International Journal of Advanced Scientific and Technical Research Issue 5 volume 2, March.-April.2015
Available online on http://www.rspublication.com/ijst/index.html ISSN 2249-9954

Soft strongly generalized closed set with respect to an ideal in soft topological space

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ABSTRACT

In this paper, we initiate the study the concepts such as soft strongly generalized closed set with respect to an and exhibit some results related to these concepts and the relationship between the soft strongly generalized closed set with respect to an ideal and the other types of soft sets in soft topological spaces with several related properties are investigated.

Key words: SSIg-closed set, SSIg-open set, Ig-closed set, Soft g-closed set.

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Corresponding Author: This paper is a part of M.Sc. Thesis of Alyasaa supervised by Assist Prof Dr. Narjis.

1. INTRODUCTION

In 1999, Molodtsov initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modelling the problems with incomplete information in engineering, physics, computer science, economics, social sciences and medical sciences. Soft set theory does not require the specification of parameters. Instead, it accommodates approximate description of an object as its starting point which makes it a natural mathematical formalism for approximate reasoning. So the application of soft set theory in other disciplines and real life problems are now catching moment. In this research we mixed the concept of soft topological space with the concept of strongly generalized closed set with respect to an ideal to get the new soft set, which we called the soft strongly generalized closed set with respect to an ideal (denoted by SSIg-closed set).

2. PRELIMINARIES

Definition 2.1 ([2]). For $A \subseteq E$, the pair (F,A) is called a Soft Set over X, where F is a mapping given by $F{:}A \rightarrow P(X)$. In other words, the soft set is a parametrized family of subsets of the set X. Every set $F(e)$, $e \in E$, from this family may be considered as the set of e-elements of the soft set (F,E) , or as the set of e-approximate elements of the soft set. Clearly, a soft set is not a set. As an illustration, let us consider the following examples.

Note 2.2([1]). In what follows by $SS(X,E)$ we denote the family of all soft sets (F,E) over X.

Definition 2.3 ([1]). For two soft sets *(F,A)* and *(G,B)* in *SS(X,E)* , we say that *(F, A)* is a soft subset of *(G, B)* if $A \subseteq B$ and $F(e) \subseteq G(e), \forall e \in E$.

Also, we say that the pairs *(F,A)* and *(G,B)* are soft equal if *(F,A)* $\tilde{\subseteq}$ *(G,B)* and *(G,B)* $\tilde{\subseteq}$ *(F,A)*. Symbolically, we write *(F,A) = (G,B)*.

Definition 2.4 ([5]). The union of two soft sets (F, A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cup B$ and for all $e \in C$,

$$H(e)=\begin{cases} F(e) & ,e\in A-B, \\ G(e) & ,e\in B-A, \\ F(e)\cup G(e) & ,e\in A\cap B. \end{cases}$$

Definition 2.5 ([5]). The intersection of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F,E) over a universe X in which all the parameter set E are same.

Definition 2.6 ([3]). The complement of a soft set (F,E), denoted by $(F,E)^c$ is defined by $(F,E)^c=(F^c,E)$, $F^c: E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$, $\forall e \in E$ and F^c is called the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F,E)^c)^c = (F,E)$.

Definition 2.7 ([3]). The difference of two soft sets (F,E) and (G,E) over the common universe X, denoted by (F,E)−(G,E) is the soft set (H,E) where for all $e \in E$, $H(e) = F(e) - G(e)$.

Definition 2.8 ([5]). Let (F,E) be a soft set over X and $x \in X$. We say that $x \tilde{\in} (F,E)$ read as x belongs to the soft set (F,E), whenever $x \in F(\alpha)$ for all $\alpha \in E$. Note that for $x \in X$, $x \tilde{\notin} (F,E)$ if $x \notin F(\alpha)$ for some $\alpha \in E$.

Definition 2.9 ([2]). A soft set (F,A) over X is said to be a null soft set, denoted by ϕ_A, if for all $e \in A$, $F(e)=\phi$ (null set), where $\phi_A(e) = \phi \quad \forall e \in A$

Definition 2.10 ([3]). A soft set (F,A) over X is said to be an absolute soft set, denoted by X_A, if for all $e \in A$, $F(e)= X$. Clearly, we have $X_A{}^c = \phi_A$ and $\phi_A{}^c = X_A$.

Definition 2.11 ([4]) . Let τ be a collection of soft sets over X with the fixed set E of parameters , then $\tau \subseteq$ SS(X,E). We say that the family τ defines a soft topology on X if the following axioms are true :

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1- X_A , $\phi_A \in \tau$,

2- If (G,*A*) , (H,*A*) $\in \tau$, then (G,*A*)$\tilde{\cap}$(H,*A*) $\in \tau$,

3- If $(G_i, A) \in \tau$ for every $i \in \Lambda$, then $\tilde{\bigcup}_{i \in \Lambda} (G_i, A) \in \tau$.

Then τ is called a Soft Topology on X and the triple (X,τ,E) is called Soft Topological Spaces over X .

Definition 2.12 ([2]). Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X,E)$. Define $\tau_{(F,E)} = \{(G,E) \tilde{\cap} (F,E) : (G,E) \in \tau\}$, which is a soft topology on (F,E). This soft topology is called soft relative topology on (F,E), then $[(F,E), \tau_{(F,E)}]$ is called soft subspace of (X,τ,E).

