A NEW MATHEMATICAL MODEL TO FUZZY TIME COST AND QUALITY TRADE OFF PROBLEMS

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ABSTRACT: Project Management is one of the most important fields in business and industry. One important aspect of the project management is to acquire the information related to an optimum balance between the project's objectives. The three interrelated and conflicting objectives of any project are time, cost and quality. These objectives are dependent on the related features of the activities of that project. The purpose of this paper is to develop mathematical models of cost, time and quality tradeoffs in conditions that time parameters of the project activities are estimated by triangular fuzzy number. The model is formulated in the form of Fuzzy Linear Programming. This paper helps practicing project engineers to have realistic expectations of the method.

KEY WORDS: Fuzzy Time Cost Quality Trade off Problem – Fuzzy Linear Mathematical Model – Triangular Fuzzy number.

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1 INTRODUCTION

Project management is one of the most important fields in business and industry. Every task in an organization can be taken into account as a project. In scheduling a project, it is generally considered to expedite the duration of some activities through expending extra budget in order to compress the project completion time. This process can be considered under either some fixed available budget or a threshold of project completion time. This problem is known as Time Cost Trade Off Problem (TCTP) in project management literature. The main objective of the TCTP is to determine the optimal amount of duration and cost assigned to the activities so that the overall cost is minimized. Hence this problem leads to a balance between the project completion time and the project total cost.

There are three main points that are the most important factors for a successful project: (1) a project must meet the customer requirements, (2) it has to be within project and (3) it has to be on time. These three criteria are often referred to as The Iron Triangle.

One important aspect of the project management is to acquire the information related to an optimum balance between the project's objectives. According to the Iron Triangle, time, cost and quality are important objectives of a project. Heretofore, extensive researchers have been conducted to develop cost-time trade off problems. Nowadays, the quality of a project is also added to the project time and cost. The aim of these problems (TCQTP) is to select a set of activities for crashing as well as an appropriate execution method for each activity such that the project cost and time is minimized while the project quality is maximized.

1.1 TIME COST AND QUALITY TRADE OFF PROBLEMS IN DIFFERENT NATURE

Babu and Suresh [5] presented the first paper considering the influence on project quality by project scheduling and developed three inter-related linear programming models to study the trade off among time, cost and quality in a deterministic CPM network. Each of the three proposed models, one of the three entries (ie) time, cost and quality by assigning desired levels to the other two entries. The linearity and deterministic assumptions led to simple solvable mathematical program which enabled the authors to investigate the idea of Time Cost and Quality Trade Off Problem (TCQTP). In Khang and Myint [14], the model proposed by Babu and Suresh [5] was applied to an actual cement factory construction project. The purpose was to evaluate the applicability of the method by highlighting the managerial insights gained, as well as pointing out key problems and difficulties faced. The problems investigated by Babu and Suresh [5], Khang and Myint [14] can be categorized in the class of continuous time, cost and quality trade off problem. Thereafter, many researchers have developed mathematical programming model for these kinds of problems.

In El-Rayes et al. [10] for the first time, the discrete Time Cost Quality Trade Off problem was investigated. They used a real world example and suggested new functions to enable the consideration of construction quality in the time, cost and quality optimization problem in construction industry. To estimate the project quality, they introduced some quality indicators, and used the weighed sum of the quality levels sassed by indicators as the project quality. In another work, a discrete model of time, cost and quality trade off was proposed by Tareghian [25] using three integer programming in which activities are performed in one of several available alternatives. The purpose of suggested model is to complete the project at a given deadline such that total accost is minimized and overall quality is maximized. Similar to Babu and Suresh [5] all the three entries are assumed to be deterministic parameters and consequently, the CPM framework were too applied to this research.

The authors employed a discrete multimode model using activity based resource utilization options to transform the time cost trade off to a time, cost and quality trade off model and solved it by genetic algorithm. Other works by Rahimi and Iranmanesh et al. [21], Johnson-Pollack and Liberatore [20] have been presented to optimize the discrete multi-mode model to TCQTP.

