

## Almost Slightly $rg$ -continuity

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**Abstract:** In this paper we discuss new type of continuous functions called almost slightly  $rg$ -continuous its properties and interrelation with other such functions are studied.

**Keywords:** almost slightly continuous functions; slightly  $rg$ -continuous functions and almost slightly  $rg$ -continuous functions.

**AMS-classification Numbers:** 54C10; 54C08; 54C05

### Introduction

In 1995 T.M.Nour introduced slightly semi-continuous functions. After him T.Noiri and G.I.Chae further studied slightly semi-continuous functions in 2000. T.Noiri individually studied about slightly  $\beta$ -continuous functions in 2001. C.W.Baker introduced slightly precontinuous functions in 2002. Erdal Ekici and M. Caldas studied slightly  $\gamma$ -continuous functions in 2004. Arse Nagli Uresin and others studied slightly  $\delta$ -continuous functions in 2007. Recently S. Balasubramanian and P.A.S. Vyjayanthi studied slightly  $\nu$ -continuous functions in 2011. Inspired with these developments we introduce in this paper slightly  $rg$ -continuous functions and study its basic properties and interrelation with other type of such functions. Throughout the paper  $(X, \tau)$  and  $(Y, \sigma)$  (or simply  $X$  and  $Y$ ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

### 1. Preliminaries

**Definition 2.1:**  $A \subseteq X$  is called

- (i)  $g$ -closed[ $rg$ -closed] if  $cl A \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (ii)  $b$ -open if  $A \subseteq (cl\{A\})^o \cap cl\{A^o\}$ .

**Definition 2.2:** A function  $f: X \rightarrow Y$  is said to be

- (i) continuous[resp: nearly-continuous;  $rg$ -continuous;  $\nu$ -continuous;  $\alpha$ -continuous; semi-continuous;  $\beta$ -continuous; pre-continuous] if inverse image of each open set is open[resp: regular-open;  $rg$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen].
- (ii) nearly-irresolute[resp:  $rg$ -irresolute;  $\nu$ -irresolute;  $\alpha$ -irresolute; irresolute;  $\beta$ -irresolute; pre-irresolute] if inverse image of each regular-open[resp:  $rg$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen] set is regular-open[resp:  $rg$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen].
- (iii) almost continuous[resp: almost nearly-continuous; almost  $rg$ -continuous; almost  $\nu$ -continuous; almost  $\alpha$ -continuous; almost semi-continuous; almost  $\beta$ -continuous; almost pre-continuous] if for each  $x$

in  $X$  and each open set  $(V, f(x))$ ,  $\exists$  an open[resp: regular-open;  $r\alpha$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen] set  $(U, x)$  such that  $f(U) \subset (cl(V))^o$ .

(iv) weakly continuous[resp: weakly nearly-continuous; weakly  $r\alpha$ -continuous; weakly  $\nu$ -continuous; weakly  $\alpha$ -continuous; weakly semi-continuous; weakly  $\beta$ -continuous; weakly pre-continuous] if for each  $x$  in  $X$  and each open set  $(V, f(x))$ ,  $\exists$  an open[resp: regular-open;  $r\alpha$ -open;  $\nu$ -open;  $\alpha$ -open; semi-open;  $\beta$ -open; preopen] set  $(U, x)$  such that  $f(U) \subset cl(V)$ .

(v) slightly continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly  $\beta$ -continuous; slightly  $\gamma$ -continuous; slightly  $\alpha$ -continuous; slightly  $r$ -continuous; slightly  $\nu$ -continuous] at  $x$  in  $X$  if for each clopen subset  $V$  in  $Y$  containing  $f(x)$ ,  $\exists U \in \tau(X)$  [ $\exists U \in SO(X)$ ;  $\exists U \in PO(X)$ ;  $\exists U \in \beta O(X)$ ;  $\exists U \in \gamma O(X)$ ;  $\exists U \in \alpha O(X)$ ;  $\exists U \in RO(X)$ ;  $\exists U \in \nu O(X)$ ] containing  $x$  such that  $f(U) \subseteq V$ .