Definition 2.13 ([1]). Let (X, τ, E) be a soft topological space and $(F,E) \in SS(X,E)$. The soft closure of (F,E), denoted by $cl(F,E)$ is the intersection of all closed soft super sets of (F,E) that to say $cl(F,E) = \tilde{\cap} \{(H,E)$; (H,E) *is* closed soft set and $(F,E) \tilde{\subseteq} (H,E)\}$.

Definition 2.14 ([2]). Let (X, τ ,E) be a soft topological space and $(G,E) \in SS(X,E)$. The soft interior of (G,E), denoted by $int(G,E)$ is the union of all open soft subsets of (G,E) that to say $int(G,E) = \tilde{\cup} \{(H,E)$; (H,E) is an open soft set and $(H,E) \tilde{\subseteq} (G,E)\}$.

Definition 2.15 : Let (A,E) be a soft set in a soft topological space (X,τ,E) with an ideal I, Then the border of (A,E) defined by $b(A,E) = (A,E) \tilde{\cap} cl(A,E)^c$. and denoted it by $b(A,E)$

Definition 2.16 ([2]). Two soft sets (A,E) and (B,E) are said to be soft separated in a soft topological space (X,τ,E) if $cl(A,E) \tilde{\cap} (B,E) = \phi_E$ and $(A,E) \tilde{\cap} cl(B,E) = \phi_E$.

Definition 2.17 ([2]). Let (A,E) be a soft set over X and (X,τ,E) be soft topological space . Then the boundary of (A,E) denoted by $bd(A,E)$ and defined as $bd(A,E) = cl(A,E) \tilde{\cap} cl(A,E)^c$.

Definition 2.18 ([5]). A soft set (A,E) is called a soft generalized closed (soft g-closed) set in a soft topological space (X,τ,E) if $cl(A,E) \tilde{\subseteq} (U, E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft open set in X . The relative complement *of* (A,E) is called a soft generalized open (soft g-open) set.

Definition 2.19 ([5]). Let E be a set of parameters, A nonempty collections I of soft subsets over X is called a soft ideal on X if the following holds

(1) If $(F,A) \in I$ and $(G,B) \tilde{\subseteq} (F,A)$ implies $(G,B) \in I$ (heredity),

(2) If (F,A) and $(G,A) \in I$, then $(F,A) \tilde{\cup} (G,A) \in I$ (additivity).

If I is ideal on X and Y is subset of X, then $I_Y = \{ Y_E \tilde{\cap} I_1 : I_1 \in I\}$ is an ideal on Y.

Proposition 2.20. Let (X,τ_1,E_1) and (Y,τ_2,E_2) be two soft topological spaces with ideals I_1 and I_2 respectively. Then $I_1 \tilde{\times} I_2 = \{(V,E_1) \tilde{\times} (U,E_2)$; $(V,E_1) \tilde{\in} I_1$,$(U,E_2) \tilde{\in} I_2\}$ is an ideal on the product soft topological space $(X \times Y, \tau_1 \times \tau_2, E_1 \times E_2)$.

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Proof. Let $(V,E_1)\tilde{\times}(U,E_2)$, $(V_1,E_1)\tilde{\times}(U_1,E_2)\tilde{\in}\ I_1\tilde{\times}I_2$.

Then $(V,E_1)\tilde{\times}(U,E_2)\tilde{\cup}(V_1,E_1)\tilde{\times}(U_1,E_2)= (V,E_1)\tilde{\cup}(V_1,E_1)\tilde{\times}(U_1,E_2)\tilde{\cup}(U_1,E_2)\in I_1\tilde{\times}I_2$.

If $(A,E_1)\tilde{\times}(B,E_2)\tilde{\subseteq}(V,E_1)\tilde{\times}(U,E_2)$, then $(A,E_1)\tilde{\times}(B,E_2)\in\ I_1\tilde{\times}I_2$.

Definition 2.21 ([4]). Let (X,τ,E) be a soft topological space with an ideal I . A soft set $(F,E)\in SS(X,E)$ is called soft generalized closed set with respect to an ideal I (soft Ig-closed) if $cl(F,E)-(G,E)\in I$ whenever $(F,E)\tilde{\subseteq}(G,E)$ and $(G,E)\in\tau$. The relative complement $(F,E)^c$ is called soft generalized open set with respect to an ideal I (soft Ig-open).

3. SOFT STRONGLY GENERALIZED CLOSED SETS WITH RESPECT TO AN IDEAL IN SOFT TOPOLOGICAL SPACES.

Definition 3.1. Let (X,τ,E) be a soft topological space with an ideal I. A soft subset (A,E) of (X,τ,E) is said to be soft strongly generalized closed set with respect to an ideal, (briefly SSIg-closed), if $cl(int(A,E))-(B,E)\in I$ whenever $(A,E)\tilde{\subseteq}(B,E)$ and (B,E) is soft open set. the relative complement $(F,E)^c$ is Soft strongly generalized open sets with respect to an ideal I, ,(briefly SSIg- closed), in soft topological space .

