X, Y, Z be topological spaces and every *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-;* π-]closed set is *r*-open in Y. Then the composition of two al-c-*g*-[al-c-*rg-;* al-c-*sg-;* al-c-*gs-;* al-c-*βg-;* al-c-*rαg-;* al-c-*rgα-*; al-c-*r-*; al-c-*rα-*; al-c-*α-;* al-c-*s-;* al-c-*p-*; al-c-*β-;* al-c-*v-;* al-c-π-]open maps is almost contra *vg*-open.
Proof: Let A be *r*-open set in X, then $f(A)$ is *sg*-closed in Y and so *r*-open in Y (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is *sg*-closed in Z. Hence gof is almost contra *vg*-open [since every *sg*-closed set is *vg*-closed].

Corollary 3.5: Let X, Y, Z be topological spaces and every *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-;* π-]closed set is open [*r*-open] in Y. Then the composition of two c-*g*-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα-*; c-*rα-*; c-*α-; c-s-; c-p-*; c-*β-; c-v-; c-*π-; c-*r*-]open maps is almost contra *vg*-open.

Example 3: Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$ and $\eta = \{\varphi, \{a\}, \{b, c\}, Z\}$. $f{:}X\to Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$ and $g{:}Y\to Z$ be defined $g(a) = b$, $g(b) = a$ and $g(c) = c$, then g, f and $g\,o\,f$ are almost contra *vg*-open.

Theorem 3.11: If $f{:}X\to Y$ is almost contra g-open[almost contra rg-open], g:$Y\to Z$ is *vg*-closed and Y is $T_{½}$ [*r*-$T_{½}$] then gof is almost contra *vg*-open.
Proof: (a) Let A be *r*-open in X. Then $f(A)$ is *g*-closed and so closed in Y as Y is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is *vg*-closed in Z (since g is *vg*-closed). Hence gof is almost contra *vg*-open.

Corollary 3.6: If $f{:}X\to Y$ is almost contra *g*-open[almost contra *rg*-open], $g{:}Y\to Z$ is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-; *r*-]closed and Y is $T_{½}$ [*r*-$T_{½}$] then gof is almost contra *vg*-open.

Theorem 3.12: If $f{:}X\to Y$ is almost g-open[almost rg-open], g:$Y\to Z$ is contra *vg*-open and Y is $T_{½}$ [*r*-$T_{½}$] then gof is almost contra *vg*-open.
Proof: (a) Let A be *r*-open in X. Then $f(A)$ is *g*-open and so open in Y as Y is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is *vg*-closed in Z (since g is contra *vg*-open). Hence gof is almost contra *vg*-open.

Theorem 3.13: If $f{:}X\to Y$ is almost g-open[almost rg-open], g:$Y\to Z$ is c-*g*-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα-*; c-*rα-*; c-*α-; c-s-; c-p-*; c-*β-; c-*π-]open and Y is $T_{½}$ [*r*-$T_{½}$] then gof is almost contra *vg*-open.

Theorem 3.14: If $f:X\to Y$ is *g*-open[*rg*-open], $g:Y\to Z$ is c-*g*-[c-*rg*-; *c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα*-; c-*rα*-; c-*α-; c-s-; c-p*-; c-*β-; c*-π-]open and *Y* is $T_{½}$[*r*-$T_{½}$], then *gof* is almost contra *vg*-open.
Proof: Let A be *r*-open set in X, then *f*(A) is *g*-closed in Y and so closed in *Y* (by assumption) ⇒ *g*(*f* (A)) = *gof*(A) is *gs*-closed in *Z*. Hence *gof* is almost contra *vg*-open [since every *gs*-closed set is *vg*-closed].

Theorem 3.15: If $f:X\to Y$ is c-*g*-open[c-*rg*-open], $g:Y\to Z$ is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα*-; *rα*-; *α-; s-; p*-; *β-;* π-]closed and *Y* is $T_{½}$[*r*-$T_{½}$], then *gof* is almost contra *vg*-open.
Proof: Let A be *r*-open set in X, then *f*(A) is *g*-closed in Y and so closed in *Y* (by assumption) ⇒ *g*(*f* (A)) = *gof*(A) is *gs*-closed in *Z*. Hence *gof* is almost contra *vg*-open [since every *gs*-closed set is *vg*-closed].