At this stage, meta heuristic methods were still regarded as an appropriate tool for solving these kinds of problems, for example in some works of Afsar et al. [2], Huang et al. [12], Yang [27], Tareghian and Taheri [26], Liberatore and Johnson-Pollack [20] and Lakshminayaranan et al. [15]

One of the most important issues in modeling this kind of problems is data uncertainty. This uncertainty is caused as the available information is often approximate or partial. The project managers require approximating the values of time, cost and quality of the activities and all these approximations deal with uncertainty. Many of the models, however, applied the crisp data as approximation of the parameters. These models neglect the inexact nature of such approximation. Some researchers include Cohen et al. [9], Abbasnia et al. [1], Ravi Shankar [19] and Zhang and Xing [28] considered the uncertainity problem of CTQT based on stochastic or fuzzy data. Meanwhile, Mokhtari et al. [17] developed a hybrid approach for the stochastic time-cost trade off problem in PERT networks. Amiri and Golozari [3] applied fuzzy multi arrtibute decision making techniques in project planning. Salmasnia et al. [4] regarded quality as an additional aspect in the traditional time cost trade off while the parameters are considered as stochastic. Seyed Hossein [24] developed a combination of fuzzy goal programming model and grey linear programming to solve the mathematical model.

Different heuristic approaches were presented by Siemens and Moselhi and Deb [18]. Meta heuristics such as genetic algorithm were exploited in solving this problem by Feng et al. [7] and Chau et al. [8]. Multi criteria techniques based on simulation model, stochastic dominance rules and a multi criteria aggregation procedure.

The research works in the field of time, cost and quality trade off, the subject of this research, can be categorized into two distinct categories:

1. Continuous trade-off problems: in this category, the relation among time, cost and quality has been defined as continuous function. In these works, one of the three variables (usually time) is considered to vary independently and the two others are defined as functions of that variable. Research works of Babu and Suresh [5], Khang and Myint [14] are some examples.
2. Discrete trade-off problems: in this class, the relation among time, cost and quality has been considered discrete. In other words, for each project activity, different modes of execution are defined, and for each mode, distinct time, cost, and quality are associated. So to trade-off among the objectives, one execution mode is selected for each activity. Works El-Rayes et al. [10], Tareghian and Taheri and Iranmanesh et al. [25] are a few to cite.

When a project manager faces a project in which there are alternatives for executing activities, and each alternative have distinct time, cost and quality, discrete models are applicable. Project manager can select among these alternatives to optimize the trio of project objectives. The discrete models get impracticable in the projects that there are many activities, or there are many number of execution mode for each activity. In cases where the total number of the modes is very high (either because of high number of activities, or because of high number of mode per activity, or a combination), discrete models lose their applicability in two aspects:

- The definition of the problem parameters and data gathering for all modes of the project activities is not practical for project managers.
- The according problem gets very complex to be solved

On the other hand, continuous models are suitable in projects that project manager faces with many alternatives, and a continuous relation (or an approximation) among the time; cost and quality of an activity can be defined. They are also applicable in cases that project activities are outsourced; therefore the time, cost and quality of each activity can be bargained: “how much it charges the company to reduce completion time of the activity or to improve its quality?”

Defining a realistic relation among completion time, cost and quality of each activity, and specifying the parameters of the continuous model is not simple. An efficient relation among the three features of an activity has not been defined in the works available in literature. By reducing the activity’s time, its quality inevitably reduces; so it is impossible for a project manager to have a high quality activity with the lowest possible time. While in real world practice, it is possible to have an activity with the highest possible quality and the lowest time by spending more money. In this paper, we try to develop a practical model that defines a more realistic relation among time, cost and quality of activities of a project, practicable for real works projects. This is actually the main motivation behind this research effort.

The purpose of this paper is to develop mathematical model with three different objectives representing cost, time and quality tradeoffs under the conditions that time parameters of the project activities are estimated by triangular fuzzy number. The model is formulated in the form of Fuzzy Linear Programming. The proposed model minimizes either the total cost or total fuzzy time or maximizes the total quality of the project and this model concerns both direct cost and indirect cost. This paper helps practicing project engineers to have realistic expectations of the method.

2 PRELIMINARIES

In this section, some basic definitions of fuzzy theory defined by Kaufmann, Gupta and Zimmermann, are presented.

Definition 2.1

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned values indicate the membership grade of the element in the set *A*.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(A, \mu_A(x) : x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2.2

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}(x) : R \to [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $(-\infty, a] \cup [c, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is strictly increasing on [a, b] and strictly decreasing on [b, c].
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in b$ where $a \le b \le c$.

