CONTRA αg-OPEN AND ALMOST CONTRA αg-OPEN MAPPINGS

S. Balasubramanian[1] and Ch. Chaitanya[2]
[1]Department of Mathematics, Government Arts College(A), Karur – 639 005, Tamilnadu
E.mail: mani55682@rediffmail.com
[2] Department of Mathematics, Krishnaveni Engineering College for Women, Narasaraopet – 522 601, Andhrapradesh
E.mail: chennupatichaitanya2@gmail.com

Abstract: The aim of this paper is to introduce and study the concepts of contra αg-open and almost contra αg-open mappings and the interrelationship between other contra-open maps.

Keywords: αg-open set, αg-open map, αg-closed map, contra-closed map, contra α-open map, contra α-closed map, contra αg-open map and almost contra αg-open map.
AMS Classification: 54C10, 54C08, 54C05

§1. INTRODUCTION:

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Open mappings are one such which are studied for different types of open sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1969, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced α-open and α-closed mappings in the year in 1982, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-closed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced β-open mappings in the year 1983. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. During the years 2010 to 2014, S. Balasubramanian together with his research scholars defined and studied a variety of open, closed, almost open and almost closed mappings for *v*-open, rp-open *gpr*-closed and *vg*–closed sets as well contra-open and contra-closed mappings for semi-open, pre-open, rp-open, β-open and gpr-closed sets. Inspired with these concepts and its interesting properties the authors of this paper tried to study a new variety of open maps called contra αg-open and almost contra αg-open maps. Throughout the paper X, Y means topological spaces (X, τ) and (Y,σ) on which no separation axioms are assured.

§2. Preliminaries:

Definition 2.1: $A\subseteq X$ is said to be

a) regular open[α-open] if $A = int(cl(A))$ [$A\subseteq int(cl(int(A)))$] and regular closed[$\alpha$-closed] if $A = cl(int(A))$[$cl(int(cl(A)))\subseteq A$]

b) g-closed[rg-closed, αg-closed] if $cl(A)\subset U$[$rcl(A)\subset U$, $\alpha\text{-}cl(A)\subset U$] whenever $A\subset U$ and U is open[r-open, α-open] in X and g-open[rg-open, αg-open] if its complement X- A is g-closed[rg-closed, αg-closed].

Remark 1: We have the following implication diagrams for closed sets.

(i)
$$\begin{array}{ccccccc} r\alpha.\text{closed} & & & & & & g.\ \text{closed} \\ \uparrow & & & & & & \downarrow \\ r.\ \text{closed} & \to & \text{closed} & \to & \alpha.\text{closed} & \to & \alpha g.\ \text{closed} \end{array}$$
None is reversible

The same relation is true for open sets also.

Definition 2.2: A function $f{:}X\rightarrow Y$ is said to be

a) continuous[resp: r-continuous, α-continuous] if the inverse image of every open set is open [resp: r-open, α-open].
b) r-irresolute [resp: α-irresolute] if the inverse image of every r-open [resp: α-open] set is r-open [resp: α-open].
c) closed[resp: r-closed, α-closed] if the image of every closed set is closed [resp: r-closed, α-closed].
d) g-continuous [resp: rg-continuous, αg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed, αg-closed].

Definition 2.3: A function $f{:}\ X\rightarrow Y$ is said to be

a) contra closed[resp: contra α-closed; contra rα-closed; contra r-closed; contra g-closed] if the image of every closed set in X is open[resp: α-open; rα-open; r-open; g-open] in Y.
b) contra open[resp: contra α-open; contra rα-open; contra r-open; contra g-open] if the image of every open set in X is closed[resp: α-closed; rα-closed; r-closed; g-closed] in Y.
c) almost contra closed[resp: almost contra α-closed; almost contra rα-closed; almost contra r-closed; almost contra g-closed] if the image of every closed set in X is open[resp: α-open; rα-open; r-open; g-open] in Y.
d) almost contra open[resp: almost contra α-open; almost contra rα-open; almost contra r-open; almost contra g-open] if the image of every open set in X is closed[resp: α-closed; rα-closed; r-closed; g-closed] in Y.

Definition 2.4: X is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

§3. CONTRA αg-OPEN MAPPINGS:

Definition 3.1: A function $f{:}\ X\rightarrow Y$ is said to be contra αg-open if the image of every open set in X is αg-closed in Y.

Theorem 3.1: We have the following interrelation among the following contra open mappings

(i)

$$\begin{array}{ccccccc} \text{c.r}\alpha\text{.open} & & & & & & \text{c.g.open} \\ \uparrow & & & & & & \downarrow \\ \text{c.r.open} & \rightarrow & \text{c.open} & \rightarrow & \text{c.}\alpha\text{.open} & \rightarrow & \text{c.}\alpha\text{g.open} \end{array}$$

None is reversible

(ii) If αGC(Y) = RC(Y), then the reverse relations hold for all almost contra open maps.

$$\begin{array}{ccccccc} \text{c.r}\alpha\text{.open} & & & & & & \text{c.g.open} \\ \uparrow\downarrow & & & & & & \uparrow\downarrow \\ \text{c.r.open} & \leftrightarrow & \text{c.open} & \leftrightarrow & \text{c.}\alpha\text{.open} & \leftrightarrow & \text{c.}\alpha\text{g.open} \end{array}$$

Example 1: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\} = \sigma$. Let $f{:}X\rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is contra αg –open.

