

b^{} -t-CONTINUOUS AND $**b$ -t-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES**

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ABSTRACT We introduce the notion of b^{**} -pre continuous, b^{**} -semi continuous, b^{**} -t-continuous, b^{**} -t*-continuous, b^{**} -B-continuous, b^{**} -B*-continuous, contra b^{**} -continuous, contra g- b^{**} -continuous, contra s- b^{**} -continuous, $**b$ -pre continuous, $**b$ -semi continuous, $**b$ -t-continuous, $**b$ -t*-continuous, $**b$ -B-continuous, $**b$ -B*-continuous, contra $**b$ -continuous, contra g- $**b$ -continuous, contra s- $**b$ -continuous functions.

Keywords : b^{**} -preopen, b^{**} -semi open, b^{**} -t-set, b^{**} -t*-set, b^{**} -B-set, b^{**} -B*-set, b^{**} -closed, g- b^{**} -closed, s- b^{**} -generalized closed, $**b$ -preopen, $**b$ -semi open, $**b$ -t-set, $**b$ -t*-set, $**b$ -B-set, $**b$ -B*-set, $**b$ -closed, g- $**b$ -closed, s- $**b$ -generalized closed.

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1. INTRODUCTION

Tong [14] introduced the concept of t -set and B -set in topological space. Indira, Rekha [6] introduced $^{**}b$ -open sets, t^{*} -set, B^{*} -set in topological space. Indira, Rekha [7, 11, 12] introduced $^{*}b$ - t -set, $^{*}b$ - t^{*} -set, $^{*}b$ - B -set, $^{*}b$ - B^{*} -set, $^{*}b$ -preopen, $^{*}b$ -semiopen set, g - $^{*}b$ -closed, s - $^{*}b$ -generalized closed, b^{**} -preopen, b^{**} -semi open, b^{**} - t -set, b^{**} - t^{*} -set, b^{**} - B -set, b^{**} - B^{*} -set, g - b^{**} -closed, s - b^{**} -generalized closed, $^{**}b$ -preopen, $^{**}b$ -semi open, $^{**}b$ - t -set, $^{**}b$ - t^{*} -set, $^{**}b$ - B -set, $^{**}b$ - B^{*} -set, $^{**}b$ -closed, g - $^{**}b$ -closed, s - $^{**}b$ -generalized closed sets in topological spaces. In this paper, we introduce the notion of b^{**} - t -continuous, b^{**} - t^{*} -continuous, b^{**} - B -continuous, b^{**} - B^{*} -continuous, contra b^{**} -continuous, contra g - b^{**} -continuous, contra s - b^{**} -continuous functions, $^{**}b$ - t -continuous, $^{**}b$ - t^{*} -continuous, $^{**}b$ - B -continuous, $^{**}b$ - B^{*} -continuous, contra $^{**}b$ -continuous, contra g - $^{**}b$ -continuous, contra s - $^{**}b$ -continuous functions. All through this paper (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. A is regular open if $A = Int(Cl(A))$ and A is regular closed if its complement is regular open; equivalently A is regular closed if $A = Cl(Int(A))$.

2. PRELIMINARIES

2.1 Definition A subset A of a space X is said to be:

1. Semi-open [9] if $A \subseteq Cl(Int(A))$
2. Preopen [10] if $A \subseteq Int(Cl(A))$
3. b -open [1] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$
4. $^{*}b$ -open [6] if $A \subseteq Cl(Int(A)) \cap Int(Cl(A))$
5. b^{**} -open [3] if $A \subseteq Int(Cl(Int(A))) \cup Cl(Int(Cl(A)))$
6. $^{**}b$ -open [6] if $A \subseteq Int(Cl(Int(A))) \cap Cl(Int(Cl(A)))$

