# Suggested Technique for Predicting Critical Inductor values of Controlled DC-DC Converters in Continuous and Discontinuous Conduction Mode Based on Constrain Simulation

**Abdulkhaleq A. AL-Naqeeb#1, Munaf Fathi Badr#2 , Alaa Majid Hamad#3**
#1 Prof. Dr., College of Health of Medical Technology, Baghdad, Iraq, (abdulkhaliq.alnaqeeb@yahoo.com)
#2 Assistant Prof., Al-Mustansiriyah University, College of Engineering, Mechanical Engineering Department, Baghdad, Iraq, (munaf_67@yahoo.com)
#3Lecturer, University of Baghdad, Ibn Al-Haitham College of Education, Department of Mathematics, Baghdad, Iraq, (alaa_073@yahoo.com)

## ABSTRACT

This paper is confined on the analysis of one member of the DC–DC converters family. The study is concerning on the mathematical model of the well-known step down buck converter loaded by resistive load to predict the critical value of the inductance. A controlled scheme of the switch mode converter under voltage mode control is illustrated and statistical model based on power distribution was proposed to verify the operation mode of the converter in either continuous conduction mode (CCM) or discontinuous conduction mode (DCM). A constrain simulated buck model was implemented and the statistical simulation process is performed to obtain very precise statistical models. The proposed statistical strategy is employed for predicting critical inductor values with minimizes white noise (luck of fit and pure error) which are more valid and reliable than inductance of the coil that estimating by conventional equation.

**Keywords:** Buck Converter, Critical Inductance, Mathematical Model, Constrain Simulation.

**Corresponding Author:** Munaf Fathi Badr

## INTRODUCTION

During the last few decades, switch mode DC-DC converters had been widely used in power electronic circuits to convert one level of electrical voltage to another level by switching action. Switching-mode regulators can be separated into three categories pulse-width modulated (PWM) regulators, resonant converters, and switched-capacitor voltage regulators. The (PWM) DC-DC regulators can be divided into three important topologies buck converter, boost converter, and buck-boost converter as shown in figure (1).

The first type (buck converter) decreases the output voltage while the second one increases it and the third one is a combination of them. They are used a lot to build power sources of electronic equipment, due to their great efficiency and reduced size, and also for electrical motor control.

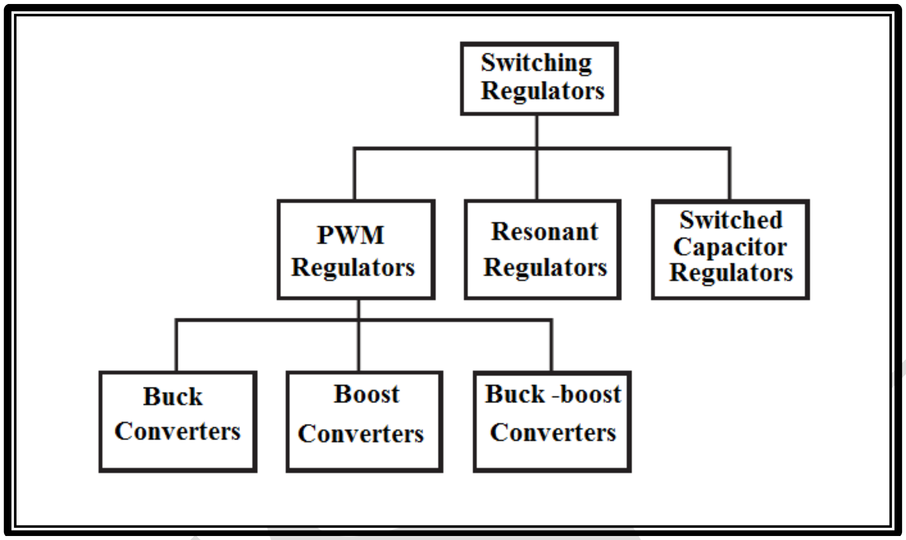


Fig 1: The Classification of the Switching Mode DC-DC Converters.