Example 2: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f{:}X\rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is not contra αg-open, contra open, contra α-open, contra rα-open and contra g-open.

Theorem 3.2:

(i) If(Y, σ) is discrete, then f is contra open of all types.

(ii) If f is contra open and g is αg-closed then gof is contra αg-open.
(iii) If f is open and g is contra αg-open then gof is contra αg-open.

Corollary 3.1: If f is contra open and g is [r-; α-; $r\alpha$-; g-] closed then gof is contra αg-open.

Corollary 3.2: If f is open[r-open] and g is c-[c-r-; c-α-; c-$r\alpha$-] open then gof is contra αg-open.

Theorem 3.3: If f: X→ Y is contra αg-open, then $\alpha g(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$
Proof: Let $A \subset X$ be open and f: X→ Y is contra αg-open gives $f(\mathrm{cl}\{A\})$ is αg-closed in Y and $f(A) \subset f(\mathrm{cl}(A))$ which in turn gives $\alpha g(\mathrm{cl}(f(A))) \subset \alpha g\mathrm{cl}(f(\mathrm{cl}(A)))$------ (1)
Since $f(\mathrm{cl}(A))$ is αg-closed in Y, $\alpha g\mathrm{cl}(f(\mathrm{cl}(A))) = f(\mathrm{cl}(A))$------ (2)
From(1) and (2) we have $\alpha g(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$ for every subset A of X.

Remark 2: Converse is not true in general.

Corollary 3.3: If f: X→ Y is c-[c-r-; c-α-; c-$r\alpha$-] open, then $\alpha g(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$

Theorem 3.4: If $f: X \to Y$ is contra αg-open and $A \subseteq X$ is open, $f(A)$ is $\tau_{\alpha g}$-closed in Y.
Proof: Let $A \subset X$ be open and f: X→ Y is contra αg-open implies $\alpha g(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$ which in turn implies $\alpha g(\mathrm{cl}(f(A))) \subset f(A)$, since $f(A) = f(\mathrm{cl}(A))$. But $f(A) \subset \alpha g(\mathrm{cl}(f(A)))$. Combaining we get $f(A) = \alpha g(\mathrm{cl}(f(A)))$. Hence $f(A)$ is $\tau_{\alpha g}$-closed in Y.

Corollary 3.4: If $f: X \to Y$ is c-[c-r-; c-α-; c-$r\alpha$-]open, then $f(A)$ is $\tau_{\alpha g}$ closed in Y if A is open set in X.

Theorem 3.5: If $\alpha g(\mathrm{cl}(f(A))) = r\mathrm{cl}(A)$ for every $A \subset Y$ and X is discrete space, then the following are equivalent:
a) f: X→ Y is contra αg-open map
b) $\alpha g(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$
Proof: (a) ⇒ (b) follows from theorem 3.3
(b) ⇒ (a) Let A be any open set in X, then $f(A) = f(\mathrm{cl}(A)) \supset \alpha g(\mathrm{cl}(f(A)))$ by hypothesis. We have $f(A) \subset \alpha g(\mathrm{cl}(f(A)))$. Combining we get $f(A) = \alpha g(\mathrm{cl}(f(A))) = r(\mathrm{cl}(f(A)))$[by given condition] which implies $f(A)$ is r-closed and hence αg-closed. Thus f is contra αg-open.

Theorem 3.6: If $\alpha(\mathrm{cl}(A)) = r\mathrm{cl}(A)$ for every $A \subset Y$ and X is discrete space, then the following are equivalent:
a) f: X→ Y is contra αg-open map
b) $\alpha g(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$

Theorem 3.7: $f: X \to Y$ is contra αg-open iff for each subset S of Y and each closed set U containing $f^{-1}(S)$, there is an αg-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.
Proof: Assume $f: X \to Y$ is contra αg-open. Let $S \subseteq Y$ and U be closed set containing $f^{-1}(S)$. Then X-U is open in X and $f(X\text{-}U)$ is αg-closed in Y as f is contra αg-open and $V = Y\text{-} f(X\text{-}U)$ is αg-open in Y. $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y\text{-} f(X\text{-}U)) = f^{-1}(Y)\text{-} f^{-1}(f(X\text{-}U)) = f^{-1}(Y)\text{-}(X\text{-}U) = X\text{-}(X\text{-}U) = U$

Conversely Let F be open in X ⇒ F^c is closed. Then $f^{-1}(f(F^c)) \subseteq F^c$. By hypothesis there exists an αg-open set V of Y, such that $f(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is αg-closed in Y. Therefore f is contra αg-open.

Remark 3: Composition of two contra αg-open maps is not contra αg-open in general

Theorem 3.8: Let X, Y, Z be topological spaces and every αg-closed set is open in Y. Then the composition of two contra αg-open maps is contra αg-open.
Proof:(a) Let f and g be contra αg-open maps. Let A be any open set in $X \Rightarrow f(A)$ is open in Y(by assumption) $\Rightarrow g(f(A)) = g \, o \, f(A)$ is αg-closed in Z. Therefore gof is contra αg-open.