2.2 Definition [7, 11, 12] A subset A of a space X is said to be:

1. $^{*}b$ - t -set if $Int(A) = Int(^{*}bCl(A))$
2. $^{*}b$ - t^{*} -set if $Cl(A) = Cl(^{*}bInt(A))$
3. $^{*}b$ - B -set if $A = U \cap V$, where $U \in \tau$ and V is a $^{*}b$ - t -set
4. $^{*}b$ - B^{*} -set if $A = U \cap V$, where $U \in \tau$ and V is a $^{*}b$ - t^{*} -set
5. $^{*}b$ -semiopen if $A \subseteq Cl(^{*}bInt(A))$

6. $*b$ -preopen if $A \subseteq Int(*bCl(A))$
7. b^{**} - t -set if $Int(A) = Int(b^{**}Cl(A))$
8. b^{**} - t^{*} -set if $Cl(A) = Cl(b^{**}Int(A))$
9. b^{**} - B -set if $A = U \cap V$, where $U \in \tau$ and V is a b^{**} - t -set
10. b^{**} - B^{*} -set if $A = U \cap V$, where $U \in \tau$ and V is a b^{**} - t^{*} -set
11. b^{**} -semiopen if $A \subseteq Cl(b^{**}Int(A))$
12. b^{**} -preopen if $A \subseteq Int(b^{**}Cl(A))$
13. $**b$ - t -set if $Int(A) = Int(**bCl(A))$
14. $**b$ - t^{*} -set if $Cl(A) = Cl(**bInt(A))$
15. $**b$ - B -set if $A = U \cap V$, where $U \in \tau$ and V is a $**b$ - t -set
16. $**b$ - B^{*} -set if $A = U \cap V$, where $U \in \tau$ and V is a $**b$ - t^{*} -set
17. $**b$ -semiopen if $A \subseteq Cl(**bInt(A))$
18. $**b$ -preopen if $A \subseteq Int(**bCl(A))$

2.3 Definition [3,6,7,11,12]

A subset A of a space X is said to be:

1. generalized $*b$ -closed set if $*bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
2. generalized b^{**} -closed set if $b^{**}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
3. generalized $**b$ -closed set if $**bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
4. s - $*b$ -generalized closed if $s(*bCl(A)) \subseteq U$ whenever $A \subseteq U$ and U is $*b$ -preopen.
5. s - b^{**} -generalized closed if $s(b^{**}Cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is b^{**} -preopen.
6. s - $**b$ -generalized closed if $s(**bCl(A)) \subseteq U$ whenever $A \subseteq U$ and U is $**b$ -preopen.
7. locally $*b$ -closed if $A = U \cap V$, where $U \in \tau$ and V is a $*b$ -closed set.
8. locally b^{**} -closed if $A = U \cap V$, where $U \in \tau$ and V is a b^{**} -closed set.
9. locally $**b$ -closed if $A = U \cap V$, where $U \in \tau$ and V is a $**b$ -closed set.

2.4 Definition [8,13]

A function $f : X \rightarrow Y$ is called $\ast b$ -pre continuous (resp. $\ast b$ -semi continuous, $\ast b$ -t-continuous, $\ast b$ -t*-continuous, $\ast b$ -B-continuous, $\ast b$ -B*-continuous, contra $\ast b$ -continuous, contra g - $\ast b$ -continuous, contra s - $\ast b$ -continuous, completely continuous, locally b^{**} -closed continuous, locally $\ast\ast b$ -closed continuous) if $f^{-1}(V)$ is $\ast b$ -preopen (resp. $\ast b$ -semiopen, $\ast b$ -t-set, $\ast b$ -t*-set, $\ast b$ -B-set, $\ast b$ -B*-set, $\ast b$ -closed, g - $\ast b$ -closed, s - $\ast b$ -generalized closed, regular open, locally b^{**} -closed, locally $\ast\ast b$ -closed) in X for each open set V of Y .

3. b^{**} -t-CONTINUOUS AND b^{**} -B-CONTINUOUS

3.1 Definition A function $f : X \rightarrow Y$ is called b^{**} -pre continuous if $f^{-1}(V)$ is b^{**} -preopen in X for each open set V of Y .

3.2 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c\}$,

$\sigma = \{Y, \phi, \{b\}, \{c\}, \{b, c\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = a$; $f(b) = a$; $f(c) = b$; $f(d) = c$.