(vi) slightly continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly  $\beta$ -continuous; slightly  $\gamma$ -continuous; slightly  $\alpha$ -continuous; slightly  $r$ -continuous; slightly  $\nu$ -continuous] if it is slightly-continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly  $\beta$ -continuous; slightly  $\gamma$ -continuous; slightly  $\alpha$ -continuous; slightly  $r$ -continuous; slightly  $\nu$ -continuous] at each  $x$  in  $X$ .

(vii) almost strongly  $\theta$ -semi-continuous[resp: strongly  $\theta$ -semi-continuous] if for each  $x$  in  $X$  and for each  $V \in \sigma(Y, f(x))$ ,  $\exists U \in SO(X, x)$  such that  $f(scl(U)) \subset scl(V)$ [resp:  $f(scl(U)) \subset V$ ].

### Lemma 2.1:

(i) Let  $A$  and  $B$  be subsets of a space  $X$ , if  $A \in RGO(X)$  and  $B \in RO(X)$ , then  $A \cap B \in RGO(B)$ .

(ii) Let  $A \subset B \subset X$ , if  $A \in RGO(B)$  and  $B \in RO(X)$ , then  $A \in RGO(X)$ .

### 3. Slightly $rg$ -continuous functions:

**Definition 3.1:** A function  $f: X \rightarrow Y$  is said to be almost slightly  $rg$ -continuous at  $x$  in  $X$  if for each  $r$ -clopen subset  $V$  in  $Y$  containing  $f(x)$ ,  $\exists U \in RGO(X)$  containing  $x$  such that  $f(U) \subseteq V$  and almost slightly  $rg$ -continuous if it is almost slightly  $rg$ -continuous at each  $x$  in  $X$ .

**Note 1:** Here after we call almost slightly  $rg$ -continuous function as al.sl. $rg$ .c function shortly.

**Example 3.1:**  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$  and  $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$ . Let  $f$  be defined as  $f(a) = b$ ;  $f(b) = c$  and  $f(c) = a$ , then  $f$  is al.sl. $rg$ .c.

**Theorem 3.1:** The following are equivalent:

- (i)  $f$  is al.sl. $rg$ .c.
- (ii)  $f^{-1}(V)$  is  $rg$ -open for every  $r$ -clopen set  $V$  in  $Y$ .
- (iii)  $f^{-1}(V)$  is  $rg$ -closed for every  $r$ -clopen set  $V$  in  $Y$ .
- (iv)  $f(rgcl(A)) \subseteq rgcl(f(A))$ .

**Corollary 3.1:** The following are equivalent.

- (i)  $f$  is al.sl. $rg$ .c.
- (ii) For each  $x$  in  $X$  and each  $r$ -clopen subset  $V \in (Y, f(x))$   $\exists U \in RGO(X, x)$  such that  $f(U) \subseteq V$ .

**Theorem 3.2:** Let  $\Sigma = \{U_i; i \in I\}$  be any cover of  $X$  by regular open sets in  $X$ . A function  $f$  is al.sl. $rg$ .c. iff  $f_{|U_i}$  is al.sl. $rg$ .c., for each  $i \in I$ .

**Proof:** Let  $i \in I$  be an arbitrarily fixed index and  $U_i \in RO(X)$ . Let  $x \in U_i$  and  $V \in RCO(Y, f_{|U_i}(x))$ . Since  $f$  is al.sl. $rg$ .c,  $\exists U \in RGO(X, x)$  such that  $f(U) \subset V$ . Since  $U_i \in RO(X)$ , by Lemma 2.1  $x \in U \cap U_i \in RGO(U_i)$  and  $(f_{|U_i})U \cap U_i = f(U \cap U_i) \subset f(U) \subset V$ . Hence  $f_{|U_i}$  is al.sl. $rg$ .c.