Example 3.2. Let X={a, b, c} be the set of three cars under consideration and, E={ e_1(costly), e_2(Luxurious)}. Let (A,E), (B,E), (C,E) be three soft sets representing the attractiveness of the car which Mr. X, Mr. Y and M. Z are going to buy, $\tau=\{\phi_E\ ,\ X_E\ ,(A,E)\ ,(B,E),(C,E)\ \}$ where $(A,E)=\{(e_1,\{b\})\ ,(e_2,\{a\})\ \}$, $(B,E)=\{(e_1,\{b,c\})\ ,(e_2,\{a,b\})\ \}$ and $(C,E)=\{(e_1,\{a,b\})\ ,\ (e_2,\{a,c\})\ \}$. Let $I=\{\phi_E\ ,(M,E)\ ,\ (H,E)\ ,\ (D,E)\ \}$ where $(M,E)=\{(e_1,\{a\}),(e_2,\phi)\ \}$and $(H,E)=\{(e_1,\ \phi),(e_2,\{c\})\ \}$ and $(D,E)=\{(e_1,\{a\}),(e_2,\{c\})\}$.

Now $(B,E)\subseteq(B,E)$ and (B,E) is soft open set. Then $int(B,E)=(B,E)$ and $cl(B,E)\ =\ X_E$. Therefore, $cl(int(B,E))-(B,E)=X_E-(B,E)=(B,E)^c=(D,E)\in I$. Hence, $cl(int(B,E))-(B,E)\ \in I$. Thus (B,E) is soft strongly generalized closed set with respect to an ideal I.

On the other hand $(A,E)\tilde{\subseteq}(A,E)$ and (A,E) is soft open set . Then $int(A,E)=(A,E)$ and $cl(A,E)\ =\ X_E$. Therefore, $cl(int(A,E))-(A,E)\ =\ X_E-(A,E)=(A,E)^c\notin I$. Hence (A,E) is not soft strongly generalized closed set with respect to an ideal I . ■

Proposition 3.3. Let (X,τ,E) be a soft topological space with an ideal I. Then every soft closed set is an SSIg-closed set.

Proof. Let (A,E) be a soft closed set in a soft topological space (X,τ,E) with an ideal I.

Let (B,E) be any soft open set in (X,τ,E) such that $(A,E)\tilde{\subseteq}(B,E)$. By definition of interior then $int(A,E)\tilde{\subseteq}(A,E)$, also By definition of closure and since (A,E) is soft closed set then *cl(int(A,E))*$\tilde{\subseteq}$*cl(A,E)= (A,E)*$\tilde{\subseteq}$*(B,E). Hence cl(int(A,E))-(B,E)*$\tilde{\subseteq}$*(A,E)-(B,E)=* $\phi_E \in I$. *Thus (A,E)* is SSIg-closed set. ■

Corollary 3.4. Let (X,τ,E) be a soft topological space with an ideal *I*. Then every soft open set is an SSIg-open set.

Proof. It is clear by Proposition 3.3. ■

Corollary 3.5. Every soft subset of a soft discrete topological space with respect to an ideal I is an SSIg-closed set .

Proof. Since every soft set in soft discrete topological space is soft closed set, so it is an SSIg-closed set by Proposition 3.3. ■

Remark 3.6. Let (X,τ,E) be a soft topological space with an ideal *I*. Then X_E and ϕ_E are SSIg-closed set.

Proof. It is clear by Proposition 3.3. ■

Theorem 3.7. Every soft g-closed set is a soft strongly generalized closed set with respect to an ideal I .

Proof. Let *(A,E)* be soft g- closed set. Suppose that *(A,E)*$\tilde{\subseteq}$*(B,E)* and *(B,E)* is soft open set . Since *(A,E)* is a soft g- closed set by hypotheses . Then *cl(A,E)*$\tilde{\subseteq}$*(B,E)* **.** Since *int(A,E)* $\tilde{\subseteq}$*(A,E)* then *cl(int(A,E))*$\tilde{\subseteq}$*cl(A,E)*$\tilde{\subseteq}$*(B,E),* therefore *cl(int(A,E))- (B,E)* = $\phi_E \in I$, hence *cl(int(A,E))- (B,E)*$\tilde{\in}$*I* whenever *(A,E)*$\tilde{\subseteq}$ *(B,E)* and *(B,E)* is soft open set . Hence, *(A,E)* is a soft strongly generalized closed set with respect to an ideal *I* . ■

Remark 3.8. The converse of the Theorem(3.7) need not to be true by the following example . **Example.** Let X= {a,b,c} and E = $\{e_1, e_2\}$ and τ = $\{\phi_E, X_E$ *,(A,E)*,(B,E) } where *(A,E)* = $\{(e_1,\{a\}),(e_2,X)\}$, where *(B,E)* = $\{(e_1,\{a,b\}), (e_2,X)\}$. Let $I = \{\phi_E$ *, (C,E) , (H,E) , (D,E)* } where *(C,E)*=$\{(e_1,\{b\}), (e_2,\phi)\}$ and (H,E)=$\{(e_1,\{b,c\}), (e_2,\phi)\}$ and (D,E)=$\{(e_1,\{c\}), (e_2,\phi)\}$. Now *(A,E)*$\tilde{\subseteq}$*(A,E)* and *(A,E)* is soft open set . Then, *(A,E) = int(A,E)* and *cl(A,E)* = X_E. Therefore, *cl(int(A,E))-(A,E)=* X_E *-(A,E)=(A,E)*^{c}*=(H,E)*$\in$*I.* Hence, *cl(int(A,E))-(A,E)*$\tilde{\in}$*I.* Thus, *(A,E)* is soft strongly generalized closed set with respect to an ideal *I.* But *cl(A,E)* = $X_E \tilde{\not\subseteq}$ *(A,E)* for *(A,E)*$\tilde{\subseteq}$*(A,E)* and *(A,E)* is soft open set . Therefore *(A,E)* is not soft g-closed set .■

Corollary 3.9. Every soft g-open set is a soft strongly generalized open set with respect to an ideal I .