Corollary 3.5: Let X, Y, Z be topological spaces and every [r-; α-; $r\alpha$-] closed set is open in Y. Then the composition of two c-[c-r-; c-α-; c-$r\alpha$-]open maps is contra αg-open.

Example 3: Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$ and $\eta = \{\varphi, \{a\}, \{b, c\}, Z\}$. $f:X\rightarrow Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$ and $g:Y\rightarrow Z$ be defined $g(a) = b$, $g(b) = a$ and $g(c) = c$, then g, f and $g \, o \, f$ are contra αg-open.

Theorem 3.9: If $f:X\rightarrow Y$ is contra g-open[contra rg-open], g:$Y\rightarrow Z$ is αg-closed and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is contra αg-open.
Proof:(a) Let A be open in X. Then $f(A)$ is g-closed and so closed in Y as Y is $T_{1/2} \Rightarrow g(f(A)) = gof(A)$ is αg-closed in Z(since g is αg-closed). Hence gof is contra αg-open.

Corollary 3.6: If $f:X\rightarrow Y$ is contra g-open[contra rg-open], $g:Y\rightarrow Z$ is [r-; α-; $r\alpha$-] closed and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is contra αg-open.

Theorem 3.10: If $f:X\rightarrow Y$ is g-open[rg-open], g:$Y\rightarrow Z$ is contra αg-open and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is contra αg-open.
Proof:(a) Let A be open in X. Then $f(A)$ is g-open and so open in Y as Y is $T_{1/2} \Rightarrow g(f(A)) = gof(A)$ is αg-closed in Z. Hence gof is contra αg-open.

Theorem 3.11: If $f:X\rightarrow Y$ is g-open[rg-open], g:$Y\rightarrow Z$ is c-[c-r-; c-α-; c-$r\alpha$-] open and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is contra αg-open.

Theorem 3.12: If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that gof is contra αg-open [contra open] then the following statements are true.
a) If f is continuous [r-continuous] and surjective then g is contra αg-open.
b) If f is g-continuous[resp: rg-continuous], surjective and X is $T_{1/2}$ [resp: r-$T_{1/2}$] then g is contra αg-open.
Proof: A open in Y, $f^{-1}(A)$ open in $X \Rightarrow (g \, o \, f)(f^{-1}(A)) = g(A)$ αg-closed in Z. Hence g is contra αg-open.

Similarly one can prove the remaining parts and hence omitted.

Corollary 3.7: If $f:X\rightarrow Y$, g:$Y\rightarrow Z$ be two mappings such that gof is c-[c-r-; c-α-; c-$r\alpha$-] open then the following statements are true.
a) If f is continuous [r-continuous] and surjective then g is contra αg-open.
b) If f is g-continuous[rg-continuous], surjective and X is $T_{1/2}$ [r-$T_{1/2}$] then g is contra αg-open.

Theorem 3.13: If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that gof is αg-closed then the following statements are true.
a) If f is contra-continuous [contra-r-continuous] and surjective then g is contra αg-open.
b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{1/2}$ [resp: r-$T_{1/2}$] then g is contra αg-open.
Proof: A open in Y, $f^{-1}(A)$ closed in $X \Rightarrow (g \, o \, f)(f^{-1}(A)) = g(A)$ αg-closed in Z. Hence g is contra αg-open.

Corollary 3.8: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that gof is [r-; α-; $r\alpha$-]closed then the following statements are true.

a) If f is contra-continuous [contra-r-continuous] and surjective then g is contra αg-open.

b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is contra αg-open.

Theorem 3.14: If X is αg-regular, f: X→ Y is r-closed, nearly-continuous, contra αg-open surjection and $\bar{A} = A$ for every αg-closed set in Y, then Y is αg-regular.

Proof: Let $p\in U\in \alpha gO(Y)$. Then there exists a point $x\in X$ such that $f(x) = p$ as f is surjective. Since X is αg-regular and f is r-continuous there exists $V\in RO(X)$ such that $x\in V\subseteq\overline{V}\subseteq f^{-1}(U)$ which implies $p\in f(V) \subseteq f(\overline{V}) \subseteq f(f^{-1}(U)) = U \to(1)$

Since f is contra αg-open, $f(\overline{V}) \subseteq U$, By hypothesis $\overline{f(\overline{V})} = f(\overline{V})$ *and* $\overline{f(\overline{V})} = \overline{f(V)} \to(2)$

By(1) &(2) we have $p\in f(V) \subseteq f(\overline{V}) \subseteq U$ and $f(V)$ is αg-open. Hence Y is αg-regular.

Corollary 3.9: If X is αg-regular, f: X→ Y is r-closed, nearly-continuous, contra αg-open surjection and $\bar{A} = A$ for every r-closed set in Y then Y is αg-regular.

Theorem 3.15: If $f:X\to Y$ is contra αg-open and $A\in RC(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is contra αg-open.

Proof: Let F be open in A. Then F = A∩E for some open set E of X and so F is open in $X \Rightarrow f(A)$ is αg-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is contra αg-open.

Theorem 3.16: If $f:X\to Y$ is contra αg-open, X is $rT_{½}$ and A is rg-open set of X then $f_A:(X,\tau(A))\to(Y,\sigma)$ is contra αg-open.