Definition 2.3

Triangular fuzzy number is a fuzzy number represented with three points as follows:

$A = (a_1, a_2, a_3)$ this representation is interpreted as membership functions (Fig5.6).

$$\mu_A(x) = \begin{cases} 0 \quad if\ x < a_1\ and\ x > a_3 \\ \dfrac{x - a_1}{a_2 - a_1} \quad if\ a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} \quad if\ a_2 \le x \le a_3 \end{cases}$$

Definition 2.4:

Let F(R) denotes the set of all triangular fuzzy numbers. Let us define a ranking function $\Re : F(R) \to R$ which maps all triangular fuzzy numbers into R. If $\tilde{A} = (a, b, c)$ is a triangular fuzzy number, then the Graded Mean Integration Representation (GMIR) method to defuzzify the number is given by,

$$\Re(\tilde{A}) = \frac{a + 2b + c}{4}$$

3 PROBLEM DESCRIPTIONS

One of the most important issues in industrial project scheduling is to determine the best amount of allocated resources to each activity, while minimizing total time and total cost of project. This problem is a known and crucial decision making issue in project management because project planners are often interested to limit project total time by spending minimum amount of budget or to minimize the project total time by a fixed available budget. During recent years, it was suggested that the quality of a project should also be taken into consideration along with the time and cost tradeoffs. The purpose of this problem is to minimize the total cost of the project while complete the project at a given deadline and the quality of the project is maximizing. In this problem, a trade off among time, cost and quality is made, hence the Time Cost and Quality Trade off Problem is referred by an acronym TCQTP. Assigning higher amount of allocated budget to the activities, in a constant level of activity duration, may increase the quality of underlying task, but the overall project cost would be increased. Also, spending more budgets on an activity, in a constant level of quality, may shorten the activity duration.

The total cost function of a project has two components: direct and indirect costs. Direct costs are incurred because of the performance of project activities, while indirect costs include those items that are not directly related to individual project activities and thus can be assessed for the entire project. In general, indirect cost increases almost linearly with the increase of project duration and usually assumed as a percentage of project direct cost. The project time cost trade off problem, thus, is reduced to determine project cost against project duration. A possible way to solve time cost and quality trade off problem is to use a mathematical programming model whose objective function is constructed so that project direct cost is minimized and the imposed constraints guarantee a desired project deadline, while the precedence requirements of the network are maintained.

A project can be represented by an activity-on-arc network G = (V, A), where V = {1, 2,…n} is the set of nodes representing the milestones and A is the set of arcs representing the activities. In the network, node 1 and n represent the start and end of the project respectively. In this paper, the normal activity durations are assumed to be uncertain variables.

3.1 THE PROPOSED MODEL AND ITS THREE FEATURES OF AN ACTIVITY

The three features of an activity of the project i.e. quality, cost and time are interrelated. As for other interrelated variables, the relation among these features can be defined in different ways by different functions. The variety may be due to the type of function, degree of freedom, and explicit or implicit nature of the function. As it was stated in the introduction, the defined relations existing in the literature lack the characteristics of real world situations. The problem is rooted in the degree of freedom considered in the relation, that is considered with one degree of freedom (only time can vary independently), which lead to a curve in space; while the real and the other is dependent on the value of these two. For instance, the direct cost needed to execute the activity is dependent on the time and quality level selected, or the quality of an activity can be determined by knowing both the time and budget level specified for it.

3.2 ASSUMPTIONS OF THE PROPOSED MODEL

The following are the assumption of the proposed model:

1. For each activity, the normal activity duration denoted by normal time, corresponding to the most efficient work method used to perform activity, and the minimal duration of an activity denoted as its crash time are defined. Associated with normal time, normal quality is defined, which is always less than the ultimate possible quality (100%).
2. The cost of an activity can be categorized in three items:

- Cost of executing the activity with normal time and normal quality

- Cost of expediting the activity (reducing time)
- Cost of improving the quality of an activity (increasing quality)

3. By reducing the time of an activity (by spending money), its quality inevitably reduces as well. So, if we want the activity to increase its quality (normal quality), the cost of the activity increases once more.
4. If we want to have the activity with a quality better than normal quality, the cost of the activity increases. The amount of this increase is more when the time of the activity is less.

To implement the mentioned concepts, a mathematical model is defined using three functions:

i. The function $Q_{ij}(c)$ determines the planned quality of activity $i \to j$. This function is considered linear. To obtain this function, we should have the quality of the activity in normal time and crash time and normal quality cost and crash quality cost.

ii. The function $C_{ij}(t)$ determines the cost of executing activity with duration t. A linear relation is considered between time t and cost $C_{ij}(t)$. This function can be obtained by having normal cost and the crash cost of activity.

iii. The function $D_{ij}(c)$ determines the planned duration of each activity.