Proof. It is clear by Proposition 3.5.■

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Theorem 3.10. Every soft Ig- closed set is a soft strongly generalized closed set with respect to a soft ideal I .

Proof. Let *(A,E)* be soft Ig- closed set. Suppose that *(A,E)* $\tilde{\subseteq}$ *(B,E)* and *(B,E)* is soft open set . Since *(A,E)* is a soft Ig- closed set by hypotheses. Then *cl(A,E)-(B,E)*$\in$*I*. Since *int(A,E)* $\tilde{\subseteq}$ *(A,E)*. Then, *cl(int(A,E))*$\tilde{\subseteq}$*cl(A,E)*. Therefore *cl(int(A,E))- (B,E)*$\tilde{\subseteq}$*cl(A,E)-(B,E)*$\tilde{\in}$*I*. By definition of an ideal we get *cl(int(A,E))- (B,E)*$\in$*I*. Hence, *cl(int(A,E))- (B,E)* $\in$*I* whenever *(A,E)* $\tilde{\subseteq}$ *(B,E)* and *(B,E)* is soft open set. Thus, *(A,E)* is a soft strongly generalized closed set with respect to an ideal *I*.■

Remark 3.11. The converse of the Theorem(3.10) need not to be true by the following example .

Example. Let X= {a,b,c} and E = $\{e_1, e_2\}$ and τ =$\{\phi_E, X_E$,*(A,E),(B,E)*}, where *(A,E)* and *(B,E)* be a soft sets such that *(A,E)*={(e_1,{a}),(e_2,*X*) }, *(B,E)*={(e_1,{a,c}),(e_2,X)}. Let I = {ϕ , (C,E) , (H,E) , (D,E)} where *(C,E)*={(e_1,{b}) , (e_2,ϕ) }, (H,E)={(e_1,{a,b}) , (e_2,ϕ) }, (D,E)={(e_1,{a}),(e_2,ϕ)}. Now *(D,E)*$\in$ *SS(X,E)* and *(D,E)*$\tilde{\subseteq}$*(A,E)* and *(A,E)* is soft open set . Then, *int(D,E)* = ϕ_E . So , *cl(int(D,E))* = ϕ_E . Therefore, *(D,E)* is soft strongly generalized closed set with respect to an ideal *I*. But *cl(D,E)* = X_E . Then *cl(D,E)- (A,E)* = X_E *-(A,E)* = *(A,E)*^{c} $\notin$ *I*. Therefore *(D,E)* is not soft Ig-closed set .■

Corollary 3.12. Every soft Ig-open set is a soft strongly generalized open set with respect to a soft ideal *I*.

Proof. It is clear by Proposition(3.7) .■

Theorem 3.13. Let (X,τ,E) be a soft topological space and *I* be an ideal . A soft subset *(A,E)* of (X,τ,E) is a SSIg-closed if and only if *(G,E)*$\tilde{\subseteq}$*cl(int(A,E))-(A,E)* and *(G,E)* is soft closed set implies *(G,E)*$\in$*I* .

Proof. Let *(A,E)* be SSIg- closed set . Assume that *(G,E)* $\tilde{\subseteq}$*cl(int(A,E))-(A,E)* and *(G,E)* is soft closed set. Then *(G,E)* $\tilde{\subseteq}$ X_E *-(A,E)* ,then *(A,E)* $\tilde{\subseteq}$ X_E *-(G,E)*. Therefore, X_E *-(G,E)* is soft open set and *(A,E)* $\tilde{\subseteq}$ X_E *-(G,E)*. But *(A,E)* is SSIg-closed set then *cl(int(A,E))–(* X_E *-(G,E))*$\tilde{\in}$*I*. But *(G,E)*$\tilde{\subseteq}$*cl(int(A,E))-(A,E)*$\tilde{\subseteq}$*cl(int(A,E))-(* X_E *-(G,E))*$\tilde{\in}$*I* . By definition of an ideal *I* we get *(G,E)*$\in$*I*.

Conversely ,

Assume that *(G,E)*$\tilde{\subseteq}$*cl(int(A,E))-(A,E)* and *(G,E)* is soft closed set implies *(G,E)*$\in$*I*. We need to prove that *(A,E)* is a SSIg-closed set. Suppose that *(A,E)*$\tilde{\subseteq}$*(G,E)* and *(G,E)* is a soft

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open set. Then *cl(int(A,E))-(G,E)* = *cl(int(A,E))* $\tilde{\cap}$ X_E *-(G,E)*. Since *(G,E)* is soft open set ,then X_E *-(G,E)* is soft closed set so for this *cl(int(A,E))* $\tilde{\cap}$ *(* X_E *-(G,E)* is soft closed set which is contained in *cl(int(A,E))-(G,E)*. By hypothecs *cl(int(A,E))-(G,E)∈I*. Therefore *(A,E)* is a SSIg-closed set . ∎

Theorem 3.14. Let (X,τ,E) be a soft topological space and *I* be an ideal . If a soft subset *(A,E)* of (X,τ,E) is a SSIg-closed and if *cl(int(A,E))-(A,E)* contains a soft closed set *(G,E)* , then *cl(int(A,E))* $\tilde{\cap}$ *(G,E)∈I* .