In this work, the buck converter is chosen for analysis. It is one of the simplest but most useful a controlled step-down converters that converts an unregulated DC input voltage to a regulated DC output at a lower voltage

The buck converter basic configuration consists of power MOSFET switch, diode, inductor and capacitor. The wide variety of circuit topologies ranges from the single transistor buck converter to complex configuration comprising two or four devices and employing soft-switching or resonant techniques to control the switching losses. On the other word, the statistical modeling is an approach to realize the effects of variations on the performance of converter circuits in terms of process parameters, similar methods of analysis and control are applied to many of these configurations [1-6].

The purpose of this paper is to present an idle buck converter model and to create a statistical model associated with mathematical analysis to predict that the values of critical inductance in the buck converter circuit in order to discriminating the operation of the converter either in (CCM) or (DCM)in relative of output power and input voltages. A simulation process has been carried out and the statistical analysis has been done to observe and to investigate the effect of the critical inductance value on the operation of the converter.

The mathematical analyses of the buck converter are presented in Section II. A suggested statistical approach involving the obtained data is applied in section III. Simulation model involved results are presented in section VI. Finally conclusions and future works are discussed in section V.

## Converter Configuration and Operation

The idle buck DC-DC converter configuration comprises of power MOSFET switch(*S*), diode (D), inductor (*L*) and capacitor (*C*) as shown in figure (2). The coil nonlinearities and parasitic capacitors at each of the switching instants are neglected. Theswitch is assumed as ideal andthis converter gives an output voltage ($V_o$ )smaller than the input voltage ($V_{in}$).

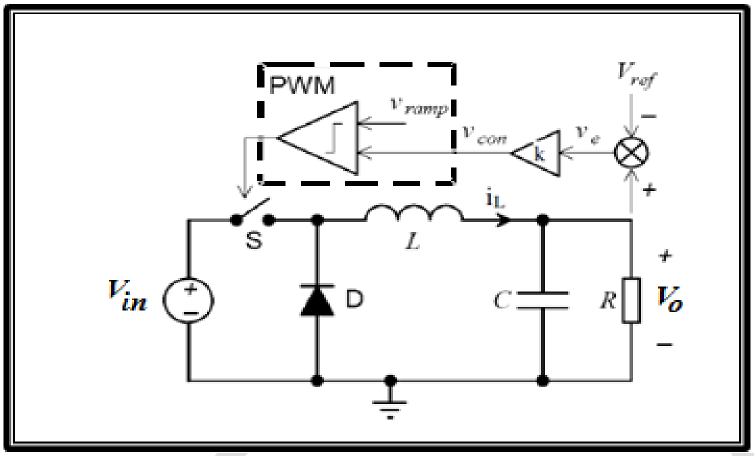


Fig 2: The Buck Converter under Voltage Mode Control.

Under voltage mode control, theswitch (*S*) is controlled by pulse-width modulatedsignal; if the switch (*S*) is closedthe diode is on inverse polarization andcan be eliminated for analysis. The resulting electrical system is described by the pair of differential equations as following [1-3]: -

$$\frac{di_l(t)}{dt} = -\frac{1}{L}v_c(t) + \frac{1}{L}v_{in}(t) \tag{1}$$

$$\frac{dv_o(t)}{dt} = \frac{1}{C}i_l(t) + \frac{1}{RC}v_o(t) \tag{2}$$

When the switch (*S*) was opening, the diode can be replaced by a conductor and deleted the sourceof voltage ($V_{in}$). The resulting circuit, with two meshes, isdescribed by the equations (3) and (4).

$$\frac{di_l(t)}{dt} = -\frac{1}{L}v_o(t) \tag{3}$$

$$\frac{dv_o(t)}{dt} = \frac{1}{C}i_l(t) - \frac{1}{RC}v_o(t) \tag{4}$$

Where

$V_{in}$ and $V_o$ are the input and output voltages respectively.
$i_l$ is the inductor current.
*L* is the inductance of the coil.
C is the output capacitance.