4 FORMULATION OF THE DEVELOPED MODEL

To formulate the time, cost and quality trade off problem, one of the important issues is to access the value of that time and the quality, total cost of the project. Clearly, project completion time can be calculated by determining the total completion time for the critical path. A critical path in a project is the path with longest duration. Moreover the total cost is equal to the sum of cost used for all activities. The main goal of this problem is in two folds: to minimize the total cost or to minimize the total fuzzy time of the project.

The primary information obtained from traditional scheduling is basically activities start and finish timings and floats. The duration and the corresponding cost for an activity are selected optimally form their utility data to satisfy the objective function and the imposed constraints. If the start time of an activity is determined, the finish time can be specified by adding the selected activity duration, and vice versa. Parameters and decision variables of model are as follows:

Parameters

n	Number of actual activities
ND_{ij}	Normal time for activity $i \to j$
CD_{ij}	Crash time for activity $i \to j$
NC_{ij}	Cost of doing activity in normal time (Normal Cost)
$Min\, Q_{ij}$	Quality of doing activity in normal duration (Minimum Quality)
$Max\, Q_{ij}$	Quality of doing activity in crash duration (Maximum Quality)
s_{ij}	Slope cost for activity $i \to j$
T	Project completion time
Q	Planned quality increase of the project
t_{ij}	Cost of increasing one percent of quality for activity $i \to j$

Decision Variables:

C_{ij} Total cost of time and quality; Q_{ij} Planned quality of the activity $i \to j$

T_i Starting time of node i; D_{ij} Planned time of the activity $i \to j$

In this paper, time parameter and starting time variables are considered in triangular fuzzy number.

- **Precedence relationship constraint:**

The completion time of project could be constrained by one of the two methods. The first approach is to allow for a precedence constraint for each immediate preceding relationship in the project network. This approach was used in almost all existing optimization techniques. The second is to allow for one constraint for each path from the first activity to the last one in the project network. In the present model, the first approach will be adopted.

The logical relationship between any two consecutive activities i and its immediate predecessor j, is expressed mathematically as $\tilde{T}_j - \tilde{T}_i - \tilde{D}_{ij} \geq 0$

- **Project completion constraint:**

Project completion is controlled by the latest finish time of ending activities. If the number of ending activities is denoted by n, the project completion constraint is given by the equation, in which T is the desired deadline of the project. $\tilde{T}_n \leq \tilde{T}$

The upper and lower bounds on T are the normal project duration and crash project duration respectively.

- **Bounded constraint of duration:**

The set of constraints mentioned below is used to constrain the values of duration of activities within the interval of crash duration and normal duration. It can be written mathematically as

$$\tilde{C}D_{ij} \leq \tilde{D}_{ij} \leq N\tilde{D}_{ij}$$

- **Bounded constraint of quality:**

The set of constraints is used to constrain the values of quality of activities within the interval crash quality and normal quality. It is mathematically notated as

$$NQ_{ij} \leq Q_{ij} \leq CQ_{ij}$$

- **Cost constraint:**

The set of constraints mentioned in this section is used to calculate the cost of each activity based on the values of fuzzy duration and quality selected for that activity where the fuzzy duration is represented in the form of triangular fuzzy number. Instead of defining this set of constraint, the objective of the model can be written as

$$C_{ij} = NC_{ij} + s_{ij}\ (N\tilde{D}_{ij} - \tilde{D}_{ij}) + t_{ij}(Q_{ij} - Min\, Q_{ij})$$

- **Quality Increase Constraint:**

If we want to increase the quality of the project so as to attain the maximum possible without exceeding the total quality cost.

$$\sum_i t_{ij}(Q_{ij} - Min\, Q_{ij}) = QCost$$

Where QCost is the budget allotted to the quality increase.