Then f is b^{**} -pre continuous.

3.3 Definition A function $f : X \rightarrow Y$ is called b^{**} -semi continuous if $f^{-1}(V)$ is b^{**} -semi open in X for each open set V of Y .

3.4 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c\}$,

$\sigma = \{Y, \phi, \{b\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = a$; $f(b) = a$; $f(c) = b$; $f(d) = c$.

Then f is b^{**} -semi continuous.

3.5 Definition A function $f : X \rightarrow Y$ is called b^{**} -t-continuous if $f^{-1}(V)$ is b^{**} -t-set in X for each open set V of Y .

3.6 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c\}$

$\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = c$; $f(b) = a$; $f(c) = b$; $f(d) = a$

Then f is b^{**} -t-continuous.

3.7 Definition A function $f : X \rightarrow Y$ is called b^{**} -t*-continuous if $f^{-1}(V)$ is b^{**} -t*-set in X for each open set V of Y .

3.8 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c, d\}$

$\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Define a map $f : X \rightarrow Y$ by

$f(a) = c$; $f(b) = d$; $f(c) = a$; $f(d) = b$. Then f is b^{**} -t*-continuous.

3.9 Definition A function $f : X \rightarrow Y$ is called b^{**} -B-continuous if $f^{-1}(V)$ is b^{**} -B-set in X for each open set V of Y .

3.10 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c, d\}$, $\sigma = \{Y, \phi, \{a, c\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = b$; $f(b) = c$; $f(c) = d$; $f(d) = a$. Then f is b^{**} -B-continuous.

3.11 Definition A function $f : X \rightarrow Y$ is called b^{**} -B*-continuous if $f^{-1}(V)$ is b^{**} -B*-set in X for each open set V of Y .

3.12 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c\}$, $\sigma = \{Y, \phi, \{a\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = c$; $f(b) = b$; $f(c) = a$; $f(d) = b$. Then f is b^{**} -B*-continuous.

3.13 Definition A function $f : X \rightarrow Y$ is called Contra b^{**} -continuous if $f^{-1}(V)$ is b^{**} -closed in X for each open set V of Y .

3.14 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define a map $f : X \rightarrow X$ by $f(a) = d$; $f(b) = c$; $f(c) = a$; $f(d) = b$. Then f is Contra b^{**} -continuous.

3.15 Definition A function $f : X \rightarrow Y$ is called Contra g - b^{**} -continuous if $f^{-1}(V)$ is g - b^{**} -closed in X for each open set V of Y .

3.16 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define a map $f : X \rightarrow X$ by $f(a) = d$; $f(b) = c$; $f(c) = a$; $f(d) = b$. Then f is Contra g - b^{**} -continuous.

3.17 Definition A function $f : X \rightarrow Y$ is called Contra s - b^{**} -continuous if $f^{-1}(V)$ is s - b^{**} -generalized closed in X for each open set V of Y .

3.18 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define a map $f : X \rightarrow X$ by $f(a) = c$; $f(b) = d$; $f(c) = a$; $f(d) = b$. Then f is Contra s - b^{**} -continuous.

3.19 Theorem

Let $f : X \rightarrow Y$ be a function. Then f is continuous if and only if it is b^{**} -pre continuous and b^{**} -B-continuous.

Proof: Assume that f is continuous. Let V be open in Y . Since f is continuous $\Rightarrow f^{-1}(V)$ is open in X [Since open $\Rightarrow b^{**}$ -preopen and a b^{**} -B-set [11]] $\Rightarrow f^{-1}(V)$ is b^{**} -preopen and a b^{**} -B-set in X . $\Rightarrow f^{-1}(V)$ is b^{**} -preopen and a b^{**} -B-set in X for each open set V of $Y \Rightarrow f$ is b^{**} -pre continuous and b^{**} -B-continuous. Assume that f is b^{**} -pre continuous and b^{**} -B-continuous. Let V be open in Y . Since f is b^{**} -pre continuous and b^{**} -B-continuous $\Rightarrow f^{-1}(V)$ is b^{**} -preopen and b^{**} -B-set in $X \Rightarrow f^{-1}(V)$ is b^{**} -preopen and b^{**} -B-set in X for each open set V of Y [Since b^{**} -preopen and a b^{**} -B-set \Rightarrow open [11]] $\Rightarrow f^{-1}(V)$ is open in X for each open set V of $Y \Rightarrow f$ is continuous.