Proof: Let F be open in A. Then F = A∩E for some open set E of X and so F is open in $X \Rightarrow f(A)$ is αg-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is contra αg-open.

Corollary 3.10: If $f:X\to Y$ is c-[c-r-; c-α-; c-$r\alpha$-] open and $A\in RC(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is contra αg-open.

Theorem 3.17: If $f_i:X_i\to Y_i$ be contra αg-open for i =1, 2. Let f: $X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) =(f_1(x_1), f_2(x_2))$. Then $f:X_1 x X_2\to Y_1 x Y_2$ is contra αg-open.

Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is open in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is αg-closed set in $Y_1 x Y_2$. Hence f is contra αg-open.

Corollary 3.11: If $f_i:X_i\to Y_i$ be c-[c-r-; c-α-; c-$r\alpha$-]open for i =1, 2. Let $f:X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) =(f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2\to Y_1 x Y_2$ is contra αg-open.

§4. ALMOST CONTRA αg-OPEN MAPPINGS:

Definition 4.1: A function f: $X\to Y$ is said to be almost contra αg-open if the image of every r-open set in X is αg-closed in Y.

Theorem 4.1: Every contra αg-open map is almost contra αg-open map but not conversely.

Theorem 4.2: We have the following interrelation among the following almost contra open mappings

(i)
$$
\begin{array}{ccccccc}
\text{al.c.r}\alpha\text{.open} & & & & \text{al.c.g.open} \\
\uparrow & & & & \downarrow \\
\text{al.c.r.open} & \to & \text{al.c.open} & \to \ \text{al.c.}\alpha\text{.open} \ \to & \text{al.c.}\alpha\text{g.open}
\end{array}
$$
None is reversible

(ii)

$$\begin{array}{ccccccc} \text{c.r}\alpha\text{.open} & & & & & & \text{c.g.open} \\ \uparrow & & & & & & \downarrow \\ \text{c.r.open} & \rightarrow & \text{c.open} & \rightarrow & \text{c.}\alpha\text{.open} & \rightarrow & \text{al.c.}\alpha\text{g.open} \end{array}$$

None is reversible

(iii) If αGC(Y) = RC(Y), then the reverse relations hold for all almost contra open maps.

$$\begin{array}{ccccccc} \text{al. c.r}\alpha\text{.open} & & & & & & \text{al.c.g.open} \\ \uparrow\downarrow & & & & & & \uparrow\downarrow \\ \text{al.c.r.open} & \leftrightarrow & \text{al.c.open} & \leftrightarrow & \text{al.c.}\alpha\text{.open} & \leftrightarrow & \text{al.c.}\alpha\text{g.open} \end{array}$$

Example 4: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\} = \sigma$. Let $f{:}X\rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is almost contra αg –open.

Example 5: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f{:}X\rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is not almost contra αg-open, almost contra open, almost contra α-open, almost contra rα-open and almost contra g-open.

Theorem 4.3:
(i) If (Y, σ) is discrete, then f is almost contra open of all types.
(ii) If f is almost contra open and g is αg-closed then gof is almost contra αg-open.
(iii) If f is almost open and g is contra αg-open then gof is almost contra αg-open.

Corollary 4.1: If f is almost contra open and g is [r-;α-; $r\alpha$-] closed then gof is almost contra αg-open.

Corollary 4.2: If f is almost open[almost r-closed] and g is c-[c-r-; c-α-; c-$r\alpha$-] open then gof is almost contra αg-open.

Theorem 4.4: If f: X→ Y is almost contra αg-open, then $\alpha g\,(cl(f(A))) \subset f(cl(A))$

Proof: Let A⊂X be r-open and f: X→ Y is almost contra αg-open gives $f(cl\{A\})$ is αg-closed in Y and $f(A) \subset f(cl(A))$ which in turn gives $\alpha g\,(cl(f(A))) \subset \alpha g cl(f(cl(A)))$----- (1)
Since $f(cl(A))$ is αg-closed in Y, $\alpha g cl(f(cl(A))) = f(cl(A))$------ (2)
From (1) and (2) we have $\alpha g\,(cl(f(A))) \subset f(cl(A))$ for every subset A of X.

Remark 4: Converse is not true in general.

Corollary 4.3: If f: X→ Y is al-c-[al-c-r-; al-c-α-; al-c-$r\alpha$-] open, then $\alpha g\,(cl(f(A))) \subset f(cl(A))$

Theorem 4.5: If $f{:}X\rightarrow Y$ is almost contra αg-open and $A\subseteq X$ is r-open, $f(A)$ is $\tau_{\alpha g}$-closed in Y.

Proof: Let A⊂X be r-open and f: X→ Y is almost contra αg-open implies $\alpha g\,(cl(f(A))) \subset f(cl(A))$ which in turn implies $\alpha g\,(cl(f(A))) \subset f(A)$, since $f(A) = f(cl(A))$. But $f(A) \subset \alpha g\,(cl(f(A)))$. Combaining we get $f(A) = \alpha g\,(cl(f(A)))$. Hence $f(A)$ is $\tau_{\alpha g}$-closed in Y.

Corollary 4.4: If $f{:}X\rightarrow Y$ is al-c-[al-c-r-; al-c-α-; al-c-$r\alpha$-] open, then $f(A)$ is $\tau_{\alpha g}$ closed in Y if A is r-open set in X.