Conversely Let  $x$  in  $X$  and  $V \in \text{RCO}(Y, f(x))$ ,  $\exists i \in I$  such that  $x \in U_i$ . Since  $f|_{U_i}$  is  $\text{al.sl.rg.c.}$ ,  $\exists U \in \text{RGO}(U_i, x)$  such that  $f|_{U_i}(U) \subset V$ . By Lemma 2.1,  $U \in \text{RGO}(X)$  and  $f(U) \subset V$ . Hence  $f$  is  $\text{al.sl.rg.c.}$

### Theorem 3.3:

- (i) If  $f$  is  $\text{rg-irresolute}$  and  $g$  is  $\text{al.sl.rg.c.}[\text{al.sl.c.}; \text{al.g.c.}]$ , then  $g \circ f$  is  $\text{al.sl.rg.c.}$
- (ii) If  $f$  is  $\text{rg-irresolute}$  and  $g$  is  $g$ -continuous, then  $g \circ f$  is  $\text{al.sl.rg.c.}$
- (iii) If  $f$  is  $\text{rg-continuous}$  and  $g$  is  $\text{al.sl.rg.c.}[\text{al.sl.c.},]$  then  $g \circ f$  is  $\text{al.sl.rg.c.}$

**Theorem 3.4:** If  $f$  is  $\text{rg-irresolute}$ ,  $\text{rg-open}$  and  $\text{RGO}(X) = \tau$  and  $g$  be any function, then  $g \circ f: X \rightarrow Z$  is  $\text{al.sl.rg.c}$  iff  $g$  is  $\text{al.sl.rg.c.}$

**Proof:** If part: Theorem 3.3(i)

Only if part: Let  $A$  be  $\text{r-clopen}$  subset of  $Z$ . Then  $(g \circ f)^{-1}(A)$  is a  $\text{rg-open}$  subset of  $X$  and hence open in  $X$  [by assumption]. Since  $f$  is  $\text{rg-open}$   $f(g \circ f)^{-1}(A)$  is  $\text{rg-open}$   $Y \Rightarrow g^{-1}(A)$  is  $\text{rg-open}$  in  $Y$ . Thus  $g$  is  $\text{al.sl.rg.c.}$

**Corollary 3.2:** If  $f$  is  $\text{rg-irresolute}$ ,  $\text{rg-open}$  and bijective,  $g$  is a function. Then  $g$  is  $\text{al.sl.rg.c.}$  iff  $g \circ f$  is  $\text{al.sl.rg.c.}$

**Theorem 3.5:** If  $g: X \rightarrow X \times Y$ , defined by  $g(x) = (x, f(x))$  for all  $x$  in  $X$  be the graph function of  $f: X \rightarrow Y$ . Then  $g: X \rightarrow X \times Y$  is  $\text{al.sl.rg.c}$  iff  $f$  is  $\text{al.sl.rg.c.}$

**Proof:** Let  $V \in \text{RCO}(Y)$ , then  $X \times V$  is  $\text{r-clopen}$  in  $X \times Y$ . Since  $g$  is  $\text{al.sl.rg.c.}$ ,  $f^{-1}(V) = f^{-1}(X \times V) \in \text{RGO}(X)$ . Thus  $f$  is  $\text{al.sl.rg.c.}$

Conversely, let  $x$  in  $X$  and  $F$  be a  $\text{r-clopen}$  subset of  $X \times Y$  containing  $g(x)$ . Then  $F \cap (\{x\} \times Y)$  is  $\text{r-clopen}$  in  $\{x\} \times Y$  containing  $g(x)$ . Also  $\{x\} \times Y$  is homeomorphic to  $Y$ . Hence  $\{y \in Y: (x, y) \in F\}$  is  $\text{r-clopen}$  subset of  $Y$ . Since  $f$  is  $\text{al.sl.rg.c.}$   $\cup \{f^{-1}(y): (x, y) \in F\}$  is  $\text{rg-open}$  in  $X$ . Further  $x \in \cup \{f^{-1}(y): (x, y) \in F\} \subseteq g^{-1}(F)$ . Hence  $g^{-1}(F)$  is  $\text{rg-open}$ . Thus  $g: X \rightarrow Y$  is  $\text{al.sl.rg.c.}$

**Theorem 3.6:** (i)  $f: \prod X_\lambda \rightarrow \prod Y_\lambda$  is  $\text{al.sl.rg.c.}$  iff  $f_\lambda: X_\lambda \rightarrow Y_\lambda$  is  $\text{al.sl.rg.c}$  for each  $\lambda \in \Gamma$ .