The buck converters operates either in continuous or discontinuous mode depending on the circuit components which includes a inductor and a capacitor along with the load for which smooth acceleration control, high efficiency and faster dynamic response is required.

In some applications, it is required that the DC-DC converter operates in CCM, while for some other applications is needed to operate in DCM, for discontinuous conduction mode:

$$\frac{di_l(t)}{dt} = 0 \quad (5)$$

$$\frac{dv_o(t)}{dt} = -\frac{1}{RC} v_o(t) \quad (6)$$

To ensure the continuous current mode of conduction, the selected value of inductance should be greater than the critical value of the inductance ($L_{critical}$) which acts as a boundary condition for the continuous and discontinuous current modes of the operations. The critical value of inductance is the smallest inductor value for which the inductor current is greater than zero at all times and under all allowed operating conditions of the converter and it's given by [3-5]:-

$$L_{critical} = \frac{(1-D)}{2f_s} R \quad (7)$$

The output voltage of the buck converter is always less than the input voltage and is given by:

$$V_o = DV_{in} \quad (8)$$

And

$$D = \frac{T_{on}}{T_s} \quad (9)$$

Re-arranging the terms in equation (7), this leads to

$$L_{critical} = \frac{(1 - \frac{V_o}{V_{in}})(V_o)^2 T_S}{2P_O} \quad (10)$$

$$L_{critical} = \frac{(V_{in} - V_o)(V_o)^2 T_S}{2P_o V_{in}} \quad (11)$$

Where

$f_s$ is the switching frequencyand ($f_s=1/T_s$).
$D$ isthe duty cycle ratio.
$T_s$ is the switching time period
$T_{on}$ is the (*ON*)time of the semiconductor switch.
$P_o$ is the output power.

Hence in order for the CCM condition to occur

$$L \geq L_{critical} \quad (12)$$

And for the DCM condition to occur

$$L < L_{critical} \quad (13)$$

From equation (11), the highest ($L_{critical}$) value occurs when ($P_o$) is at its minimum value and ($V_{in}$) is at its maximum value, which is considered to be the worst case. While the lowest

($L_{critical}$) value occurs when ($P_o$) is at its maximum value and ($V_{in}$) is at its minimum value, which is considered to be the worst case. For summarizes preceding conditions, worst cases of $L_{critical}$ occurs could be written as in table (1).

Table (1): The lowest critical inductor values ($L_{critical}$) occurs in CCM & DCM Modes.

| $L_{critical}$ | | $P_o$ | $V_{in}$ |
|---|---|---|---|
| Mode | CCM | Max. | Min. |
| | DCM | Min. | Max. |

## SUGGESTED TECHNIQUE (*)

The predicting critical value ($\hat{L}_{critical}$) of inductor in the buck converter circuit had been done, based on the suggested technique(*), taking into account the data errors. This aspectis considered with thefollowing procedures:-

i. Equation (11) could be written equivalent to Power Distribution Function (*PDF*) as following:-

$$\hat{L}_{critical} = b_0 * \left(T_s^{\,b_1}\right) \quad (14)$$

And it could be written by:

$$\text{Ln}\left(\hat{L}_{critical}\right) = \text{Ln}\,(b_0) + \left(b_1 * \text{Ln}\,(T_s)\right) \quad (15)$$

Where

$$b_0 = \frac{(V_{in} - V_o)(V_o)^2}{2P_o V_{in}}, b_1 = 1$$

Scatter diagram of simulated ($\hat{L}_{critical}$) included residuals term ($e_i$), then (15) should be written by:

$$\text{Ln}\,(L_{critical}) = \text{Ln}\,(\beta_o) + \left(\beta_1 * \text{Ln}\,(T_s)\right) + e_i \quad (16)$$

ii. Equation ( 16) is a simple linear regression equation with[$e\sim N(0,\sigma^2)$]of switching time periods in natural logarithm transmitted as independent variable and inductance of the coil in natural logarithm transmitted as dependent variable, as well as intercept of equation belong to natural logarithm of a constant factors in relative to each switchingtime period, and voltages input.