Theorem 4.6: If $f{:}X\rightarrow Y$ is c-[c-r-; c-α-; c-$r\alpha$-] open and $A\subseteq X$ is r-open, $f(A)$ is $\tau_{\alpha g}$-closed in Y.

Proof: For A⊂X is r-open and f:X→Y is c-r-open, $f(A)$ is τ_r-closed in Y and so $f(A)$ is $\tau_{\alpha g}$-closed in Y. [since r-closed set is αg-closed]. Similarly we can prove the remaining results.

Theorem 4.7: If αg (cl(f(A))) = rcl(A) for every A$\subset$Y and X is discrete space, then the following are equivalent:

a) f: X$\rightarrow$ Y is almost contra αg-open map

b) αg (cl(f(A))) $\subset$ f(cl(A))

Proof: (a) $\Rightarrow$ (b) follows from theorem 4.4

(b) $\Rightarrow$ (a) Let A be any r-open set in X, then f(A) = f(cl(A)) $\supset$ αg (cl(f(A))) by hypothesis. We have f(A) $\subset$ αg (cl(f(A))). Combining we get f(A) = αg (cl(f(A))) = r(cl(f(A)))[by given condition] which implies f(A) is r-closed and hence αg-closed. Thus f is almost contra αg-open.

Theorem 4.8: If α(cl(A)) = rcl(A) for every A$\subset$Y and X is discrete space, then the following are equivalent:

a) f: X$\rightarrow$ Y is almost contra αg-open map

b) αg (cl(f(A))) $\subset$ f(cl(A))

Proof: (a) $\Rightarrow$ (b) follows from theorem 4.4

(b) $\Rightarrow$ (a) Let A be any r-open set in X, then f(A) = f(cl(A)) $\supset$ αg (cl(f(A))) by hypothesis. We have f(A) $\subset$ αg (cl(f(A))). Combining we get f(A) = αg (cl(f(A))) = rcl(f(A))[by given condition] which implies f(A) is r-closed and hence αg-closed. Thus f is almost contra αg-open.

Theorem 4.9: $f:X\rightarrow Y$ is almost contra αg-open iff for each subset S of Y and each U$\in$RC(X, f^{-1}(S)), there is an αg-open set V of Y such that S$\subseteq$V and f^{-1}(V) $\subseteq$ U.

Proof: Assume $f:X\rightarrow Y$ is almost contra αg-open. Let S $\subseteq$ Y and U$\in$RC(X, f^{-1}(S)). Then X-U is r-open in X and f(X-U) is αg-closed in Y as f is almost contra αg-open and V=Y- f(X-U) is αg-open in Y. f^{-1}(S) $\subseteq$ U $\Rightarrow$ S$\subseteq$ f(U) $\Rightarrow$ S $\subseteq$ V and f^{-1}(V) = f^{-1}(Y- f(X-U)) = f^{-1}(Y)- f^{-1}(f (X-U)) = f^{-1}(Y)-(X-U) = X-(X-U) = U

Conversely Let F be r-open in X $\Rightarrow$ F^c is r-closed. Then $f^{-1}(f(F^c)) \subseteq F^c$. By hypothesis there exists an αg-open set V of Y, such that $f(F^c) \subseteq$ V and $f^{-1}(V) \supset F^c$ and so F $\subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus f (F) is αg-closed in Y. Thus f is almost contra αg-open.

Remark 5: Composition of two almost contra αg-open maps is not almost contra αg-open in general.

Theorem 4.10: Let X, Y, Z be topological spaces and every αg-closed set is r-open in Y. Then the composition of two almost contra αg-open maps is almost contra αg-open.

Proof: (a) Let f and g be almost contra αg-open maps. Let A be any r-open set in X $\Rightarrow$ f(A) is r-open in Y (by assumption) $\Rightarrow$ g(f (A)) = g o f(A) is αg-closed in Z. Therefore gof is almost contra αg-open.

Theorem 4.11: Let X, Y, Z be topological spaces and every [r-; α-; $r\alpha$-]closed set is r-open in Y. Then the composition of two al-c-[al-c-r-; al-c-α-; al-c-$r\alpha$-]open maps is almost contra αg-open.

Proof: Let A be r-open set in X, then f(A) is r-closed in Y and so r-open in Y (by assumption) $\Rightarrow$ g(f (A)) = gof(A) is r-closed in Z. Hence gof is almost contra αg-open.

Corollary 4.5: Let X, Y, Z be topological spaces and every [r-; α-; $r\alpha$-]closed set is open [r-open] in Y. Then the composition of two c-[c-r-; c-α-; c-$r\alpha$-]open maps is almost contra αg-open.

Example 6: Let X = Y = Z = {a, b, c}; τ = {φ, {a}, {a, b}, X}; σ = {φ, {a, c}, Y} and η = {φ, {a}, {b, c}, Z}. $f:X\rightarrow Y$ be defined f(a) = c, f(b) = b and f(c) = a and $g:Y\rightarrow Z$ be defined g(a) = b, g(b) = a and g(c) = c, then g, f and g o f are almost contra αg-open.