Theorem 3.16: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that *gof* is contra *vg*-open then the following statements are true.
a) If *f* is continuous [*r*-continuous] and surjective then *g* is almost contra *vg*-open.
b) If *f* is g-continuous[resp: rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is almost contra *vg*-open.
Proof: (a) For A *r*-open in *Y*, f^{-1}(A) open in *X* ⇒ (*g o f*)(f^{-1}(A)) = *g*(A) *vg*-closed in *Z*. Hence *g* is almost contra *vg*-open.

Similarly one can prove the remaining parts and hence omitted.

Corollary 3.7: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that *gof* is contra *vg*-open then the following statements are true.
a) If *f* is almost continuous [almost *r*-continuous] and surjective then *g* is almost contra *vg*-open.
b) If *f* is almost g-continuous[resp: almost rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is almost contra *vg*-open.

Corollary 3.8: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that *gof* is c-g-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα*-; c-*rα*-; c-*α-; c-s-; c-p*-; c-*β-; c-v-; c*-π-; c-*r*-]open then the following statements are true.
a) If *f* is continuous [*r*-continuous] and surjective then *g* is almost contra *vg*-open.
b) If *f* is g-continuous[rg-continuous], surjective and *X* is $T_{½}$ [*r*-$T_{½}$] then *g* is almost contra *vg*-open.

Corollary 3.9: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that *gof* is c-g-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα*-; c-*rα*-; c-*α-; c-s-; c-p*-; c-*β-; c-v-; c*-π-; c-*r*-]open then the following statements are true.
a) If *f* is almost continuous [almost *r*-continuous] and surjective then *g* is almost contra *vg*-open.
b) If *f* is almost g-continuous[almost rg-continuous], surjective and *X* is $T_{½}$ [*r*-$T_{½}$] then *g* is almost contra *vg*-open.

Theorem 3.17: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that *gof* is *vg*-closed then the following statements are true.
a) If *f* is contra-continuous [contra-*r*-continuous] and surjective then *g* is almost contra *vg*-open.
b) If *f* is contra-g-continuous[contra-rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is almost contra *vg*-open.
Proof: (a) For A *r*-open in *Y*, f^{-1}(A) closed in *X* ⇒ (*g o f*)(f^{-1}(A)) = *g*(A) *vg*-closed in *Z*. Hence *g* is almost contra *vg*-open.

Corollary 3.10: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that *gof* is *vg*-closed then the following statements are true.
a) If *f* is almost contra-continuous [almost contra-*r*-continuous] and surjective then *g* is almost contra *vg*-open.

b) If f is almost contra-g-continuous[almost contra-rg-continuous], surjective and X is $T_{½}$ [resp: r-$T_{½}$] then g is almost contra vg-open.

Proof: (a) For A r-open in Y, f^{-1}(A) closed in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ vg-closed in Z. Hence g is almost contra vg-open.

Corollary 3.11: If f:X→Y, g:Y→Z be two mappings such that gof is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; v-; π-; r-]closed then the following statements are true.
a) If f is contra-continuous [contra-r-continuous] and surjective then g is almost contra vg-open.
b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is almost contra vg-open.

Corollary 3.12: If f:X→Y, g:Y→Z be two mappings such that gof is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; v-; π-; r-]closed then the following statements are true.
a) If f is almost contra-continuous [almost contra-r-continuous] and surjective then g is almost contra vg-open.
b) If f is almost contra-g-continuous[almost contra-rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is almost contra vg-open.

Theorem 3.18: If X is vg-regular, f: X→ Y is r-closed, nearly-continuous, almost contra vg-open surjection and $\bar{A} = A$ for every vg-closed set in Y, then Y is vg-regular.

Corollary 3.13: If X is vg-regular, f: X→ Y is r-closed, nearly-continuous, almost contra vg-open surjection and $\bar{A} = A$ for every r-closed set in Y then Y is vg-regular.