iii. For each measured of switching time periods interpolated, generated a white noise values (Random errors) regarding to their mean values, and standard deviation estimates in front of each output power.

iv. For different output powers, (Random errors) are added to equation (16) in order to obtain natural logarithm for the inductance of the coil as in equation (17).

$$\text{Ln}\,(L_{critical}) = \text{Ln}\left(\hat{L}_{critical}\right) + e_i \quad (17)$$

(*)Cogitation and implementation by the author.

v. Finally taken exponential of both sided for estimating inductance of the coil, which is more valid and reliable than inductance of the coil estimating by equation (11).
Now equation (16) could be written in term of $b_1$:

$$b_1 = \frac{\left(\mathrm{Ln}\,(L_{\,critical}\,) - \mathrm{Ln}\,(\beta o)\right) - e_{\mathrm{i}}}{\mathrm{Ln}\,(T_s)} \qquad (18)$$

## SIMULATION & RESULTS [(*)]

The various parameters of the buck converter which had been employed using equation (11) and to establish a general scheme of measurement to obtain the desired data through the measured variables are listed as shown in table (2).

Table (2) the Various Parameters of the Buck Converter.

| Output Power ($P_o$)&Current ($I_o$) | $V_{in}$ (V) | $V_o$ (V) | $T_s$ (sec) | $f_s$ (kHz) | $D=Vo/Vin$ |
|---|---|---|---|---|---|
| $I_o$=0.6A , $P_o$= 3W | 8 | 5 | $3.33\times10^{-6}$ | 300 | 0.6250 |
| | 48 | 5 | $6.66\times10^{-6}$ | 150 | 0.1042 |
| | 60 | 5 | $9.99\times10^{-6}$ | 100 | 0.0833 |
| $I_o$=1A , $P_o$ = 5W | 8 | 5 | $3.33\times10^{-6}$ | 300 | 0.6250 |
| | 48 | 5 | $6.66\times10^{-6}$ | 150 | 0.1042 |
| | 60 | 5 | $9.99\times10^{-6}$ | 100 | 0.0833 |
| $I_o$=1.2A , $P_o$= 6W | 8 | 5 | $3.33\times10^{-6}$ | 300 | 0.6250 |
| | 48 | 5 | $6.66\times10^{-6}$ | 150 | 0.1042 |
| | 60 | 5 | $9.99\times10^{-6}$ | 100 | 0.0833 |
| $I_o$=1.4A , $P_o$ = 7W | 8 | 5 | $3.33\times10^{-6}$ | 300 | 0.6250 |
| | 48 | 5 | $6.66\times10^{-6}$ | 150 | 0.1042 |
| | 60 | 5 | $9.99\times10^{-6}$ | 100 | 0.0833 |
| $I_o$=1.6A , $P_o$ = 8W | 8 | 5 | $3.33\times10^{-6}$ | 300 | 0.6250 |
| | 48 | 5 | $6.66\times10^{-6}$ | 150 | 0.1042 |
| | 60 | 5 | $9.99\times10^{-6}$ | 100 | 0.0833 |

[(*)]All statistical calculations have been accomplished via applying the following statistical software packages:-[Statgraphic (Ver.14), SPSS (Ver.10) & (NCSS and PASS) copyright 2007].

On application of three transmitted switching time period (3.33µs, 6.66µs and 9.99µs) on each output power ($P_o$=3W, 5W, 6W, 7W and 8W) with contrasts of input voltages ($V_{in}$=8V, 48V and 60V) and constant output voltage ($V_o$=5V), interpolated of three decimals for switching time periods formed along integers numbers [0.001 – 2.999] /or (0 – 3) through multiplicative by preceding switching time period.Table (3)represents mean and standard deviation values of simulated random errors for different output powers.