- **Objective function:**

Our main objective of this linear programming model is to minimize the total cost of the project (both direct and indirect). The linear programming objective will be

$$Min \quad \sum_i \sum_j C_{ij} + \sum_n K_n + ICost * Duedate$$

Thus the C**omplete Fuzzy Mathematical Model** can be summarized as follows:

Model 1:

$$
\begin{aligned}
& Min \quad \sum_i \sum_j C_{ij} + \sum_n K_n + ICost * Due\tilde{date} \\
& subject \quad to \\
& \tilde{T}_1 = 0 \\
& C\tilde{D}_{ij} \le \tilde{D}_{ij} \le N\tilde{D}_{ij} \\
& \tilde{T}_i - \tilde{T}_j + \tilde{D}_{ij} \le 0 \\
& \tilde{T}_n - \tilde{T}_1 \le Due\tilde{date} \\
& Min Q_{ij} \le Q_{ij} \le Max Q_{ij} \\
& \sum_i t_{ij}(Q_{ij} - Min\, Q_{ij}) = QCost \\
& C_{ij} = \sum_i \sum_j NC_{ij} + s_{ij}(N\tilde{D}_{ij} - \tilde{x}_{ij}) + t_{ij}(Q_{ij} - Min\, Q_{ij})
\end{aligned}
\qquad (P_1)
$$

5 ALGORITHM TO SOLVE THE FUZZY TIME COST AND QUALITY TRADE OFF PROBLEM

In this section, a new algorithm for solving fuzzy Time, Cost and Quality Trade Off Problems through Fuzzy Linear Programming technique has presented.

1. Formulate the chosen fuzzy time, cost and quality trade off problem (P) into the following Fuzzy Linear Programming Model (1) (i.e. (P_1)) as mentioned in section (4).
2. Convert the reduced Fuzzy Linear Programming Problem (P_1) into Crisp Linear Programming Problem (P_2) using Ranking function as given in the following:

$$
\begin{aligned}
& Min \quad \sum_i \sum_j C_{ij} + \sum_n K_n + ICost*(0.25*(T_{n1} + 2*T_{n2} + T_{n3})) \\
& subject \quad to \\
& T_{11} = 0, \quad T_{12} = 0, \quad T_{13} = 0 \\
& CD_{ij1} \le D_{ij1} \le ND_{ij1} \\
& CD_{ij2} \le D_{ij2} \le ND_{ij2} \\
& CD_{ij3} \le D_{ij3} \le ND_{ij3} \\
& T_{i1} - T_{j1} + D_{ij1} \le 0 \\
& T_{i2} - T_{j2} + D_{ij2} \le 0 \qquad\qquad (P2) \\
& T_{i3} - T_{j3} + D_{ij3} \le 0 \\
& T_{n1} - T_{11} \le Duedate_1 \\
& T_{n2} - T_{12} \le Duedate_2 \\
& T_{n3} - T_{13} \le Duedate_3 \\
& Min Q_{ij} \le Q_{ij} \le Max Q_{ij} \\
& \sum_i t_{ij}(Q_{ij} - Min\, Q_{ij}) = QCost \\
& C_{ij} = \sum_i \sum_j NC_{ij} + s_{ij}*(0.25*\{(ND_{ij1} - x_{ij1}) + 2*(ND_{ij2} - x_{ij2}) + (ND_{ij3} - x_{ij3})\}) + t_{ij}(Q_{ij} - Min\, Q_{ij})
\end{aligned}
$$

3. Solve the above Crisp Linear Programming Problem using LINGO software package.
4. Substitute the obtained values from Crisp Linear Programming Problem in the Fuzzy Linear Programming Problem, the solution of (P) can be obtained.

6 NUMERICAL ILLUSTRATION

To illustrate the developed Mathematical model, consider the simple example project used by RaviShankar [19] and depicted by the precedence network shown in Fig. 1. Table 1 presents the description of the project. Critical path of the project is 1-3-4-6-7-8 and project fuzzy duration is (99, 103, 107) days and indirect cost of the project is Rs. 2000. In this project, time parameter and starting time variables are considered in triangular fuzzy number form. Activities information is given in Table 2 and Table 3.

Table 1: Project Description

Activity	Description	Activity	Description
$1 \to 2$	Plan approval	$4 \to 5$	Raising walls from foundation to windows level
$1 \to 3$	Site preparation	$4 \to 6$	Making doors, windows and fitting them
$1 \to 4$	Laying foundation	$5 \to 7$	Roofing
$2 \to 5$	Sanitary work	$5 \to 8$	Electrical wiring
$3 \to 4$	Raising walls from foundation to windows level	$6 \to 7$	Plastering
$3 \to 6$	Interior arrangements	$7 \to 8$	White washing

Fig. 1 Project Network:

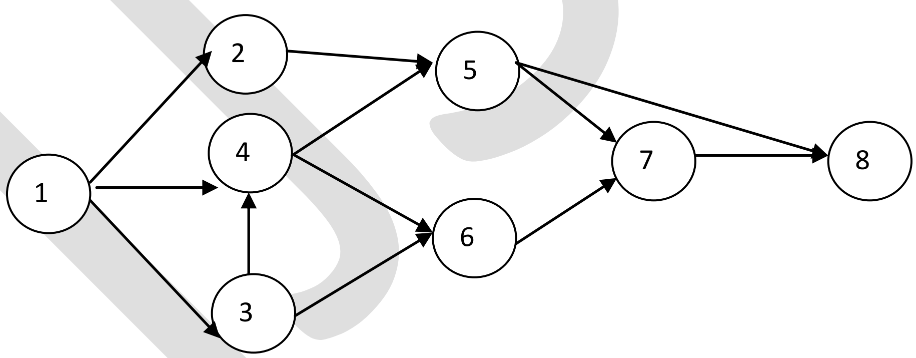


Table 2: Details of Fuzzy time and Cost

Activity	Normal Duration	Crash Duration	Normal Cost, Crash Cost, Slope Cost
$1 \to 2$ (A)	(12, 14, 16)	(9, 10, 11)	10000, 16000, 1500
$1 \to 3$ (B)	(17, 19, 21)	(15, 17, 19)	10000, 12000, 1000
$1 \to 4$ (C)	(17, 18, 19)	(14, 15, 16)	40000, 45400, 1800
$2 \to 5$ (D)	(12, 15, 18)	(11, 13, 15)	2000, 4400, 1200
$3 \to 4$ (E)	(18, 18, 18)	(15, 15, 15	160000, 175000, 5000
$3 \to 6$ (F)	(17, 19, 21)	(15, 16, 17)	21000, 24900, 1300
$4 \to 5$ (G)	(20, 22, 24)	(18, 20, 22)	40000, 46000, 3000

$4 \rightarrow 6$ (H)	(20, 24, 28)	(20, 24, 28)	12000, 12000, nil
$5 \rightarrow 7$ (I)	(27, 27, 27)	(24, 24, 24)	50000, 54500, 1500
$5 \rightarrow 8$ (J)	(18, 20, 22)	(14, 16, 18)	20000, 22000, 500
$6 \rightarrow 7$ (K)	(21, 22, 23)	(17, 18, 19)	14000, 19000, 1250
$7 \rightarrow 8$ (L)	(17, 18, 19)	(15, 15, 15)	7000, 11500, 1500

Table 3: Details of Quality of the Sample Project

Activity	**Max. Quality**	**Min. Quality**	**Cost per quality increase**
$1 \rightarrow 2$ (A)	80	60	250
$1 \rightarrow 3$ (B)	85	95	250
$1 \rightarrow 4$ (C)	75	50	400
$2 \rightarrow 5$ (D)	90	80	120
$3 \rightarrow 4$ (E)	95	80	1000
$3 \rightarrow 6$ (F)	88	76	400
$4 \rightarrow 5$ (G)	90	80	200
$4 \rightarrow 6$ (H)	92	92	-
$5 \rightarrow 7$ (I)	95	90	500
$5 \rightarrow 8$ (J)	91	82	125
$6 \rightarrow 7$ (K)	94	88	500
$7 \rightarrow 8$ (L)	92	84	300

The complete Fuzzy Linear Mathematical Model (1) of the example has been formulated. The Fuzzy Linear Model on hand consists of 81 variables and 150 constraints.

$$
\begin{aligned}
& Min \quad \sum_{i=1}^{7}\sum_{j=2}^{8} C_{ij} + \sum_{n} K_n + I * \tilde{T}_8 \\
& Subject\,to \\
& \tilde{T}_1 = 0 \\
& \tilde{T}_j - \tilde{T}_i - \tilde{D}_{ij} \geq 0, \quad i, j = 1,2,\ldots 8 \\
& \tilde{T}_8 \leq \tilde{T} \\
& C\tilde{D}_{ij} \leq \tilde{D}_{ij} \leq N\tilde{D}_{ij}, \quad i, j = 1,2\ldots 8 \qquad (P_1) \\
& C_{ij} = NC_{ij} + s_{ij}(N\tilde{D}_{ij} - \tilde{D}_{ij}) + t_{ij}(Q_{ij} - Min\,Q_{ij}) \\
& Min\,Q_{ij} \leq Q_{ij} \leq Max\,Q_{ij}, \quad i, j = 1,2\ldots 8 \\
& \sum_{i=1}^{7}\sum_{j=2}^{8} t_{ij}(Q_{ij} - Min\,Q_{ij}) = QCost
\end{aligned}
$$

