

Somewhat αg -closed functions

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Abstract: In this paper authors tried to introduce a new variety of closed functions called Somewhat αg -closed functions, almost somewhat αg -closed and somewhat M - αg -closed functions. Basic properties were discussed.

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1. Introduction

b -open sets are introduced by Andrijevic in 1996. K.R.Gentry introduced somewhat continuous functions in the year 1971. V.K.Sharma and the present authors of this paper defined and studied basic properties of ν -open sets and ν -continuous functions in the year 2006 and 2010 respectively. T.Noiri and N.Rajesh introduced somewhat b -continuous functions in the year 2011. S. Balasubramanian defined and studied somewhat closed and somewhat \ast -closed functions in the year 2014. S. Balasubramanian, C. Sandhya and P. A. S. Vyjayanthi defined and studied few properties of somewhat ν -closed functions in the same year. Inspired with these developments we introduce in this paper Somewhat αg -closed functions, almost somewhat αg -closed and somewhat M - αg -closed functions which are new in the literature of General Topology and study their basic properties and interrelation with other type of such functions available in the literature. Throughout the paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2. Preliminaries

For $A \subset (X, \tau)$, $cl(A)$ and A° denote the closure of A and the interior of A in X , respectively.

Definition 2.1: $A \subset X$ said to be b -open if $A \subset int(cl(A)) \cap cl(int(A))$.

Definition 2.2: A function f is said to be

- (i) somewhat continuous[resp: somewhat b -continuous] if for $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$, there exists an open[resp: b -open] set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.
- (ii) somewhat open[resp: somewhat b -open] provided that if $U \in \tau$ and $U \neq \emptyset$, then there exists a open[resp: b -open] set V in Y such that $V \neq \emptyset$ and $V \subset f(U)$.
- (iii) somewhat closed[resp: Somewhat \ast -closed functions [$\ast = r$ -; semi-; pre-; α -; β -; $\alpha\alpha$ -; b -; γ -] provided that if $U \in \tau$ and $U \neq \emptyset$, then there exists a closed[resp: \ast -closed] set V in Y such that $V \neq \emptyset$ and $f(U) \subset V$.

Definition 2.3: If τ and σ are topologies on X , then τ is said to be equivalent[resp: b-equivalent] to σ provided if $U \in \tau$ and $U \neq \emptyset$, then there is an open[resp: b-open] set V in X such that $V \neq \emptyset$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \emptyset$, then there is an open[resp: b-open] set V in (X, τ) such that $V \neq \emptyset$ and $U \supset V$.

Definition 2.4: $A \subset X$ is said to be dense in X if there is no proper closed set C in X such that $M \subset C \subset X$.

Now, consider the identity function f and assume that τ and σ are αg -equivalent. Then f and f^{-1} are somewhat continuous. Conversely, if the identity function f is somewhat continuous in both directions, then τ and σ are αg -equivalent.

3. Somewhat αg -closed function:

Definition 3.1: A function f is said to be somewhat αg -closed provided that if $U \in C(\tau)$ and $U \neq \emptyset$, then there exists a non-empty αg -closed set V in Y such that $f(U) \subset V$.

Example 1: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. The function $f: (X, \tau) \rightarrow (X, \sigma)$ defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ is somewhat r -closed; somewhat closed; somewhat α -closed; somewhat αg -closed.

Example 2: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, X\}$. The function $f: (X, \tau) \rightarrow (X, \sigma)$ defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is somewhat $r\alpha$ -closed; but not somewhat r -closed; somewhat closed; somewhat α -closed; somewhat αg -closed.

Theorem 3.1: Let f be a closed function and g somewhat αg -closed. Then $g \circ f$ is somewhat αg -closed.

Theorem 3.2: For a bijective function f , the following are equivalent:

- (i) f is somewhat αg -closed.
- (ii) If $C \in \tau$, such that $f(C) \neq Y$, then there is a $D \in \alpha GO(Y)$ such that $D \neq Y$ and $D \subset f(C)$.

Proof: (i) \Rightarrow (ii): Let $C \in \tau$ such that $f(C) \neq Y$. Then $X - C$ is closed and $X - C \neq \emptyset$. Since f is somewhat αg -closed, there exists $V \neq \emptyset \in \alpha GC(Y)$ such that $f(X - C) \subset V$. Put $D = Y - V$. Clearly $D \in \alpha GO(Y)$ and we claim $D \neq Y$. If $D = Y$, then $V = \emptyset$, which is a contradiction. Since $f(X - C) \subset V$, $D = Y - V \subset (Y - f(X - C)) = f(C)$.