Theorem 3.19: If f:X→Y is almost contra vg-open and $A \in RO(X)$, then f_A:(X,τ(A))→(Y,σ) is almost contra vg-open.

Proof: Let F be an r-open set in A. Then F = A∩E for some r-open set E of X and so F is r-open in $X \Rightarrow$ f(A) is vg-closed in Y. But f(F) = f_A(F). Therefore f_A is almost contra vg-open.

Theorem 3.20: If f:X→Y is contra vg-open and $A \in RO(X)$, then f_A:(X,τ(A))→(Y,σ) is almost contra vg-open.

Theorem 3.21: If f:X→Y is almost contra vg-open, X is rT½ and A is rg-open set of X then f_A: (X,τ(A))→(Y,σ) is almost contra vg-open.

Proof: Let F be a r-open set in A. Then F = A∩E for some r-open set E of X and so F is r-open in $X \Rightarrow$ f(A) is vg-closed in Y. But f(F) = f_A(F). Therefore f_A is almost contra vg-open.

Theorem 3.22: If f:X→Y is contra vg-open, X is rT½ and A is rg-open set of X then f_A:(X,τ(A))→(Y,σ) is almost contra vg-open.

Corollary 3.14: If f:X→Y is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-r-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-v-; c-π-] open and $A \in RO(X)$, then f_A:(X,τ(A))→(Y,σ) is almost contra vg-open.

Corollary 3.15: If f:X→Y is al-c-g-[al-c-rg-; al-c-sg-; al-c-gs-; al-c-βg-; al-c-$r\alpha g$-; al-c-$rg\alpha$-; al-c-r-; al-c-$r\alpha$-; al-c-α-; al-c-s-; al-c-p-; al-c-β-; al-c-v-; al-c-π-] open and $A \in RO(X)$, then f_A:(X,τ(A))→(Y,σ) is almost contra vg-open.

Theorem 3.23: If f_i:X_i→Y_i be almost contra vg-open for i =1, 2. Let f: X_1xX_2→Y_1xY_2 be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then f:X_1xX_2→Y_1xY_2 is almost contra vg-open.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is r-open in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is vg-closed set in Y_1xY_2. Hence f is almost contra vg-open.

Corollary 3.16: If $f_i:X_i\rightarrow Y_i$ be contra vg-open for i =1, 2. Let f: $X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f:X_1xX_2\rightarrow Y_1xY_2$ is almost contra vg-open.

Corollary 3.17: If $f_i:X_i\rightarrow Y_i$ be al-c-g-[al-c-rg-; al-c-sg-; al-c-gs-; al-c-βg-; al-c-$r\alpha g$-; al-c-$rg\alpha$-; al-c-r-; al-c-$r\alpha$-; al-c-α-; al-c-s-; al-c-p-; al-c-β-; al-c-v-; al-c-π-]open for i =1, 2. Let $f:X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1xX_2\rightarrow Y_1xY_2$ is almost contra vg-open.

Corollary 3.18: If $f_i:X_i\rightarrow Y_i$ be c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-r-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-v-; c-π-]open for i =1, 2. Let $f:X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1xX_2\rightarrow Y_1xY_2$ is almost contra vg-open.

§4. ALMOST CONTRA *vg*-CLOSED MAPPINGS:

Definition 4.1: A function f: $X\rightarrow Y$ is said to be almost contra vg-closed if the image of every r-closed set in X is vg-open in Y.

Example 4: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f:X\rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is almost contra vg-closed, almost contra rg-closed and almost contra rgα-closed but not almost contra closed, almost contra semi-closed, almost contra pre-closed, almost contra α-closed, almost contra rα-closed, almost contra v-closed, almost contra π-closed, almost contra β-closed, almost contra g-closed, almost contra sg-closed, almost contra gs-closed, almost contra pg-closed, almost contra gp-closed and almost contra βg-closed .

Example 5: Let $X = Y = \{a, b, c, d\}$; $\tau = \{\varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let $f:X\rightarrow Y$ be defined $f(a) = b$, $f(b) = a$, $f(c) = d$ and $f(d) = c$. Then f is almost contra vg-closed but not contra vg-closed.