Theorem 4.12: If $f:X\rightarrow Y$ is almost contra g-open[almost contra rg-open], g:$Y\rightarrow Z$ is αg-closed and Y is $T_{½}$ [r-$T_{½}$] then gof is almost contra αg-open.
Proof: (a) Let A be r-open in X. Then f(A) is g-closed and so closed in Y as Y is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is αg-closed in Z (since g is αg-closed). Hence gof is almost contra αg-open.

Corollary 4.6: If $f:X\rightarrow Y$ is almost contra g-open[almost contra rg-open], $g:Y\rightarrow Z$ is [r-; α-; $r\alpha$-]closed and Y is $T_{½}$ [r-$T_{½}$] then gof is almost contra αg-open.

Theorem 4.13: If $f:X\rightarrow Y$ is almost g-open[almost rg-open], g:$Y\rightarrow Z$ is contra αg-open and Y is $T_{½}$ [r-$T_{½}$] then gof is almost contra αg-open.
Proof: (a) Let A be r-open in X. Then f(A) is g-open and so open in Y as Y is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is αg-closed in Z (since g is contra αg-open). Hence gof is almost contra αg-open.

Theorem 4.14: If $f:X\rightarrow Y$ is almost g-open[almost rg-open], g:$Y\rightarrow Z$ is c-[c-r-; c-α-; c-$r\alpha$-]open and Y is $T_{½}$ [r-$T_{½}$] then gof is almost contra αg-open.

Theorem 4.15: If $f:X\rightarrow Y$ is g-open[rg-open], $g:Y\rightarrow Z$ is c-[c-r-; c-α-; c-$r\alpha$-]open and Y is $T_{½}$[r-$T_{½}$], then gof is almost contra αg-open.
Proof: Let A be r-open set in X, then f(A) is g-closed in Y and so closed in Y (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is r-closed in Z. Hence gof is almost contra αg-open.

Theorem 4.16: If $f:X\rightarrow Y$ is c-g-open[c-rg-open], $g:Y\rightarrow Z$ is [r-; α-; $r\alpha$-]closed and Y is $T_{½}$[r-$T_{½}$], then gof is almost contra αg-open.
Proof: Let A be r-open set in X, then f(A) is g-closed in Y and so closed in Y (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is r-closed in Z. Hence gof is almost contra αg-open.

Theorem 4.17: If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that gof is contra αg-open then the following statements are true.
a) If f is continuous [r-continuous] and surjective then g is almost contra αg-open.
b) If f is g-continuous[resp: rg-continuous], surjective and X is $T_{½}$ [resp: r-$T_{½}$] then g is almost contra αg-open.
Proof: (a) For A r-open in Y, $f^{-1}(A)$ open in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ αg-closed in Z. Hence g is almost contra αg-open.

Similarly one can prove the remaining parts and hence omitted.

Corollary 4.7: If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that gof is contra αg-open then the following statements are true.
a) If f is almost continuous [almost r-continuous] and surjective then g is almost contra αg-open.
b) If f is almost g-continuous[resp: almost rg-continuous], surjective and X is $T_{½}$ [resp: r-$T_{½}$] then g is almost contra αg-open.

Corollary 4.8: If $f:X\rightarrow Y$, g:$Y\rightarrow Z$ be two mappings such that gof is c-[c-r-; c-α-; c-$r\alpha$-]open then the following statements are true.
a) If f is continuous [r-continuous] and surjective then g is almost contra αg-open.
b) If f is g-continuous[rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is almost contra αg-open.

Corollary 4.9: If $f:X\rightarrow Y$, g:$Y\rightarrow Z$ be two mappings such that gof is c-[c-r-; c-α-; c-$r\alpha$-]open then the following statements are true.

a) If f is almost continuous [almost r-continuous] and surjective then g is almost contra αg-open.
b) If f is almost g-continuous[almost rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is almost contra αg-open.

Theorem 4.18: If f:X→Y, g:Y→Z be two mappings such that gof is αg-closed [r-closed] then the following statements are true.
a) If f is contra-continuous [contra-r-continuous] and surjective then g is almost contra αg-open.
b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{½}$ [resp: r-$T_{½}$] then g is almost contra αg-open.
Proof: (a) For A r-open in Y, f^{-1}(A) closed in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ αg-closed in Z. Hence g is almost contra αg-open.

Corollary 4.10: If f:X→Y, g:Y→Z be two mappings such that gof is αg-closed [r-closed] then the following statements are true.
a) If f is almost contra-continuous [almost contra-r-continuous] and surjective then g is almost contra αg-open.
b) If f is almost contra-g-continuous[almost contra-rg-continuous], surjective and X is $T_{½}$ [resp: r-$T_{½}$] then g is almost contra αg-open.

Corollary 4.11: If f:X→Y, g:Y→Z be two mappings such that gof is [r-; α-; $r\alpha$-]closed then the following statements are true.
a) If f is contra-continuous [contra-r-continuous] and surjective then g is almost contra αg-open.
b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is almost contra αg-open.

Corollary 4.12: If f:X→Y, g:Y→Z be two mappings such that gof is [r-; α-; $r\alpha$-]closed then the following statements are true.
a) If f is almost contra-continuous [almost contra-r-continuous] and surjective then g is almost contra αg-open.
b) If f is almost contra-g-continuous[almost contra-rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is almost contra αg-open.