(ii) \Rightarrow (i): Let $U \neq \emptyset$ be any closed set in X . Then $C = X - U \in \tau$ and $f(X - U) = f(C) = Y - f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a $D \in \alpha GO(Y)$ such that $D \neq Y$ and $D \subset f(C)$. Clearly $V = Y - D \neq \emptyset \in \alpha GC(Y)$. Also, $V = Y - D \supseteq Y - f(C) = Y - f(X - U) = f(U)$.

Theorem 3.3: The following statements are equivalent:

- (i) f is somewhat αg -closed.
- (ii) If A is a αg -dense subset of Y , then $f^{-1}(A)$ is a αg -dense subset of X .

Proof: (i) \Rightarrow (ii): Suppose A is a αg -dense set in Y . If $f^{-1}(A)$ is not αg -dense in X , then there exists a closed set B in X such that $f^{-1}(A) \subset B \subset X$. Since f is somewhat αg -closed and $X - B \in \alpha GO(X)$, there exists a $C \neq \emptyset \in \alpha GO(Y)$ such that $C \subset f(X - B)$. Therefore, $C \subset f(X - B) \subset f(f^{-1}(Y - A)) \subset Y - A$. That is, $A \subset Y - C \subset Y$. Now, $Y - C \neq \emptyset \in \alpha GC(Y)$ and $A \subset Y - C \subset Y$. This implies that A is not a αg -dense set in Y , which is a contradiction. Therefore, $f^{-1}(A)$ is a αg -dense set in X .

(ii) \Rightarrow (i): Suppose $A \neq \emptyset \in \alpha GO(X)$. We want to show that $\alpha g(f(A)) \neq \emptyset$. Suppose $\alpha g(f(A)) = \emptyset$. Then, $\alpha GCl(f(A)) = Y$. Therefore, by (ii), $f^{-1}(Y - f(A))$ is αg -dense in X . But $f^{-1}(Y - f(A)) \subset X - A$. Now, $X - A \neq \emptyset \in \alpha GC(X)$. Therefore, $f^{-1}(Y - f(A)) \subset X - A$ gives $X = \alpha GCl(f^{-1}(Y - f(A))) \subset X - A$. This implies that $A = \emptyset$, which is contrary to $A \neq \emptyset$. Therefore, $\alpha g(f(A)) \neq \emptyset$. Hence f is somewhat αg -closed.

Theorem 3.4: Let f be somewhat αg -closed and $A \in RC(X)$. Then $f|_A$ is somewhat αg -closed.

Proof: Let $U \neq \emptyset$ be closed in τ_A . Since U is closed in A and A is closed in X , U is closed in X and since f is somewhat αg -closed, there exists a $V \in \alpha GC(Y)$, such that $f(U) \subset V$. Thus, for any closed set $U \neq \emptyset$ in τ_A , there exists a $V \in \alpha GC(Y)$ such that $f(U) \subset V$ which implies $f|_A$ is somewhat αg -closed.

Theorem 3.5: Let f be a function and $X = A \cup B$, where $A, B \in RC(X)$. If the restriction functions $f|_A$ and $f|_B$ are somewhat αg -closed, then f is somewhat αg -closed.

Proof: Let $U \neq \emptyset$ be closed in X . Since $X = A \cup B$, either $A \cap U \neq \emptyset$ or $B \cap U \neq \emptyset$ or both $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$. Since U is closed in X , U is closed in both A and B .

Case (i): If $A \cap U \neq \emptyset$ is closed in A . Since $f|_A$ is somewhat αg -closed, there exists $V \in \alpha GC(Y)$ such that $f(U \cap A) \subset f(U) \subset V$, which implies that f is somewhat αg -closed.

Case (ii): If $B \cap U \neq \emptyset$ is closed in B . Since $f|_B$ is somewhat αg -closed, there exists $V \in \alpha GC(Y)$ such that $f(U \cap B) \subset f(U) \subset V$, which implies that f is somewhat αg -closed.

Case (iii): If both $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$. Then by case (i) and (ii) f is somewhat αg -closed.

Remark 1: Two topologies τ and σ for X are said to be αg -equivalent if and only if the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is somewhat αg -closed in both directions.