Theorem 4.1: Every contra vg-closed map is almost contra vg-closed map, but not conversely.

Theorem 4.2: We have the following interrelation among the following almost contra closed mappings

(i)

al.c.g.closed   al.c.gs.closed
↓   ↓
al.c.rαg.closed → al.c.rgα.closed → al.c.rg.closed → al.c.***vg*.closed** ← al.c.sg.closed ← al.c.βg.closed
↑   ↑   ↑   ↑   ↑
→ → al.c.rα.closed → → al.c.v.closed →
↑   ↓   ↑   ↑
al.c.r.closed → al.c.π.closed → al.c.closed → al.c.α.closed → al.c.s.closed → al.c.β.closed
↓   ↓   ↓
al.c.πg.closed   al.c.p.closed → al.c.ω.closed   al.c.gα.closed
↓   ↓
al.c.gp.closed ← al.c.pg.closed   al.c.rω.closed   None is reversible

(ii)

c.g.closed   c.gs.closed
↓   ↓
c.rαg.closed → c.rgα.closed → c.rg.closed → al.c.***vg*.closed** ← c.sg.closed ← c.βg.closed

↑ ↑ ↑ ↑ ↑
→ → c.rα.closed → → c.*v*.closed →
↑ ↓ ↑ ↑
c.r.closed → c.π.closed → c.closed → c.α.closed → c.s.closed → c.β.closed
None is reversible

(iii) If *v*GO(Y) = RO(Y), then the reverse relations hold for all almost contra closed maps.

al.c.g.closed al.c.gs.closed
↑↓ ↑↓
al.c.rαg.closed ↔ al.c.rgα.closed ↔ al.c.rg.closed ↔ al.c.***vg*.closed** ↔ al.c.sg.closed ↔ al.c.βg.closed
↑↓ ↑↓ ↑↓ ↑↓ ↑↓
↔ ↔ al.c.rα.closed ↔ ↔ ↔ al.c.*v*.closed ↔
↑↓ ↑↓ ↑↓ ↑↓
al.c.r.closed ↔ al.c.π.closed ↔ al.c.closed ↔ al.c.α.closed ↔ al.c.s.closed ↔ al.c.β.closed

Theorem 4.3:
(i) If (Y, σ) is discrete, then *f* is almost contra closed of all types.
(ii) If *f* is almost contra closed and *g* is *vg*-open then *gof* is almost contra *vg*-closed.
(iii) If *f* is almost closed and *g* is contra *vg*-closed then *gof* is almost contra *vg*-closed.

Corollary 4.1: If *f* is almost contra closed and *g* is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα*-; *rα*-; *α-; s-; p*-; *β-; v-;* π-; *r*-] open then *gof* is almost contra *vg*-closed.

Corollary 4.2: If *f* is almost closed[almost *r*-closed] and *g* is c-*g*-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα*-; c-*rα*-; c-*α-; c-s-; c-p*-; c-*β-;c*-π-] closed then *gof* is almost contra *vg*-closed.

Theorem 4.4: If *f:* X→ Y is almost contra *vg*-closed, then $f(A^o) \subset vg(f(A))^o$
Proof: Let A⊆ X be *r*-closed and *f:* X→ Y is almost contra *vg*-closed gives $f(A^o)$ is *vg*-open in Y and $f(A^o) \subset f(A)$ which in turn gives $vg(f(A^o))^o \subset vg(f(A))^o$ - - - (1)
Since $f(A^o)$ is *vg*-open in Y, $vg(f(A^o))^o = f(A^o)$ - - - - - - - - - - - -(2)
combining (1) and (2) we have $f(A^o) \subset vg(f(A))^o$ for every subset A of X.

Remark 4: Converse is not true in general.