Table (3): Mean and Standard Deviation Values (Std. dev.) of Simulated Random Errors for Different Output Powers.

| Distribution: Normal | Parameters | |
|---|---|---|
| | Mean | Std. dev. |
| $e \sim N(\mu, \sigma^2)$ | 0.000020237 | 0.000009309 |
| | 0.000012142 | 0.000005586 |
| | 0.000010118 | 0.000004655 |
| | 0.000008671 | 0.000003976 |
| | 0.000008671 | 0.000003491 |
| $e \sim N(0, \sigma^2)$ | 0.000000000 | 0.000009309 |
| | 0.000000000 | 0.000005586 |
| | 0.000000000 | 0.000005586 |
| | 0.000000000 | 0.000003976 |
| | 0.000000000 | 0.000003491 |

Figures (3.1) and (3.2)represent curves frequency plot of simulated random errors which indicating that random errors are shifted to the left side with increasing output powers, as well as figure (3.2) represents curves frequency plot of simulated random errors with [$e \sim N(\mu, \sigma^2)$].Table (4) represents mean square error (MSE) indicator for different input voltages.Then mean values in relative of output power and input voltages estimates beta parameters (the slopes) of the predicted equation (18), and as illustrated in table (5).

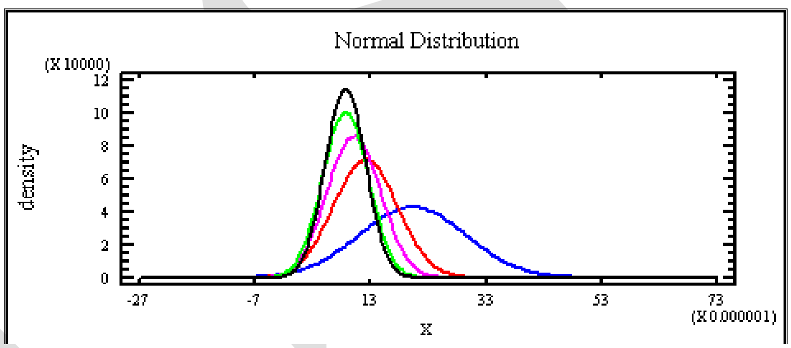


Fig (3.1): Represents Curves Frequency Plot of Constrain Simulated Random Errors with$e \sim N(\mu, \sigma^2)$, for Different Output Powers.

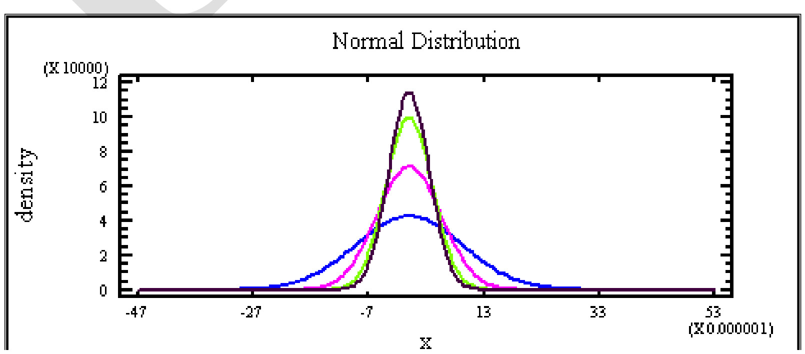


Figure (3.2): Represents Curves Frequency Plot of Constrain Simulated Random Errors with [$e \sim N(0, \sigma^2)$], for Different Output Powers.

Table (4): Mean Square Error (MSE) Indicator for Different Input Voltages.

| Input Voltages (V) | Simulated random errors with [e ~N(μ,$\sigma^2$)] | Simulated random errors with [e ~N(0,$\sigma^2$)] |
|---|---|---|
| 8 | 0.00000723034636 | 0.00000602001284 |
| 48 | 0.00000725967881 | 0.00000586867219 |
| 60 | 0.00000710526102 | 0.00000583984201 |

Table (5): Estimates Beta Parameters (the slopes) of Predicted Equation of Lowest Critical Inductor Values ($L_{critical}$) Occurs in (CCM & DCM) Modes in Relative of Output Power.