The project requires a total duration of (99, 103, 107) days to complete if all the activities are performed at their normal durations. However, the all crash solution produces a project completion time of (86, 89, 92) days. The all normal and all crash project durations are the two extreme project time limits. The project indirect cost is assumed to be Rs. 2000 per day. Procedure to solve fuzzy time cost trade off problems presented in section should be used in

order to solve this problem. As mentioned in step 2, problem (P_1) can be changed in to Crisp Linear Programming Problem (P_2) using ranking function mentioned in step 2.
Similarly, the Complete Fuzzy Linear Mathematical Model (2) of the example has been formulated. The Fuzzy Linear Model (2) on hand consists of 81 variables and 150 constraints. The values of minimum total cost and planned fuzzy duration and planned quality of the project have been determined using LINGO solver. A computer package called LINGO (LINGO 2000) is used on a personal computer to solve the mathematical model of the example project. LINGO is a commercial package using the power of linear and non-linear optimization to formulate large problems concisely, solve them, and analyze the solution. In all tested runs, the linear mathematical model of the example project requires less than one second on LINGO to obtain the optimal solution.

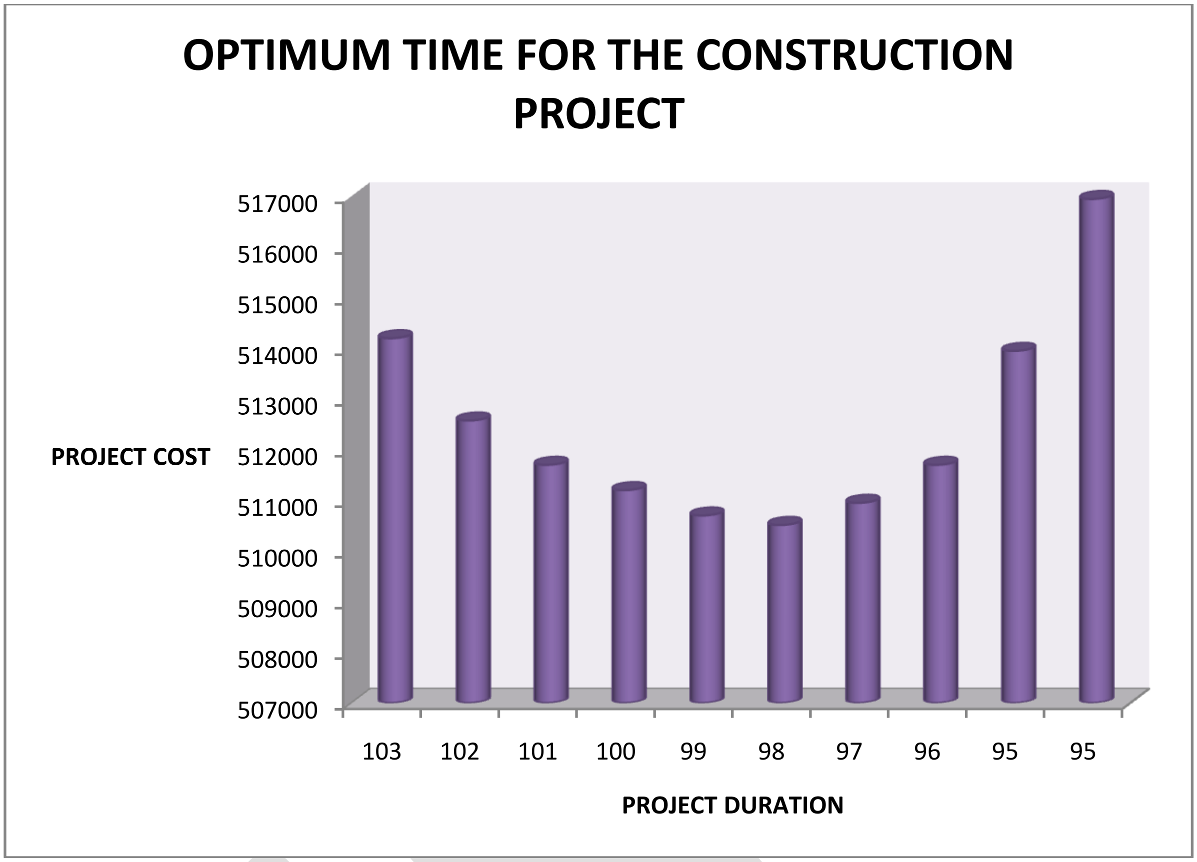
6.1 RESULT ANALYSIS

In the present fuzzy time cost and quality trade off problem, the time parameter and starting time of activity and specified project completion time are defined as triangular fuzzy numbers. The sample project is solved easily using Fuzzy Linear Programming technique and the computational results are tabulated. It is observed from the results obtained from Table 4 that the optimal total cost of this project is obtained at (94, 98, 102) days.