Corollary 4.3: If *f:* X→ Y is al-c-*g*-[al-c-*rg-;* al-c-*sg-;* al-c-*gs-;* al-c-*βg-;* al-c-*rαg-;* al-c-*rgα*-; al-c-*r*-; al-c-*rα*-; al-c-*α-;* al-c-*s-;* al-c-*p*-; al-c-*β-;* al-c-*v-;* al-c-π-] closed, then $f(A^o) \subset vg(f(A))^o$

Theorem 4.5: If *f:X*→*Y* is almost contra *vg*-closed and A⊆*X* is *r*-closed, *f*(A) is τ_{vg}-open in *Y.*
Proof*:* Let A⊂X be *r*-closed and *f:*X→Y is almost contra *vg*-closed ⇒ $f(A^o) \subset vg(f(A))^o \Rightarrow f(A) \subset vg(f(A))^o$, since $f(A) = f(A^o)$. But $vg(f(A))^o \subset f(A)$. Combining we get $f(A) = vg(f(A))^o$. Hence *f*(A) is τ_{vg}-open in Y.

Corollary 4.4: If *f:X*→*Y* is al-c-*g*-[al-c-*rg-;* al-c-*sg-;* al-c-*gs-;* al-c-*βg-;* al-c-*rαg-;* al-c-*rgα*-; al-c-*r*-; al-c-*rα*-; al-c-*α-;* al-c-*s-;* al-c-*p*-; al-c-*β-;* al-c-*v-;* al-c-π-]closed, then *f*(A) is τ_{vg} open in *Y* if A is *r*-closed set in *X.*

Theorem 4.6: If $vg(A)^o = r(A)^o$ for every A⊂ Y, then the following are equivalent:
a) *f:* X→ Y is almost contra *vg*-closed map
b) $f(A^o) \subset vg(f(A))^o$

Proof: (a) $\Rightarrow$(b) follows from theorem 4.4.

(b) $\Rightarrow$ (a) Let A be any r-closed set in X, then $f(A) = f(A^o) \subset vg(f(A))^o$ by hypothesis. We have $f(A) \subset vg(f(A))^o$, which implies $f(A)$ is vg-open. Therefore f is almost contra vg-closed.

Theorem 4.7: If $v(A)^o = r(A)^o$ for every $A \subset Y$, then the following are equivalent:

a) $f: X \rightarrow Y$ is contra vg-closed map

b) $f(A^o) \subset vg(f(A))^o$

Proof: (a) $\Rightarrow$(b) follows from theorem 4.4.

(b) $\Rightarrow$ (a) Let A be any r-closed set in X, then $f(A) = f(A^o) \subset vg(f(A))^o$ by hypothesis. We have $f(A) \subset vg(f(A))^o$, which implies $f(A)$ is vg-open. Therefore f is almost contra vg-closed.

Theorem 4.8: $f: X \rightarrow Y$ is almost contra vg-closed iff for each subset S of Y and each $U \in RO(X, f^{-1}(S))$, there is an vg-closed set V of Y such that $S \subseteq V$ and $f^{1}(V) \subseteq U$.

Proof: Assume $f: X \rightarrow Y$ is almost contra vg-closed. Let $S \subseteq Y$ and $U \in RO(X, f^{-1}(S))$. Then X-U is r-closed in X and $f(X\text{-}U)$ is vg-open in Y as f is almost contra vg-closed and $V = Y\text{-} f(X\text{-}U)$ is vg-closed in Y. $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y\text{-} f(X\text{-}U)) = f^{-1}(Y)\text{-} f^{-1}(f(X\text{-}U)) = f^{-1}(Y)\text{-}(X\text{-}U) = X\text{-}(X\text{-}U) = U$

Conversely Let F be r-closed in $X \Rightarrow F^c$ is r-open. Then $f^{-1}(f(F^c)) \subseteq F^c$. By hypothesis there exists an vg-closed set V of Y, such that $f(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is vg-open in Y. Therefore f is almost contra vg-closed.

Remark 5: Composition of two almost contra vg-closed maps is not almost contra vg-closed in general.

Theorem 4.9: Let X, Y, Z be topological spaces and every vg-open set is r-closed in Y. Then the composition of two almost contra vg-closed maps is almost contra vg-closed.