| Output Power ( $P_o$ ) &Current ($I_o$) | Input Voltages (V) | Beta Parameter (The Slope) |
|---|---|---|
| $I_o$=0.6A , $P_o$ = 3W | 8 | 1.32375068914543 |
| | 48 | 1.47361798607196 |
| | 60 | 1.49230270218890 |
| $I_o$ =1A , $P_o$ = 5W | 8 | 1.36647137134933 |
| | 48 | 1.51634059068965 |
| | 60 | 1.53502723077961 |
| $I_o$ =1.2A , $P_o$ = 6W | 8 | 1.38171906184408 |
| | 48 | 1.53158894428785 |
| | 60 | 1.55027625478261 |
| $I_o$ =1.4A , $P_o$ = 7W | 8 | 1.39461072970015 |
| | 48 | 1.54448118766117 |
| | 60 | 1.56316915689655 |
| $I_o$ =1.6A , $P_o$ = 8W | 8 | 1.40577804104948 |
| | 48 | 1.55564902667166 |
| | 60 | 1.57433744458771 |

Finally the Table (6)shows inductance of the coil [$\hat{L}_{critical}$ (⍰H) ] estimating by equation (14) which is more valid and reliable than inductance of the coil [$L_{critical}$(μH)] estimating by equation (11).

Table (6): Predicted lowest critical inductor values ($\hat{L}_{critical}$) occurs in CCM & DCM Modes in relative output power.

| Output Power( $P_o$ ) &Current ($I_o$) | Input Voltages (V) | $L_{critical}$ (H) | $\hat{L}_{critical}$ (H) |
|---|---|---|---|
| $I_o$=0.6A,$P_o$= 3W | 8 | $5.21\times10^{-6}$ | $8.77\times10^{-8}$ |
| | 48 | $2.49\times10^{-5}$ | $2.62\times10^{-6}$ |
| | 60 | $3.82\times10^{-5}$ | $1.32\times10^{-7}$ |
| $I_o$=1A,$P_o$ = 5W | 8 | $3.12\times10^{-6}$ | $3.07\times10^{-8}$ |
| | 48 | $1.49\times10^{-5}$ | $1.04\times10^{-6}$ |
| | 60 | $2.29\times10^{-5}$ | $4.84\times10^{-8}$ |
| $I_o$=1.2A,$P_o$= 6W | 8 | $2.60\times10^{-6}$ | $2.11\times10^{-8}$ |
| | 48 | $1.24\times10^{-5}$ | $7.5010^{-7}$ |
| | 60 | $1.91\times10^{-5}$ | $3.39\times10^{-8}$ |
| $I_o$=1.4A,$P_o$ = 7W | 8 | $2.23\times10^{-6}$ | $1.54\times10^{-8}$ |
| | 48 | $1.07\times10^{-5}$ | $5.6810^{-7}$ |
| | 60 | $1.64\times10^{-5}$ | $2.50\times10^{-8}$ |
| $I_o$=1.6A,$P_o$ = 8W | 8 | $1.95\times10^{-6}$ | $1.15\times10^{-8}$ |
| | 48 | $9.33\times10^{-6}$ | $4.4610^{-7}$ |
| | 60 | $1.43\times10^{-5}$ | $1.92\times10^{-8}$ |

## CONCLUSION

- In this work, an attempt to predict the critical inductance values of the coil in the step down dc-dc voltage (buck) converter which acts as a boundary condition for the continuous and discontinuous current operation modes have been carried out. So the designer can select appropriate values of the inductor element (coil) and discriminate the operation of the converter.
- The suggested statistical approach involving different parameters of the buck converter and it can be applied to various dc–dc converters topologies (e.g. buck, boost and buck–boost).
- Inductance of the coil that has been estimated by power distribution function(*PDF*) [$\hat{L}_{critical}$ (H)] are more valid and reliable than inductance of the coil [$L_{critical}$(H)], which were calculating by monotonic conventional function, since (*PDF*) had considered that inductance of the coil is univariate random variable, having normal distribution function, as well as white noise in that function interpolated lag information, such as pure error and luck of fit.Therefore the proposed statistical design is a very powerful tool for obtaining information and highly reliable results.

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