Theorem 3.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a somewhat αg -closed function. Let τ^* and σ^* be topologies for X and Y , respectively such that τ^* is equivalent to τ and σ^* is αg -equivalent to σ . Then $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ is somewhat αg -closed.

4. Almost somewhat αg -closed function:

Definition 4.1: A function f is said to be almost somewhat αg -closed provided that if $U \in RC(\tau)$ and $U \neq \emptyset$, then there exists a non-empty αg -closed set V in Y such that $f(U) \subset V$.

Example 3: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b, c\}, X\}$. The function $f: (X, \tau) \rightarrow (X, \sigma)$ defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is almost somewhat αg -closed, somewhat αg -closed and somewhat closed.

Example 4: Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b, c\}, X\} = \sigma$. The function $f: (X, \tau) \rightarrow (X, \sigma)$ defined by $f(a) = d$, $f(b) = c$, $f(c) = b$ and $f(d) = a$ is not almost somewhat αg -closed.

Theorem 4.1: Let f be r -closed and g somewhat αg -closed. Then $g \circ f$ is almost somewhat αg -closed.

Theorem 4.2: For a bijective function f , the following are equivalent:

(i) f is almost somewhat αg -closed.

(ii) If $C \in RO(X)$, such that $f(C) \neq Y$, then there is a $D \in \alpha GO(Y)$ such that $D \neq Y$ and $D \subset f(C)$.

Proof: (i) \Rightarrow (ii): Let $C \in RO(X)$ such that $f(C) \neq Y$. Then $X - C \in RC(X)$ and $X - C \neq \emptyset$. Since f is almost somewhat αg -closed, there exists $V \neq \emptyset \in \alpha GC(Y)$ such that $f(X - C) \subset V$. Put $D = Y - V$. Clearly $D \in \alpha GO(Y)$ and we claim $D \neq Y$. If $D = Y$, then $V = \emptyset$, which is a contradiction. Since $f(X - C) \subset V$, $D = Y - V \subset (Y - f(X - C)) = f(C)$.

(ii) \Rightarrow (i): Let $U \neq \emptyset \in RC(X)$. Then $C = X - U \in RO(X)$ and $f(X - U) = f(C) = Y - f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a $D \in \alpha GO(Y)$ such that $D \neq Y$ and $D \subset f(C)$. Clearly $V = Y - D \neq \emptyset \in \alpha GC(Y)$. Also, $V = Y - D \supseteq Y - f(C) = Y - f(X - U) = f(U)$.

Theorem 4.3: The following statements are equivalent:

(i) f is almost somewhat α g-closed.

(ii) If A is a α g-dense subset of Y , then $f^{-1}(A)$ is a α g-dense subset of X .

Proof: (i) \Rightarrow (ii): Suppose A is a α g-dense set in Y . If $f^{-1}(A)$ is not α g-dense in X , then there exists a $B \in \alpha GC(X)$ such that $f^{-1}(A) \subset B \subset X$. Since f is almost somewhat α g-closed and $X-B \in \alpha GO(X)$, there exists a $C \neq \emptyset \in \alpha GO(Y)$ such that $C \subset f(X-B)$. Therefore, $C \subset f(X-B) \subset f(f^{-1}(Y-A)) \subset Y-A$. That is, $A \subset Y-C \subset Y$. Now, $Y-C \neq \emptyset \in \alpha GC(Y)$ and $A \subset Y-C \subset Y$. This implies that A is not a α g-dense set in Y , which is a contradiction. Therefore, $f^{-1}(A)$ is a α g-dense set in X .

(ii) \Rightarrow (i): Suppose $A \neq \emptyset \in \alpha GO(X)$. We want to show that $\alpha g(f(A)) \neq \emptyset$. Suppose $\alpha g(f(A)) = \emptyset$. Then, $\alpha GCl(f(A)) = Y$. Therefore, by (ii), $f^{-1}(Y - f(A))$ is α g-dense in X . But $f^{-1}(Y - f(A)) \subset X-A$. Now, $X-A \neq \emptyset \in \alpha GC(X)$. Therefore, $f^{-1}(Y - f(A)) \subset X-A$ gives $X = \alpha GCl(f^{-1}(Y - f(A))) \subset X-A$. This implies that $A = \emptyset$, which is contrary to $A \neq \emptyset$. Therefore, $\alpha g(f(A)) \neq \emptyset$. Hence f is almost somewhat α g-closed.

Theorem 4.4: Let f be almost somewhat α g-closed and $A \in RC(X)$. Then $f|_A$ is almost somewhat α g-closed.