Proof. Assume that *(G,E)* is soft closed set such that *(G,E)* $\tilde{\subseteq}$ *cl(int(A,E))-(A,E)* .

Then *(G,E)* $\tilde{\subseteq}$ X_E *-(A,E)*, so *(A,E)* $\tilde{\subseteq}$ X_E *-(G,E)*. Therefore, X_E *-(G,E)* is soft open set and *(A,E)* $\tilde{\subseteq}$ X_E *-(G,E)*. But *(A,E)* is SSIg-closed set. Then *cl(int(A,E))–(* X_E *-(G,E))* *∈I*. Therefore , *cl(int(A,E))* $\tilde{\cap}$ *(G,E)∈I* . ∎

Remark 3.15. The converse of the Theorem(3.13) need not to be true by the following example .

Example. Let X= {a,b,c} and E = { e_1, e_2 }. τ ={ ϕ_E , X_E ,*(A,E),(B,E)*} such that *(A,E), (G,E)* and *(B,E)* be a soft sets such that *(A,E)*={(e_1 ,{a}),(e_2 ,{a,b}) }, *(B,E)*={(e_1 ,{a,c}),(e_2 ,X)}. Let *I*={ ϕ , *(G,E)*} where *(G,E)*={(e_1 ,{b}),(e_2 , ϕ) }, then τ^c = { ϕ_E , X_E , $(A,E)^c$, $(B,E)^c$ }. Now *int(A,E)= (A,E)*, so *cl(int(A,E))=* X_E and since *(A,E)* is soft open set then *(A,E)* $\tilde{\subseteq}$ *(A,E)*, but *cl(int(A,E))- (A,E)=* $(A,E)^c \notin I$, therefore *(A,E)* is not SSIg-closed. But on the other hand we have *(G,E)=* $(B,E)^c$, which it is soft closed set such that *(G,E)* $\tilde{\subseteq}$ *cl(int(A,E))- (A,E)* and *cl(int(A,E))))* $\tilde{\cap}$ *(G,E)= (G,E)∈ I*. ∎

Theorem 3.16. Let (X,τ,E) be a soft topological space and with an ideal *I*. If any soft subset *(A,E)* which is an SSIg-closed and *(A,E)* $\tilde{\subseteq}$ *(G,E)* $\tilde{\subseteq}$ *cl(int(A,E))*, then *(G,E)* is SSIg-closed set.

Proof. Assume that *(A,E)* is an SSIg-closed and *(A,E)* $\tilde{\subseteq}$ *(G,E)* $\tilde{\subseteq}$ *cl(int(A,E))*. To show that *(G,E)* is SSIg-closed set. Suppose that *(G,E)* $\tilde{\subseteq}$ *(F,E)* such that *(F,E)* is soft open set, *but (A,E)* $\tilde{\subseteq}$ *(G,E)* $\tilde{\subseteq}$ *(F,E), then (A,E)* $\tilde{\subseteq}$ *(F,E)* since *(A,E)* is SSIg-closed set, then *cl(int(A,E))-(F,E)* $\tilde{\in}$ *I*. Now *(G,E)* $\tilde{\subseteq}$ *cl(int(A,E))*, then *cl(int(G,E))* $\tilde{\subseteq}$ *cl(int(A,E))*. This implies that *cl(int(G,E))-(F,E)* $\tilde{\subseteq}$ *cl(int(A,E))-(F,E)* $\tilde{\in}$ *I*. Therefore, *cl(int(G,E))-(F,E)* $\tilde{\in}$ *I*. thus *(G,E)* is SSIg-closed set. ∎

Remark 3.17. The converse of the Theorem 3.16. need not to be true by the following example .

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Example. Let X= {a,b,c} and E = $\{e_1, e_2\}$ and τ =$\{\phi_E, X_E\}$, let *(A,E)* and *(G,E)* be a soft sets such that *(G,E)*=$\{(e_1,\{a\}),(e_2,X)\}$, *(A,E)*=$\{(e_1,\{a\}),(e_2,\{a,b\})\}$. Let $I=\{\phi_E\}$. Then both *(A,E)* and *(G,E)* are SSIg-closed sets , but *cl(int(A,E))*= ϕ_E , thus *(A,E)* $\tilde{\subseteq}$ *(G,E)* $\tilde{\not\subseteq}$ *cl(int(A,E))*. On other hand if we consider that τ =$\{\phi_E, X_E, (G,E)\}$, then *(G,E)* is SSIg-closed set and *(A,E)*$\tilde{\subseteq}$*(G,E)*$\tilde{\subseteq}$*cl(int(A,E))*,but *(G,E)* is not SSIg-closed set. ■

Corollary 3.18. Let (X,τ,E) be a soft topological space and *I* be an ideal. If *int(F,E)*$\tilde{\subseteq}$*(G,E)*$\tilde{\subseteq}$ *(F,E)* and if *(F,E)* is SSIg-open set ,then *(G,E)* is SSIg-open set .