Table 4: Obtained Total Project Cost at different Fuzzy Duration

Project Fuzzy duration	Project Cost
(99, 103, 107)	5,14,200
(98, 102, 106)	5,12,575
(97, 101, 105)	5,11,700
(96, 100, 104)	5,11,200
(95, 99, 103)	5,10,700
(94, 98, 102)	**5,10,513**
(93, 97, 101)	5,10,950
(92, 96, 100)	5,11,700
(91, 95, 99)	5,13,950
(90, 94, 98)	5,16,950

The results clearly reveal that Fuzzy Time Cost and Quality Trade off Problem can be solved effectively by using Fuzzy Linear Programming model. The proposed model has the advantage of saving time, cost and computing effort. Therefore, the Linear Programming approach is an efficient way to obtain an optimal solution to the Fuzzy Time Cost and Quality trade off problems.



The above concept can also be extended to the following two models that concerns cost minimization and quality maximization as given below:

Model 2:

$$Min \quad T_n - T_1$$
$$subject \quad to$$
$$\tilde{T}_1 = 0$$
$$CD_{ij} \le D_{ij} \le ND_{ij}$$
$$T_i - T_j + D_{ij} \le 0$$
$$Min\,Q_{ij} \le Q_{ij} \le Max\,Q_{ij}$$
$$\sum_i t_{ij}(Q_{ij} - Min\,Q_{ij}) = QCost$$
$$\sum_i \sum_j C_{ij} + \sum_n K_n + ICost * Duedate \le C$$
$$C_{ij} = \sum_i \sum_j NC_{ij} + s_{ij}(ND_{ij} - x_{ij}) + t_{ij}(Q_{ij} - Min\,Q_{ij})$$

Model 3:

$$
\begin{aligned}
& Max \quad \frac{\sum_{i,j} Q_{ij}}{n} \\
& Subject\,to \\
& \tilde{T}_1 = 0 \\
& \tilde{T}_j - \tilde{T}_i - \tilde{D}_{ij} \geq 0, \quad i, j = 1,2,\ldots n \\
& \sum_{i=1}^{n-1}\sum_{j=2}^{n} C_{ij} + \sum_{n} K_n + I * \tilde{T}_n \leq C \\
& C\tilde{D}_{ij} \leq \tilde{D}_{ij} \leq N\tilde{D}_{ij}, \quad i, j = 1,2\ldots n \\
& C_{ij} = NC_{ij} + s_{ij}(N\tilde{D}_{ij} - \tilde{D}_{ij}) + t_{ij}(Q_{ij} - Min\,Q_{ij}) \\
& \tilde{T}_n - \tilde{T}_1 \leq \tilde{T} \\
& Min\,Q_{ij} \leq Q_{ij} \leq Max\,Q_{ij}, \quad i, j = 1,2\ldots n \\
& \sum_{i=1}^{n-1}\sum_{j=2}^{n} t_{ij}(Q_{ij} - Min\,Q_{ij}) = QCost
\end{aligned}
$$

7 CONCLUSIONS

A fuzzy mathematical model has been developed which links the CPM with least cost optimization, mathematical programming in order to optimize the traditional time cost and quality trade off problem. The developed model is a stand-alone piece of generic technique which may well be applied to projects of any kind; provided the projects can be defined within the boundaries of the techniques used (i.e. project is being divided into precedence related activities, each with normal and crash time and cost data.)

This paper investigated the time cost and quality trade off problem in the project network with several fuzzy parameters which makes this problem being complicated. The underlying idea is based on linear programming formulation.

The proposed model is also applicable to more complicated project networks in real world. Clearly, the proposed approach is not confined to the fuzzy parameters of triangular type. This is illustrated by successfully solving an example with fuzzy parameters. Other types such as trapezoidal type and interval type are also applicable. The proposed model suits well for the fuzzy time cost trade off problem involving both direct costs and indirect costs.

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