Proof: Let $U \neq \emptyset \in RC(\tau_A)$. Since $U \in RC(A)$ and $A \in RC(X)$, $U \in RC(X)$ and since f is almost somewhat α g-closed, there exists a $V \in \alpha GC(Y)$, such that $f(U) \subset V$. Thus, for any $U \neq \emptyset \in RC(\tau_A)$, there exists a $V \in \alpha GC(Y)$ such that $f(U) \subset V$ which implies $f|_A$ is almost somewhat α g-closed.

Theorem 4.5: Let f be a function and $X = A \cup B$, where $A, B \in RC(X)$. If the restriction functions $f|_A$ and $f|_B$ are almost somewhat α g-closed, then f is almost somewhat α g-closed.

Proof: Let $U \neq \emptyset \in RC(X)$. Since $X = A \cup B$, either $A \cap U \neq \emptyset$ or $B \cap U \neq \emptyset$ or both $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$. Since $U \in RC(X)$, $U \in RC(A)$ and $U \in RC(B)$.

Case (i): If $A \cap U \neq \emptyset \in RC(A)$. Since $f|_A$ is almost somewhat α g-closed, there exists $V \in \alpha GC(Y)$ such that $f(U \cap A) \subset f(U) \subset V$, which implies that f is almost somewhat α g-closed.

Case (ii): If $B \cap U \neq \emptyset \in RC(B)$. Since $f|_B$ is almost somewhat α g-closed, there exists $V \in \alpha GC(Y)$ such that $f(U \cap B) \subset f(U) \subset V$, which implies that f is almost somewhat α g-closed.

Case (iii): If both $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$. Then by case (i) and (ii) f is almost somewhat α g-closed.

Remark 2: Two topologies τ and σ for X are said to be α g-equivalent if and only if the identity function $f: (X, \tau) \rightarrow (X, \sigma)$ is almost somewhat α g-closed in both directions.

Theorem 4.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a almost somewhat α g-closed function. Let τ^* and σ^* be topologies for X and Y , respectively such that τ^* is equivalent to τ and σ^* is α g-equivalent to σ . Then $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ is almost somewhat α g-closed.

5. Somewhat M- α g-closed function:

Definition 5.1: A function f is said to be somewhat M- α g-closed provided that if $U \in \alpha GC(\tau)$ and $U \neq \emptyset$, then there exists a non-empty α g-closed set V in Y such that $f(U) \subset V$.

Example 5: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b, c\}, X\}$. The function $f: (X, \tau) \rightarrow (X, \sigma)$ defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is somewhat M- α g-closed, somewhat α g-closed and somewhat closed.

Example 6: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\} = \sigma$. The function $f: (X, \tau) \rightarrow (X, \sigma)$ defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is not somewhat M- α g-closed.

Theorem 5.1: Let f be an r -closed function and g somewhat αg -closed. Then $g \bullet f$ is somewhat M - αg -closed.

Theorem 5.2: For a bijective function f , the following are equivalent:

(i) f is somewhat M - αg -closed.

(ii) If $C \in \alpha GO(X)$, such that $f(C) \neq Y$, then there is a $D \in \alpha GO(Y)$ such that $D \neq Y$ and $D \subset f(C)$.

Proof: (i) \Rightarrow (ii): Let $C \in \alpha GO(X)$ such that $f(C) \neq Y$. Then $X - C \in \alpha GC(X)$ and $X - C \neq \emptyset$. Since f is somewhat M - αg -closed, there exists $V \neq \emptyset \in \alpha GC(Y)$ such that $f(X - C) \subset V$. Put $D = Y - V$. Clearly $D \in \alpha GO(Y)$ and we claim $D \neq Y$. If $D = Y$, then $V = \emptyset$, which is a contradiction. Since $f(X - C) \subset V$, $D = Y - V \subset (Y - f(X - C)) = f(C)$.

(ii) \Rightarrow (i): Let $U \neq \emptyset \in \alpha GC(X)$. Then $C = X - U \in \alpha GO(X)$ and $f(X - U) = f(C) = Y - f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a $D \in \alpha GO(Y)$ such that $D \neq Y$ and $D \subset f(C)$. Clearly $V = Y - D \neq \emptyset \in \alpha GC(Y)$. Also, $V = Y - D \supseteq Y - f(C) = Y - f(X - U) = f(U)$.