Proof. Suppose that *int(F,E)*$\tilde{\subseteq}$ *(G,E)*$\tilde{\subseteq}$*(F,E)* and *(F,E)* is SSIg-open set . We need to show that *(G,E)* is SSIg-open set . Then, X_E*-(F,E)*$\tilde{\subseteq}$ X_E*-(G,E)*$\tilde{\subseteq}$*cl(int(*X_E*-(F,E)))* and X_E-*(F,E)* is SSIg-closed set . By Theorem (3.16) we get X_E*-(G,E)* is SSIg-closed set .Therefore, *(G,E)* is SSIg-open set . ■

Theorem 3.19. Let (X,τ,E) be a soft topological space and I be an ideal . Then *(A,E)* is a SSIg-open set if and only if *(G,E)-(U,E)* $\tilde{\subseteq}$*int(A,E)* for some *(U,E)∈I* whenever *(G,E)* $\tilde{\subseteq}$ *(A,E)* and *(G,E)* is soft closed set .

Proof. Suppose that *(A,E)* be a SSIg-open set and *(G,E)* is soft closed set such that *(G,E)* $\tilde{\subseteq}$ *(A,E)* . We need to show that *(G,E)-(U,E)*$\tilde{\subseteq}$*int(A,E)* for some (*U,E*) *∈I*. Since *(G,E)*$\tilde{\subseteq}$*(A,E)*. Then, X_E*-(A,E)*$\tilde{\subseteq}$ X_E*-(G,E)*, Since X_E*-(A,E)* is SSIg-closed set and *(G,E)* is soft open set. Then, *cl(int(*X_E*-(A,E)))* $\tilde{\subseteq}$ *(*X_E*-(G,E))*$\tilde{\cup}$*(U,E)* for some *(U,E)*$\tilde{\in}$*I* . This implies X_E-*((*X_E*-(G,E))*$\tilde{\cup}$*(U,E))*$\tilde{\subseteq}$ X_E*-cl(int(*X_E*-(A,E)))*. Hence, *(G,E)-(U,E)*$\tilde{\subseteq}$*int(A,E)* for some *(U,E)∈I*.

Conversely,

Assume that *(G,E)*$\tilde{\subseteq}$*(A,E)* and *(G,E)* is soft closed set implies that *(G,E)-(U,E)* $\tilde{\subseteq}$*int(A,E)* for some *(U,E) ∈I*. We need to show that *(A,E)* is a SSIg-open, that to say X_E-*(A,E)* is a SSIg-closed set .

Consider a soft open set *(V,E)* such that X_E*-(A,E)*$\tilde{\subseteq}$*(V,E)* . Then, X_E*-(V,E)*$\tilde{\subseteq}$*(A,E)*. Therefore, *(*X_E*-(V,E))-(U,E)*$\tilde{\subseteq}$*int(A,E)* = X_E*-cl(int(*X_E*-(A,E)))* for some *(U,E) ∈I*. This gives that, X_E*-((V,E)*$\tilde{\cup}$ *(U,E))* $\tilde{\subseteq}$ X_E- *cl(int(*X_E*-(A,E)))*. Then, *cl(int(*X_E*-(A,E))*$\tilde{\subseteq}$ *(V,E)*$\tilde{\cup}$ *(U,E)* for some *(U,E) ∈I*. Therefore, *cl(int(*X_E*-(A,E)) -(V,E) ∈I*. Hence, X_E*-(A,E)* is a SSIg-closed set . Therefore, *(A,E)* is a SSIg-open set . ■

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Theorem 3.20. Let $Y \tilde{\subseteq} X$ and *(A,E)* be a soft set in (Y,τ_Y,E). If *(A,E)* is a SSIg-closed set in (X,τ,E). then *(A,E)* is a SSIg-closed relative to the soft space in (Y,τ_Y,E) with respect to an ideal I_Y .

Proof. Suppose that *(A,E)* be a soft set in (Y,τ_Y,E) such that *(A,E)* is a SSIg-closed set in (X,τ,E). Let $(A,E)\tilde{\subseteq}(B,E)$. Then $(B,E) = (U,E)\tilde{\cap} Y_E$, where *(U,E)* is soft open set in (X,τ,E). Since $(U,E)\tilde{\cap} Y_E \tilde{\subseteq} (U,E)$,then $(A,E)\tilde{\subseteq} (U,E)$. Since *(A,E)* is a SSIg-closed set in (X,τ,E), then $cl(int(A,E))-(U,E)\in I$. Therefore, $cl(int(A,E))-(B,E)=(cl(int(A,E))-((U,E)\tilde{\cap} Y_E) =(cl(int(A,E))-(U,E))\tilde{\cap} Y_E\tilde{\subseteq}(cl(int(A,E))-(U,E)) \in I$. By definition of an ideal we get $(cl(int(A,E)))- (B,E) \in I_Y$. Thus *(A,E)* is an SSIg-closed relative to the soft space in (Y,τ_Y,E) with respect to an ideal I_Y .■

Theorem 3.21. Let (X,τ,E) be a soft topological space and *I* be an ideal. If *(A,E)* and *(G,E)* are two soft subsets of (X,τ,E) which are an SSIg-closed, then $(A,E)\tilde{\cup}(G,E)$ is a SSIg-closed set .