Theorem 4.19: If X is αg-regular, f: X→ Y is r-closed, nearly-continuous, almost contra αg-open surjection and $\bar{A} = A$ for every αg-closed set in Y, then Y is αg-regular.
Proof: Let $p \in U \in \alpha gO(Y)$. Then there exists a point x∈X such that f(x) = p as f is surjective. Since X is αg-regular and f is r-continuous there exists $V \in RO(X)$ such that $x \in V \subseteq \overline{V} \subseteq f^{-1}(U)$ which implies $p \in f(V) \subseteq f(\overline{V}) \subseteq f(f^{-1}(U)) = U \rightarrow (1)$
Since f is almost contra αg-open, $f(\overline{V}) \subseteq$ U, By hypothesis $\overline{f(\overline{V})} = f(\overline{V})$ and $\overline{f(\overline{V})} = \overline{f(V)} \rightarrow (2)$
By (1) & (2) we have $p \in f(V) \subseteq f(\overline{V}) \subseteq U$ and $f(V)$ is αg-open. Hence Y is αg-regular.

Corollary 4.13: If X is αg-regular, f: X→ Y is r-closed, nearly-continuous, almost contra αg-open surjection and $\bar{A} = A$ for every r-closed set in Y then Y is αg-regular.

Theorem 4.20: If f:X→Y is almost contra αg-open and $A \in RO(X)$, then $f_A:(X,\tau(A)) \rightarrow (Y,\sigma)$ is almost contra αg-open.
Proof: Let F be an r-open set in A. Then F = A∩E for some r-open set E of X and so F is r-open in $X \Rightarrow$ f(A) is αg-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is almost contra αg-open.

Theorem 4.21: If $f:X\to Y$ is almost contra αg-open, X is $rT_{½}$ and A is rg-open set of X then $f_A:(X,\tau(A))\to(Y,\sigma)$ is almost contra αg-open.
Proof: Let F be a r-open set in A. Then F = A∩E for some r-open set E of X and so F is r-open in $X \Rightarrow f(A)$ is αg-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is almost contra αg-open.

Corollary 4.14: If $f:X\to Y$ is c-[c-r-; c-α-; c-$r\alpha$-] open and $A\in RO(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is almost contra αg-open.

Corollary 4.15: If $f:X\to Y$ is al-c-[al-c-r-; al-c-α-; al-c-$r\alpha$-] open and $A\in RO(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is almost contra αg-open.

Theorem 4.22: If $f_i:X_i\to Y_i$ be almost contra αg-open for i =1, 2. Let $f: X_1xX_2\to Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f:X_1xX_2\to Y_1xY_2$ is almost contra αg-open.
Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is r-open in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is αg-closed set in Y_1xY_2. Hence f is almost contra αg-open.

Corollary 4.16: If $f_i:X_i\to Y_i$ be al-c-[al-c-r-; al-c-α-; al-c-$r\alpha$-] open for i =1, 2.
Let $f:X_1xX_2\to Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1xX_2\to Y_1xY_2$ is almost contra αg-open.

Corollary 4.17: If $f_i:X_i\to Y_i$ be c-[c-r-; c-α-; c-$r\alpha$-] open for i =1, 2. Let $f:X_1xX_2\to Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1xX_2\to Y_1xY_2$ is almost contra αg-open.

CONCLUSION:

In this paper the authors introduced the concepts of contra αg-open mappings, almost contra αg-open mappings, studied their basic properties and interrelationship between other such contra open maps.

Acknowledgement: The Authors would like to thank the referees for their critical comments, suggestions and corrections towards the development of the paper.

References:

[1.] Abd El-Monsef. M.E., El-Deeb. S.N. and Mahmoud. R.A. β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut. Univ. A 12(1983), no.1, 77 – 90.
[2.] Asit Kumar Sen and Bhattcharya.P., On preclosed mappings, Bull.Cal.Math.Soc.,85, 409-412(1993).
[3.] Baker.C.W., Contra-open functions and contra-closed functions, Math. Today (Ahmedabad) 15 (1997), 19–24.
[4.] Balasubramanian.S., and Ch. Chaitanya αg-Separation axioms–I.J.M.A,Vol 3, No.3 (2012)855 – 863.
[5.] Balasubramanian.S., and Ch. Chaitanya on αg-Separation axioms–I.J.M.A,Vol3, No.3 (2012)877 – 888.
[6.] Balasubramanian.S., and Chaitanya.Ch.,Minimal αg-open sets – Aryabhatta J.M.I., Vol.4(1)(2012)83 – 94.
[7.] Balasubramanian.S., and Ch. Chaitanya Somewhat almost αg-continuous functions, somewhat almost αg-open functions – I.J.M.E.R., Vol.2, Issue.4, (July-Aug. 2012) 2774 – 2778.
[8.] Balasubramanian.S., and Ch. Chaitanya Slightly αg-continuous functions, somewhat αg-continuous functions – IJCMI, Vol.5,No.2(2012) 500 – 509.
[9.] Balasubramanian.S., Aruna Swathi Vyjayanthi.P., and Sandhya.C., Contra v-closed map – International Journal of Engineering Sciences and Research Technology, Vol.2,Issue 3,(2013)544 – 549
[10.] Balasubramanian.S., Sandhya.C., and Aruna Swathi Vyjayanthi.P., Contra v-open map – International Journal of Engineering Sciences and Research Technology, Vol.2,Issue 3,(2013)561 – 566
[11.] Balasubramanian.S., Aruna Swathi Vyjayanthi.P., and Sandhya.C., Almost contra v-closed map – Indian Journal of Engineering, Vol.3, No.6(April-13)24 – 29.
[12.] Balasubramanian.S., Sandhya.C., and Aruna Swathi Vyjayanthi.P., Almost contra v-open map – Indian Journal of Engineering, Vol.3, No.6(April-13)30 – 34.