(ii) If  $f: X \rightarrow \prod Y_\lambda$  is  $\text{al.sl.rg.c.}$ , then  $P_\lambda \circ f: X \rightarrow Y_\lambda$  is  $\text{al.sl.rg.c}$  for each  $\lambda \in \Gamma$ , where  $P_\lambda: \prod Y_\lambda$  onto  $Y_\lambda$ .

### Remark 1:

- (i) Composition of two  $\text{al.sl.rg.c}$  functions is not in general  $\text{al.sl.rg.c.}$
- (ii) Algebraic sum and product of  $\text{al.sl.rg.c}$  functions is not in general  $\text{al.sl.rg.c.}$
- (iii) The pointwise limit of a sequence of  $\text{al.sl.rg.c}$  functions is not in general  $\text{al.sl.rg.c.}$

However we can prove the following:

**Theorem 3.7:** The uniform limit of a sequence of  $\text{al.sl.rg.c}$  functions is  $\text{al.sl.rg.c.}$

**Note 2:** Pasting Lemma is not true for  $\text{al.sl.rg.c}$  functions. However we have the following weaker versions.

**Theorem 3.8:** Let  $X$  and  $Y$  be topological spaces such that  $X = A \cup B$  and let  $f_A: A \rightarrow Y$  and  $g_B: B \rightarrow Y$  are  $\text{al.sl.r.c}$  maps such that  $f(x) = g(x)$  for all  $x \in A \cap B$ . Suppose  $A$  and  $B$  are  $\text{r-open}$  sets in  $X$  and  $\text{RO}(X)$  is closed under finite unions, then the combination  $\alpha: X \rightarrow Y$  is  $\text{al.sl.rg.c}$  continuous.

**Theorem 3.9: Pasting Lemma** Let  $X$  and  $Y$  be spaces such that  $X = A \cup B$  and let  $f_A: A \rightarrow Y$  and  $g_B: B \rightarrow Y$  are  $\text{al.sl.rg.c}$  maps such that  $f(x) = g(x)$  for all  $x \in A \cap B$ . Suppose  $A, B$  are  $\text{r-open}$  sets in  $X$  and  $\text{RGO}(X)$  is closed under finite unions, then the combination  $\alpha: X \rightarrow Y$  is  $\text{al.sl.rg.c.}$

**Proof:** Let  $F \in \text{RCO}(Y)$ , then  $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$ , where  $f^{-1}(F) \in \text{RGO}(A)$  and  $g^{-1}(F) \in \text{RGO}(B) \Rightarrow f^{-1}(F); g^{-1}(F) \in \text{RGO}(X) \Rightarrow f^{-1}(F) \cup g^{-1}(F) \in \text{RGO}(X)$  [by assumption]. Therefore  $\alpha^{-1}(F) \in \text{RGO}(X)$ . Hence  $\alpha: X \rightarrow Y$  is al.sl.rg.c.

#### 4. Covering and Separation properties of al.sl.rg.c. functions:

**Theorem 4.1:** If  $f$  is al.sl.rg.c.[resp: al.sl.rg.c] surjection and  $X$  is  $rg$ -compact, then  $Y$  is compact.

**Proof:** Let  $\{G_i; i \in I\}$  be any open cover for  $Y$ . Then each  $G_i$  is open in  $Y$  and hence each  $G_i$  is  $r$ -clopen in  $Y$ . Since  $f$  is al.sl.rg.c.,  $f^{-1}(G_i)$  is  $rg$ -open in  $X$ . Thus  $\{f^{-1}(G_i)\}$  forms a  $rg$ -open cover for  $X$  and hence have a finite subcover, since  $X$  is  $rg$ -compact. Since  $f$  is surjection,  $Y = f(X) = \bigcup_{i=1}^n G_i$ . Therefore  $Y$  is compact.

**Corollary 4.1:** If  $f$  is al.sl.sp.c.[resp: al.sl.r.c] surjection and  $X$  is  $rg$ -compact, then  $Y$  is compact.

**Theorem 4.2:** If  $f$  is al.sl.rg.c., surjection and  $X$  is  $rg$ -compact[ $rg$ -lindeloff] then  $Y$  is mildly compact[mildly lindeloff].