Proof: (a) Let f and g be almost contra vg-closed maps. Let A be any r-closed set in $X \Rightarrow f(A)$ is r-closed in Y (by assumption) $\Rightarrow g(f(A)) = g \circ f(A)$ is vg-open in Z. Therefore gof is almost contra vg-closed.

Theorem 4.10: Let X, Y, Z be topological spaces and every g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; π-]open set is r-closed in Y. Then the composition of two al-c-g-[al-c-rg-; al-c-sg-; al-c-gs-; al-c-βg-; al-c-$r\alpha g$-; al-c-$rg\alpha$-; al-c-r-; al-c-$r\alpha$-; al-c-α-; al-c-s-; al-c-p-; al-c-β-; al-c-v-; al-c-π-]open maps is almost contra vg-closed.

Proof: Let A be r-closed set in X, then $f(A)$ is sg-open in Y and so r-closed in Y (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is sg-open in Z. Hence gof is almost contra vg-closed [since every sg-open set is vg-open].

Corollary 4.5: Let X, Y, Z be topological spaces and every g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; π-]open set is closed [r-closed] in Y. Then the composition of two c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-v-; c-π-; c-r-]closed maps is almost contra vg-closed.

Example 6: Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$ and $\eta = \{\varphi, \{a\}, \{b, c\}, Z\}$. $f: X \rightarrow Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$ and $g: Y \rightarrow Z$ be defined $g(a) = b$, $g(b) = a$ and $g(c) = c$, then g, f and $g \circ f$ are almost contra vg-closed.

Theorem 4.11: If $f: X \rightarrow Y$ is almost contra g-closed[almost contra rg-closed], $g: Y \rightarrow Z$ is vg-open and Y is $T_{½}$ [r-$T_{½}$] then gof is almost contra vg-closed.

Proof: (a) Let A be *r*-closed in *X*. Then *f*(A) is *g*-open and so open in *Y* as *Y* is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is *vg*-open in *Z* (since *g* is *vg*-open). Hence *gof* is almost contra *vg*-closed.

Corollary 4.6: If $f:X\rightarrow Y$ is almost contra *g*-closed[almost contra *rg*-closed], $g:Y\rightarrow Z$ is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα*-; *rα*-; *α-; s-; p*-; *β-; v-;* π-; *r*-]open and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is almost contra *vg*-open.

Theorem 4.12: If $f:X\rightarrow Y$ is almost g-closed[almost rg-closed], $g:Y\rightarrow Z$ is contra *vg*-closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is almost contra *vg*-closed.
Proof: (a) Let A be *r*-closed in *X*. Then *f*(A) is *g*-closed and so closed in *Y* as *Y* is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is *vg*-open in *Z* (since *g* is contra *vg*-closed). Hence *gof* is almost contra *vg*-closed.

Theorem 4.13: If $f:X\rightarrow Y$ is almost g-closed[almost rg-closed], $g:Y\rightarrow Z$ is c-*g*-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα*-; c-*rα*-; c-*α-; c-s-; c-p*-; c-*β-; c*-π-]closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is almost contra *vg*-closed.

Theorem 4.14: If $f:X\rightarrow Y$ is *g*-closed[*rg*-closed], $g:Y\rightarrow Z$ is c-*g*-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα*-; c-*rα*-; c-*α-; c-s-; c-p*-; c-*β-; c*-π-]closed and *Y* is $T_{½}$[*r*-$T_{½}$], then *gof* is almost contra *vg*-closed.
Proof: Let A be *r*-closed set in X, then *f*(A) is *g*-open in Y and so open in *Y* (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is *gs*-open in *Z*. Hence *gof* is almost contra *vg*-closed [since every *gs*-open set is *vg*-open].

Theorem 4.15: If $f:X\rightarrow Y$ is c-*g*-closed[c-*rg*-closed], $g:Y\rightarrow Z$ is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα*-; *rα*-; *α-; s-; p*-; *β-;* π-]open and *Y* is $T_{½}$[*r*-$T_{½}$], then *gof* is almost contra *vg*-closed.
Proof: Let A be *r*-closed set in X, then *f*(A) is *g*-open in Y and so open in *Y* (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is *gs*-open in *Z*. Hence *gof* is almost contra *vg*-closed [since every *gs*-open set is *vg*-open].