[13.] Balasubramanian.S., Sellamuthu.M., Sandhya.C., and Vyjayanthi.P.A.S., Contra *rp*-open map – Indian J. Sci., Vol.3,No.6,(2013),20 – 24.

[14.] Balasubramanian.S., Almost Contra-pre-open map and Almost contra-pre-closed map – International Journal of Engineering Sciences and Research Technology, Vol.2(4)(2013)697 – 705.

[15.] Balasubramanian.S., and Sandhya.C., Almost contra- β-open map and almost contra-β-closed map – International Journal of Engineering Sciences and Research Technology, Vol.2(4)(2013)811 – 819.

[16.] Balasubramanian.S., Almost Contra-semi-open map and Almost contra-semi-closed map – Indian Journal of Science, Vol.3, No.7(2013)59- 64.

[17.] Balasubramanian.S., and Sandhya.C., Contra-β-open map and contra-β-closed map – Bull. Kerala Math. Association, Vol.10, No.1(2013)77 – 88.

[18.] Balasubramanian.S., Krishnamurthy.T.K., Vyjayanthi.P.A.S., and Sandhya.C., Contra *rp*-closed map – Inter. J. Math. Archive, Vol.4, No. 9(2013)184 – 191.

[19.] Balasubramanian.S., Almost Contra-open map and Almost contra-closed map–Bull. Kerala Math. Association, Vol.10, No.2(2013)151-161

[20.] Balasubramanian.S. and Chaitanya. Ch., somewhat αg-closed mappings, I.J.A.S.T.R., Vol.6 (4),(2014) ,493 – 498.

[21.] Balasubramanian.S., and Chaitanya. Ch., Almost slightly αg-continuous mappings, I.J.M.A.,Vol.6(1) (2015) 112-120.

[22.] Balasubramanian.S., Almost Contra *vg*-open and almost contra *vg*-closed mappings, International journal of Advanced scientific and Technical Research, Issue 2 volume 5, March-April(2015),(In Press).

[23.] Balasubramanian.S., Contra *vg*-open and contra *vg*-closed mappings, Aryabhatta Journal of Mathematics and Informatics,(Communicated)

[24.] Biswas.N., On some mappings in topological spaces, Bull. Cal. Math. Soc., 61, (1969) 127 – 135.

[25.] Caldas,M., and Baker,C.W., Contra Pre-semiopen Maps, Kyungpook Math. Journal, 40 (2000), 379 – 389.

[26.] Cammaroto.F., and Noiri.T., Almost irresolute functions, I.J.P.A.M.,20(1989)5, 472 – 482.

[27.] Di Maio. G., and Noiri.T, I. J. P. A. M., 18(3) (1987) 226-233.

[28.] Dontchev.J., Mem.Fac.Sci.Kochi Univ.ser.A., Math., 16(1995), 35-48.

[29.] Dunham.W., $T_{1/2}$ Spaces, Kyungpook Math. J.17(1977), 161-169.

[30.] Long.P.E., and Herington.L.L., Basic Properties of Regular Closed Functions, Rend. Cir. Mat. Palermo, 27(1978), 20-28.

[31.] Malghan.S.R., Generalized closed maps, J. Karnatak Univ. Sci., 27(1982), 82-88.

[32.] Mashour.A.S., Hasanein.I.A., and El.Deep.S.N., *α*-continuous and *α*-open mappings, Acta Math. Hungar., 41 (1983), 213-218.

[33.] Mashhour.A.S., Abd El-Monsef. M.E. and El-Deeb. S.N., On pre continuous and weak pre continuous mappings, Proc. Math. Phy. Soc., Eqypt, 53(1982)47 – 53..

[34.] Navalagi. G.B., on semi-precontinuous functions and properties of generalized sem-pre closed sets in topological spaces, I.J.M.M.S 29.2(2002),85 – 98.

[35.] Navalagi.G.B., Vidya N.Telageri, Some more properties of β-continuous functions(2012)IJMCA, Vol.4,No.2, July-Dec(2012)133 – 139.

[36.] Noiri.T., A generalization of closed mappings, Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., 54 (1973),412-415.

[37.] Noiri.T., Almost *α*g-closed functions and separation axioms, Acta Math. Hungar. 82(3) (1999),193-205.

[38.] Palaniappan.N., Studies on Regular-Generalized Closed Sets and Maps in Topological Spaces, Ph. D Thesis, Alagappa University, Karikudi, (1995).

[39.] Vadivel.A., and Vairamanickam.K., *rgα*-Closed and *rgα*-open maps in topological spaces. Int. Journal of Math. Analysis, 2010, 4(10): 453- 468.