**Proof:** Let  $\{U_i; i \in I\}$  be  $r$ -clopen cover for  $Y$ . For each  $x$  in  $X$ ,  $\exists \alpha_x \in I$  such that  $f(x) \in U_{\alpha_x}$  and  $\exists V_x \in \text{RGO}(X, x)$  such that  $f(V_x) \subset U_{\alpha_x}$ . Since the family  $\{V_i; i \in I\}$  is a cover of  $X$  by  $rg$ -open sets of  $X$ ,  $\exists$  a finite subset  $I_0$  of  $I$  such that  $X \subset \{V_x; x \in I_0\}$ . Therefore  $Y \subset \bigcup \{f(V_x); x \in I_0\} \subset \bigcup \{U_{\alpha_x}; x \in I_0\}$ . Hence  $Y$  is mildly compact.

#### Corollary 4.2:

- (i) If  $f$  is al.sl.rg.c.[resp: al.sl.r.c] surjection and  $X$  is  $rg$ -compact[ $rg$ -lindeloff] then  $Y$  is mildly compact[mildly lindeloff].
- (ii) If  $f$  is al.sl.rg.c.[resp: al.sl.c; al.sl.r.c] surjection and  $X$  is locally  $rg$ -compact{resp:  $rg$ -Lindeloff; locally  $rg$ -lindeloff}, then  $Y$  is locally compact{resp: Lindeloff; locally lindeloff}.
- (iii) If  $f$  is al.sl.rg.c.[al.sl.r.c.], surjection and  $X$  is locally  $rg$ -compact{resp:  $rg$ -lindeloff; locally  $rg$ -lindeloff} then  $Y$  is locally mildly compact{resp: locally mildly lindeloff}.

**Theorem 4.3:** If  $f$  is al.sl.rg.c., surjection and  $X$  is  $s$ -closed then  $Y$  is mildly compact[mildly lindeloff].

**Proof:** Let  $\{V_i; V_i \in \text{RCO}(Y); i \in I\}$  be a cover of  $Y$ , then  $\{f^{-1}(V_i); i \in I\}$  is  $rg$ -open cover of  $X$  [by Thm 3.1] and so there is finite subset  $I_0$  of  $I$ , such that  $\{f^{-1}(V_i); i \in I_0\}$  covers  $X$ . Therefore  $\{V_i; i \in I_0\}$  covers  $Y$  since  $f$  is surjection. Hence  $Y$  is mildly compact.

**Corollary 4.3:** If  $f$  is al.sl.r.c., surjection and  $X$  is  $s$ -closed then  $Y$  is mildly compact[mildly lindeloff].

**Theorem 4.4:** If  $f$  is al.sl.rg.c.,[resp: al.sl.rg.c.; al.sl.r.c.] surjection and  $X$  is  $rg$ -connected, then  $Y$  is connected.

**Proof:** If  $Y$  is disconnected, then  $Y = A \cup B$  where  $A$  and  $B$  are disjoint  $r$ -clopen sets in  $Y$ . Since  $f$  is al.sl.rg.c. surjection,  $X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B)$  where  $f^{-1}(A) f^{-1}(B)$  are disjoint  $rg$ -open sets in  $X$ , which is a contradiction for  $X$  is  $rg$ -connected. Hence  $Y$  is connected.

**Corollary 4.4:** The inverse image of a disconnected space under a al.sl.rg.c.,[resp: al.sl.r.c.] surjection is  $rg$ -disconnected.

**Theorem 4.5:** If  $f$  is al.sl.rg.c.,[resp: al.sl.c.], injection and  $Y$  is  $UT_i$ , then  $X$  is  $rg_i$   $i = 0, 1, 2$ .

**Proof:** Let  $x_1 \neq x_2 \in X$ . Then  $f(x_1) \neq f(x_2) \in Y$  since  $f$  is injective. For  $Y$  is  $UT_2 \exists V_j \in \text{RCO}(Y)$  such that  $f(x_j) \in V_j$  and  $\bigcap V_j = \emptyset$  for  $j = 1, 2$ . By Theorem 3.1,  $x_j \in f^{-1}(V_j) \in \text{RGO}(X)$  for  $j = 1, 2$  and  $\bigcap f^{-1}(V_j) = \emptyset$  for  $j = 1, 2$ . Thus  $X$  is  $rg_2$ .