Theorem 5.3: The following statements are equivalent:

(i) f is somewhat M - αg -closed.

(ii) If A is a αg -dense subset of Y , then $f^{-1}(A)$ is a αg -dense subset of X .

Proof: (i) \Rightarrow (ii): Suppose A is a αg -dense set in Y . If $f^{-1}(A)$ is not αg -dense in X , then there exists a $B \in \alpha GC(X)$ such that $f^{-1}(A) \subset B \subset X$. Since f is somewhat M - αg -closed and $X - B \in \alpha GO(X)$, there exists a $C \neq \emptyset \in \alpha GO(Y)$ such that $C \subset f(X - B)$. Therefore, $C \subset f(X - B) \subset f(f^{-1}(Y - A)) \subset Y - A$. That is, $A \subset Y - C \subset Y$. Now, $Y - C \neq \emptyset \in \alpha GC(Y)$ and $A \subset Y - C \subset Y$. This implies that A is not a αg -dense set in Y , which is a contradiction. Therefore, $f^{-1}(A)$ is a αg -dense set in X .

(ii) \Rightarrow (i): Suppose $A \neq \emptyset \in \alpha GO(X)$. We want to show that $\alpha g(f(A)) \neq \emptyset$. Suppose $\alpha g(f(A)) = \emptyset$. Then, $\alpha GC(f(A)) = Y$. Therefore, by (ii), $f^{-1}(Y - f(A))$ is αg -dense in X . But $f^{-1}(Y - f(A)) \subset X - A$. Now, $X - A \neq \emptyset \in \alpha GC(X)$. Therefore, $f^{-1}(Y - f(A)) \subset X - A$ gives $X = \alpha GC(f^{-1}(Y - f(A))) \subset X - A$. This implies that $A = \emptyset$, which is contrary to $A \neq \emptyset$. Therefore, $\alpha g(f(A)) \neq \emptyset$. Hence f is somewhat M - αg -closed.

Theorem 5.4: Let f be somewhat M - αg -closed and $A \in RC(X)$. Then $f|_A$ is somewhat M - αg -closed.

Proof: Let $U \neq \emptyset \in \alpha GC(\tau_A)$. Since $U \in \alpha GC(A)$ and $A \in RC(X)$, $U \in \alpha GC(X)$ and since f is somewhat αg -closed, there exists a $V \in \alpha GC(Y)$, such that $f(U) \subset V$. Thus, for any $U \neq \emptyset \in \alpha GC(A)$, there exists a $V \in \alpha GC(Y)$ such that $f(U) \subset V$ which implies $f|_A$ is somewhat M - αg -closed.

Theorem 5.5: Let f be a function and $X = A \cup B$, where $A, B \in RC(X)$. If the restriction functions $f|_A$ and $f|_B$ are somewhat M - αg -closed, then f is somewhat M - αg -closed.

Proof: Let $U \neq \emptyset \in \alpha GC(X)$. Since $X = A \cup B$, either $A \cap U \neq \emptyset$ or $B \cap U \neq \emptyset$ or both $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$. Since $U \in \alpha GC(X)$, $U \in \alpha GC(A)$ and $U \in \alpha GC(B)$.

Case (i): If $A \cap U \neq \emptyset \in \alpha GC(A)$. Since $f|_A$ is somewhat M - αg -closed, there exists $V \in \alpha GC(Y)$ such that $f(U \cap A) \subset f(U) \subset V$, which implies that f is somewhat M - αg -closed.

Case (ii): If $B \cap U \neq \emptyset \in \alpha GC(B)$. Since $f|_B$ is somewhat M - αg -closed, there exists $V \in \alpha GC(Y)$ such that $f(U \cap B) \subset f(U) \subset V$, which implies that f is somewhat M - αg -closed.

Case (iii): If both $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$. Then by case (i) and (ii) f is somewhat M - αg -closed.

Remark 3: Two topologies τ and σ for X are said to be αg -equivalent if and only if the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is somewhat M - αg -closed in both directions.

Theorem 5.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a somewhat M - αg -closed function. Let τ^* and σ^* be topologies for X and Y , respectively such that τ^* is equivalent to τ and σ^* is αg -equivalent to σ . Then $f: (X; \tau^*) \rightarrow (Y; \sigma^*)$ is somewhat M - αg -closed.

Conclusion: In this paper authors defined Somewhat αg -closed functions, almost somewhat αg -closed and somewhat M- αg -closed mappings, studied some of their basic properties.

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