Theorem 4.16: If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that *gof* is contra *vg*-closed then the following statements are true.
a) If *f* is continuous [*r*-continuous] and surjective then *g* is almost contra *vg*-closed.
b) If *f* is g-continuous[resp: rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is almost contra *vg*-closed.
Proof: (a) For A *r*-closed in *Y*, $f^{1}(A)$ closed in $X \Rightarrow (g\ o\ f)(f^{1}(A)) = g(A)$ *vg*-open in *Z*. Hence *g* is almost contra *vg*-closed.

Similarly one can prove the remaining parts and hence omitted.

Corollary 4.7: If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that *gof* is contra *vg*-closed then the following statements are true.
a) If *f* is almost continuous [almost *r*-continuous] and surjective then *g* is almost contra *vg*-closed.
b) If *f* is almost g-continuous[resp: almost rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is almost contra *vg*-closed.

Corollary 4.8: If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that *gof* is g-[*rg-; sg-; gs-; βg-; rαg-; rgα*-; *rα*-; *α-; s-; p*-; *β-; v-;* π-; *r*-] closed then the following statements are true.
a) If *f* is continuous [*r*-continuous] and surjective then *g* is almost contra *vg*-closed.
b) If *f* is g-continuous[rg-continuous], surjective and *X* is $T_{½}$ [*r*-$T_{½}$] then *g* is almost contra *vg*-closed.

Corollary 4.9: If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that *gof* is c-g-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα*-; c-*rα*-; c-*α-; c-s-; c-p*-; c-*β-; c-v-; c*-π-; c-*r*-]closed then the following statements are true.
a) If *f* is continuous [*r*-continuous] and surjective then *g* is almost contra *vg*-closed.
b) If *f* is g-continuous[rg-continuous], surjective and *X* is $T_{½}$ [*r*-$T_{½}$] then *g* is almost contra *vg*-closed.

Theorem 4.17: If f:X→Y, g:Y→Z be two mappings such that *gof* is *vg*-open then the following statements are true.
a) If f is contra-continuous [contra-*r*-continuous] and surjective then g is almost contra *vg*-closed.
b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{½}$ [resp: r-$T_{½}$] then g is almost contra *vg*-closed.
Proof: (a) For A *r*-closed in Y, f^{-1}(A) open in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ *vg*-open in Z. Hence g is almost contra *vg*-closed.

Corollary 4.10: If f:X→Y, g:Y→Z be two mappings such that *gof* is *vg*-open then the following statements are true.
a) If f is almost contra-continuous [almost contra-*r*-continuous] and surjective then g is almost contra *vg*-closed.
b) If f is almost contra-g-continuous[almost contra-rg-continuous], surjective and X is $T_{½}$ [resp: r-$T_{½}$] then g is almost contra *vg*-closed.

Corollary 4.11: If f:X→Y, g:Y→Z be two mappings such that *gof* is g-[*rg*-; *sg*-; *gs*-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s*-; *p*-; β-; ν-; π-; *r*-]open then the following statements are true.
a) If f is contra-continuous [contra-*r*-continuous] and surjective then g is almost contra *vg*-closed.
b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is almost contra *vg*-closed.

Corollary 4.12: If f:X→Y, g:Y→Z be two mappings such that *gof* is g-[*rg*-; *sg*-; *gs*-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; *s*-; *p*-; β-; ν-; π-; *r*-]open then the following statements are true.
a) If f is almost contra-continuous [almost contra-*r*-continuous] and surjective then g is almost contra *vg*-closed.
b) If f is almost contra-g-continuous[almost contra-rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is almost contra *vg*-closed.

Theorem 4.18: If X is *vg*-regular, f:X→Y is *r*-open, *r*-continuous, almost contra *vg*-closed surjective and $A^o = A$ for every *vg*-open set in Y then Y is *vg*-regular.