**Theorem 4.6:** If  $f$  is al.sl.rg.c., injection; closed and  $Y$  is  $UT_i$ , then  $X$  is  $rgg_i$   $i = 3, 4$ .

**Proof:** (i) Let  $x$  in  $X$  and  $F$  be disjoint closed subset of  $X$  not containing  $x$ , then  $f(x)$  and  $f(F)$  be disjoint closed subset of  $Y$  not containing  $f(x)$ , since  $f$  is closed and injection. Since  $Y$  is ultraregular,  $f(x)$  and  $f(F)$  are separated by disjoint  $r$ -clopen sets  $U$  and  $V$  respectively. Hence  $x \in f^{-1}(U)$ ;  $F \subseteq f^{-1}(V)$ ,  $f^{-1}(U)$ ;  $f^{-1}(V) \in RGO(X)$  and  $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ . Thus  $X$  is  $rgg_3$ .

(ii) Let  $F_j$  and  $f(F_j)$  are disjoint closed subsets of  $X$  and  $Y$  respectively for  $j = 1, 2$ , since  $f$  is closed and injection. For  $Y$  is ultranormal,  $f(F_j)$  are separated by disjoint  $r$ -clopen sets  $V_j$  respectively for  $j = 1, 2$ . Hence  $F_j \subseteq f^{-1}(V_j)$  and  $f^{-1}(V_j) \in RGO(X)$  and  $\cap f^{-1}(V_j) = \emptyset$  for  $j = 1, 2$ . Thus  $X$  is  $rgg_4$ .

**Theorem 4.7:** If  $f$  is al.sl.rg.c.[resp: al.sl.c.], injection and

(i)  $Y$  is  $UC_i$ [resp:  $UD_i$ ] then  $X$  is  $rgC_i$ [resp:  $rgD_i$ ]  $i = 0, 1, 2$ .

(ii)  $Y$  is  $UR_i$ , then  $X$  is  $rg-R_i$   $i = 0, 1$ .

**Theorem 4.8:** If  $f$  is al.sl.rg.c.[resp: al.sl.c; al.sl.r.c] and  $Y$  is  $UT_2$ , then the graph  $G(f)$  of  $f$  is  $rg$ -closed in  $X \times Y$ .

**Proof:** Let  $(x, y) \notin G(f)$  implies  $y \neq f(x)$  implies  $\exists$  disjoint  $V$ ;  $W \in RCO(Y)$  such that  $f(x) \in V$  and  $y \in W$ . Since  $f$  is al.sl.rg.c.,  $\exists U \in RGO(X)$  such that  $x \in U$  and  $f(U) \subset W$  and  $(x, y) \in U \times V \subset X \times Y - G(f)$ . Hence  $G(f)$  is  $rg$ -closed in  $X \times Y$ .

**Theorem 4.9:** If  $f$  is al.sl.rg.c.[resp: al.sl.c; al.sl.r.c] and  $Y$  is  $UT_2$ , then  $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$  is  $rg$ -closed in  $X \times X$ .

**Proof:** If  $(x_1, x_2) \in X \times X - A$ , then  $f(x_1) \neq f(x_2)$  implies  $\exists$  disjoint  $V_j \in RCO(Y)$  such that  $f(x_j) \in V_j$ , and since  $f$  is al.sl.rg.c.,  $f^{-1}(V_j) \in RGO(X, x_j)$  for  $j = 1, 2$ . Thus  $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in RGO(X \times X)$  and  $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X - A$ . Hence  $A$  is  $rg$ -closed.

**Theorem 4.10:** If  $f$  is al.sl.r.c.[resp: al.sl.c.];  $g$  is al.sl.rg.c[resp: al.sl.c.]; and  $Y$  is  $UT_2$ , then  $E = \{x \in X; f(x) = g(x)\}$  is  $rg$ -closed in  $X$ .

**CONCLUSION:** In this paper we defined almost slightly- $rg$ -continuous functions, studied its properties and their interrelations with other types of almost slightly-continuous functions.

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