Proof. Suppose that *(A,E)* and *(G,E)* are two SSIg-closed sets . We need to show that $(A,E)\tilde{\cup}(G,E)$ is a SSIg-closed set . Let *(U,E)* be a soft open set such that $(A,E)\cup(G,E) \tilde{\subseteq} (U,E)$. Then $(A,E) \tilde{\subseteq} (U,E)$ and $(G,E) \tilde{\subseteq} (U,E)$. But both of them are SSIg-closed set, so $cl(int(A,E))-(U,E)\in I$ and $cl(int(G,E))-(U,E)\in I$. Then $cl(int((A,E)\tilde{\cup}(G,E)))-(U,E)\tilde{\subseteq} cl(int(A,E)\tilde{\cup} int(G,E))-(U,E) = cl(int(A,E))\tilde{\cup} cl(int(G,E))-(U,E)= cl(int(A,E))-(U,E)\tilde{\cup} cl(int(G,E))-(U,E)$. But $cl(int(A,E))-(U,E)\in I$ *and* $cl(int(G,E))-(U,E) \in I$. By definition of an ideal we get, $cl(int(A,E))-(U,E)\tilde{\cup} cl(int(G,E))-(U,E)\in I$. Then $cl(int((A,E)\tilde{\cup}(G,E)))-(U,E)\in I$. Therefore, $(A,E)\tilde{\cup}(G,E)$ is a SSIg-closed set.■

Remark 3.22. The infinite union of SSIg-closed sets need not to be SSIg-closed set in general as the following example shows.

Example. Let $X= \{1,2,3,....\}$, $E = \{e_1,e_2\}$, $I=\{\phi_E\}$and $\tau = \{X_E,\phi_E\}\tilde{\cup}\{(G_n,E)$; $n=1,2,3,\dots \}$, where, (G_n,E) is a soft sets such that $(G_n,E) =\{(e_1,\{n,n+1,n+2,\dots\}), (e_2,\phi)\}$. Let (H_m,E) be a soft sets such that $(H_m,E) =\{(e_1,\{2,3,4,\dots,m\}), (e_2,\phi)\}$, $n \geq 10$. For each soft open set *(B,E)* such that $(H_m,E)\subseteq(B,E)$, $m \geq 10$.

Then $int(H_m,E) = \phi_E$, $m \geq 10$. Hence $cl(int(H_m,E)) = \phi_E$, $m \geq 10$. Therefore, $cl(int((H_m,E))-(B,E)=\phi_E \in I$, $m \geq 10$. *Thus* (H_m,E) is a SSIg-closed set for each $n \geq 10$.

On the other hand $\tilde{\bigcup}_{m\geq 10}(H_m,E)=\{(e_1,\{2,3,4,\ldots\}),(e_2,\phi)\}=$ *(G₂,E). Then* $(G_2,E)\tilde{\subseteq}(H_2,E)$ and (G_2,E) is soft open set. Then, $int(G_2,E)=(G_2,E)$. Therefore, $cl(int((H_2,E))$-$(H_2,E)=\{(e_1,\{1\}),(e_2,X)\}\notin I$. *Thus,* $\tilde{\bigcup}_{m\geq 10}(H_m,E)$ is not SSIg-closed set.

Corollary 3.23. Let (X,τ,E) be a soft topological space and with an ideal *I*. Then the intersection of two SSIg-open sets is SSIg-open.

Proof. It is clear by Theorem(2.21) .

Remark 3.24. The intersection of two SSIg-closed sets need not to be SSIg-closed set in general as the following example shows.

Example. Let X= {a,b,c} , E = $\{e_1,e_2\}$ and τ = $\{\phi_E, X_E, (A,E)\}$, where *(A,E)* ,*(C,E)* and *(B,E)* are a soft sets such that $(A,E)=\{(e_1,\{b\}),(e_2,\phi)\}$, $(B,E)=\{(e_1,\{a,b\}),(e_2,\phi)\}$ and $(C,E)=\{(e_1,\{b,c\}),(e_2,\phi)\}$. Then $\tau^c=\{\phi_E, X_E, (A,E)^c\}$ where $(A,E)^c=\{(e_1,\{a,c\}),(e_2,X)\}$. Let $I=\{\phi_E\}$. $(B,E)\tilde{\subseteq}X_E$ and X_E is soft open set. Then, $int(B,E)=(A,E)$ and $cl(int(B,E))=cl(A,E)=X_E$. Therefore, $cl(int((B,E))-X_E=X_E-X_E=\phi_E\in I$. Hence, *(B,E)* is a SSIg-closed set $(C,E)\tilde{\subseteq}X_E$. Hence, *(C,E)* is a SSIg-closed set. Now, $(B,E)\tilde{\cap}(C,E)=\{(e_1,\{b\}),(e_2,\phi)\}=(A,E)$ is not SSIg-closed set. Therefore The intersection of two SSIg-closed sets is not SSIg-closed set .■

Theorem 3.25. Let (X,τ,E) be a soft topological space and I be an ideal . Let *(A,E)* be a SSIg-closed set and *(G,E)* is soft closed set in (X,τ,E), Then $(A,E)\tilde{\cap}(G,E)$ is a SSIg-closed set.