Corollary 4.13: If X is *vg*-regular, f:X→Y is *r*-open, *r*-continuous, almost contra *vg*-closed, surjective and $A^o = A$ for every *r*-closed set in Y then Y is *vg*-regular.

Theorem 4.19: If f:X→Y is almost contra *vg*-closed and $A \in RC(X)$, then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost contra *vg*-closed.
Proof: Let F be an *r*-closed set in A. Then F = A∩E for some *r*-closed set E of X and so F is *r*-closed in X $\Rightarrow f(A)$ is *vg*-open in Y. But $f(F) = f_A(F)$. Therefore f_A is almost contra *vg*-closed.

Theorem 4.20: If f:X→Y is almost contra *vg*-closed, X is r$T_{½}$ and A is rg-closed set of X then f_A: $(X,\tau(A))\rightarrow(Y,\sigma)$ is almost contra *vg*-closed.
Proof: Let F be a *r*-closed set in A. Then F = A∩E for some *r*-closed set E of X and so F is *r*-closed in X $\Rightarrow f(A)$ is *vg*-open in Y. But $f(F) = f_A(F)$. Therefore f_A is almost contra *vg*-closed.

Corollary 4.14: If f:X→Y is contra *vg*-closed and,
(i) $A \in RC(X)$, then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost contra *vg*-closed.
(ii) X is r$T_{½}$ and A is rg-closed set of X then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost contra *vg*-closed.

Corollary 4.15: If $f:X\to Y$ is c-*g*-[c-*rg*-; c-*sg*-; c-*gs*-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-*r*-; c-$r\alpha$-; c-α-; c-*s*-; c-*p*-; c-β-; c-*v*-; c-π-] open and $A\in RC(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is almost contra *vg*-open.

Corollary 4.16: If $f:X\to Y$ is al-c-*g*-[al-c-*rg*-; al-c-*sg*-; al-c-*gs*-; al-c-βg-; al-c-$r\alpha g$-; al-c-$rg\alpha$-; al-c-*r*-; al-c-$r\alpha$-; al-c-α-; al-c-*s*-; al-c-*p*-; al-c-β-; al-c-*v*-; al-c-π-] closed and $A\in RC(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is almost contra *vg*-closed.

Theorem 4.21: If $f_i:X_i\to Y_i$ be almost contra *vg*-closed for i =1, 2. Let $f: X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f:X_1 x X_2\to Y_1 x Y_2$ is almost contra *vg*-closed.
Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is *r*-closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)\ x\ f_2(U_2)$ is *vg*-open set in $Y_1 x Y_2$. Hence *f* is almost contra *vg*-closed.

Corollary 4.17: If $f_i:X_i\to Y_i$ be contra *vg*-closed for i =1, 2. Let $f: X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f:X_1 x X_2\to Y_1 x Y_2$ is almost contra *vg*-closed.
Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is *r*-closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)\ x\ f_2(U_2)$ is *vg*-open set in $Y_1 x Y_2$. Hence *f* is almost contra *vg*-closed.

Corollary 4.18: If $f_i:X_i\to Y_i$ be al-c-*g*-[al-c-*rg*-; al-c-*sg*-; al-c-*gs*-; al-c-βg-; al-c-$r\alpha g$-; al-c-$rg\alpha$-; al-c-*r*-; al-c-$r\alpha$-; al-c-α-; al-c-*s*-; al-c-*p*-; al-c-β-; al-c-*v*-; al-c-π-]closed for i =1, 2.
Let $f:X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2\to Y_1 x Y_2$ is almost contra *vg*-closed.

Corollary 4.19: If $f_i:X_i\to Y_i$ be c-*g*-[c-*rg*-; c-*sg*-; c-*gs*-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-*r*-; c-$r\alpha$-; c-α-; c-*s*-; c-*p*-; c-β-; c-*v*-; c-π-]closed for i =1, 2. Let $f:X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2\to Y_1 x Y_2$ is almost contra *vg*-closed.

CONCLUSION:

In this paper the author introduced the concepts of almost contra *vg*-open mappings, almost contra *vg*-closed mappings, studied their basic properties and interrelationship between other such contra open and contra closed maps.

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