Proof. Let *(A,E)* be a SSIg-closed set and *(G,E)* is soft closed set. We need to show that $(A,E)\tilde{\cap}(G,E)$ is a SSIg-closed . Let *(U,E)* be a soft open set such that $(A,E)\tilde{\cap}(G,E)\tilde{\subseteq}(U,E)$. Then, $(A,E)\tilde{\subseteq}(U,E)\tilde{\cup}(X_E-(G,E))$ and $(U,E)\tilde{\cup}(X_E-(G,E))$ is a soft open set. But *(A,E)* is a SSIg-closed set . Then $cl(int(A,E))-\{(U,E)\tilde{\cup}(X_E-(G,E))\}=\{cl(int(A,E))-(U,E)\}\tilde{\cup}\{cl(int(A,E))-(X_E-(G,E))\}\in I$. Therefore, $cl(int(A,E))-(X_E-(G,E))\in I$. Therefore, $cl(int((A,E)\tilde{\cap}(G,E)))\tilde{\subseteq}cl(int((A,E)))\tilde{\cap}(G,E)=(cl(int((A,E)))\tilde{\cap}(G,E))-(X_E-(G,E))$. Hence, $cl(int((A,E)\tilde{\cap}(G,E)))-(U,E)\tilde{\subseteq}(cl(int((A,E)))\tilde{\cap}(G,E))-(U,E)\tilde{\cap}(X_E-(G,E))\tilde{\subseteq}cl(int((A,E))-((U,E)\tilde{\cup}(X_E-(G,E))\in I$. By definition of an ideal we get $cl(int((A,E)\tilde{\cap}(G,E)))-(U,E)\in I$. Thus, $(A,E)\tilde{\cap}(G,E)$ is a SSIg-closed set . ■

Proposition 3.26. Let (X,τ_1,E_1) and (Y,τ_2,E_2) be two soft topological spaces with ideals I_1 and I_2 respectively. If (F,E_1) is an SS I_1g-closed set and (G,E_2) is an SS I_2g-closed set in (X,τ_1,E_1)

and (Y,τ_2,E_2) respectively, then $(F,E_1)\ \tilde{\times}\ (G,E_2)$ is a SS$(I_1\tilde{\times}I_2)$g-closed set in $(X\times Y,\tau_1\times\tau_2,E_1\times E_2)$.

Proof. Let $(V,E_1)\ \tilde{\times}\ (U,E_2)$ be a soft open set in $(X\times Y,\tau_1\times\tau_2,E_1\times E_2)$ such that $(F,E_1)\tilde{\times}(G,E_2)\tilde{\subseteq}(V,E_1)\tilde{\times}(U,E_2)$, Then $cl(int((F,E_1)\tilde{\times}(G,E_2)))\text{-}(V,E_1)\tilde{\times}(U,E_2)= cl(int((F,E_1)\tilde{\times}int((G,E_2)))\text{-}(V,E_1)\tilde{\times}(U,E_2)=cl(int((F,E_1)))\tilde{\times}cl(int((G,E_2)))\text{-}(V,E_1)\tilde{\times}(U,E_2)= cl(int((F,E_1)))\ \text{-}(V,E_1)\tilde{\times}cl(int((G,E_2)))\text{-}(U,E_2)\ \tilde{\in}\ I_1\tilde{\times}I_2$. Hence $cl(int((F,E_1)\tilde{\times}(G,E_2)))\text{-}\ (V,E_1)\tilde{\times}(U,E_2)\ \in I_1\tilde{\times}I_2$. Thus, $(F,E_1)\tilde{\times}(G,E_2)$ is a SS$(I_1\tilde{\times}I_2)$g-closed set in $(X\times Y,\tau_1\times\tau_2,E_1\times E_2)$.■

Theorem 3.27. Let (X,τ,E) be a soft topological space and I be an ideal. If *(F,E)* and *(G,E)* are separated soft SSIg-open sets ,then $(F,E)\tilde{\cup}(G,E)$ is SSIg-open set .

Proof. Suppose that *(F,E)* and *(G,E)* are separated soft SSIg-open sets in (X,τ,E) and *(U,E)* be a soft closed subset of $(F,E)\tilde{\cup}(G,E)$. Then , $(U,E)\tilde{\cap}cl(int(F,E))\tilde{\subseteq}(F,E)$ and $(U,E)\tilde{\cap}cl(int(G,E))\tilde{\subseteq}(G,E)$. By hypotheses and Theorem(2.19) we get that ;
$(U,E)\tilde{\cap}cl(int(F,E))\text{-}(V,E)\tilde{\subseteq}int(F,E)$ and $(U,E)\tilde{\cap}cl(int(G,E))\text{-}(M,E)\tilde{\subseteq}int(G,E)$ for some *(V,E)* and *(M,E)* in *I* . This mean that $(U,E)\tilde{\cap}cl(int(F,E))\text{-}int(F,E)\ \in I$ and $(U,E)\tilde{\cap}cl(int(G,E))\text{-}int(G,E)\ \in I$. Hence, $((U,E)\tilde{\cap}cl(int(F,E))\text{-}int(F,E))\tilde{\cup}cl(int(G,E)))\text{-}(int(F,E)\tilde{\cup}int(G,E))\in I$. But , $(U,E) = (U,E)\tilde{\cap}((F,E)\tilde{\cup}(G,E))\tilde{\subseteq}(U,E)\tilde{\cap}cl(int((F,E)\tilde{\cup}(G,E)))$ and we have,$(U,E)\text{-}(int((F,E)\tilde{\cup}(G,E)))\tilde{\subseteq}(U,E)\tilde{\cap}cl(int((F,E)\tilde{\cup}(G,E)))\text{-}int((F,E)\tilde{\cup}(G,E))\tilde{\subseteq}(U,E)\tilde{\cap}cl(int((F,E)\ \tilde{\cup}(G,E)))\text{-}int(F,E)\tilde{\cup}int(G,E)\in I$. Hence , $(U,E)\text{-}(H,E)\tilde{\subseteq}int((F,E)\tilde{\cup}(G,E))$ for some $(H,E)\ \in I$. Therefore, $(F,E)\tilde{\cup}(G,E)$ is SSIg-open set .■

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