3.20 Theorem

Let $f : X \rightarrow Y$ be a function. Then f is completely continuous if and only if f is b^{**} -pre continuous and b^{**} -t-continuous.

Proof: Assume that f is completely continuous. Let V be open in Y . Since f is completely continuous. $\Rightarrow f^{-1}(V)$ is regular open in X . [Since regular open $\Rightarrow b^{**}$ -preopen and b^{**} -t-set [11]] $\Rightarrow f^{-1}(V)$ is b^{**} -preopen and b^{**} -t-set in $X \Rightarrow f$ is b^{**} -pre continuous and b^{**} -t-continuous. Assume that f is b^{**} -pre continuous and b^{**} -t-continuous. Let V be open in Y . Since f is b^{**} -pre continuous and b^{**} -t-continuous $\Rightarrow f^{-1}(V)$ is b^{**} -preopen and b^{**} -t-set in X [Since b^{**} -preopen and a b^{**} -t-set \Rightarrow regular open [11]] $\Rightarrow f^{-1}(V)$ is regular open in X for each open set V of $Y \Rightarrow f$ is completely continuous.

3.21 Theorem

Let $f : X \rightarrow Y$ be a function. Then f is completely continuous if and only if f is b^{**} -pre continuous and contra s - b^{**} -continuous.

Proof: Assume that f is completely continuous. Let V be open in Y . Since f is completely continuous. $\Rightarrow f^{-1}(V)$ is regular open $\Rightarrow f^{-1}(V)$ is b^{**} -preopen and s - b^{**} -generalized closed. [Since regular open $\Rightarrow b^{**}$ -preopen and s - b^{**} -generalized closed [11]] $\Rightarrow f$ is b^{**} -pre continuous and contra s - b^{**} -continuous. Assume that f is b^{**} -pre continuous and contra s - b^{**} -continuous. Let V be open in Y . Since f is b^{**} -pre continuous and contra s - b^{**} -continuous $\Rightarrow f^{-1}(V)$ is b^{**} -preopen and s - b^{**} -generalized closed. [Since b^{**} -preopen and s - b^{**} -generalized closed \Rightarrow regular open [11]] $\Rightarrow f^{-1}(V)$ is regular open in X for each open set V of $Y \Rightarrow f$ is completely continuous.

3.22 Theorem

Let $f : X \rightarrow Y$ be a function. Then f is contra b^{**} -continuous if and only if it is locally b^{**} -closed-continuous and contra g - b^{**} -continuous.

Proof: Assume that, f is contra b^{**} -continuous. Let V be open in Y . Since f is contra b^{**} -continuous $\Rightarrow f^{-1}(V)$ is b^{**} -closed in X . [Since b^{**} -closed $\Rightarrow g$ - b^{**} -closed and locally b^{**} -closed [11] $\Rightarrow f^{-1}(V)$ is g - b^{**} -closed and locally b^{**} -closed in $X \Rightarrow f$ is contra- g - b^{**} -continuous and locally b^{**} -closed continuous. Conversely, Assume that f is contra g - b^{**} -continuous and locally b^{**} -closed continuous. Let V be open in Y . Since f is contra- g - b^{**} -continuous and locally b^{**} -closed continuous $\Rightarrow f^{-1}(V)$ is g - b^{**} -closed and locally b^{**} -closed in X [Since g - b^{**} -closed and locally b^{**} -closed $\Rightarrow b^{**}$ -closed [11] $\Rightarrow f^{-1}(V)$ is b^{**} -closed in $X \Rightarrow f^{-1}(V)$ is b^{**} -closed in X for each open set V of $Y \Rightarrow f$ is contra b^{**} -continuous.

4. **$**b$ -t-CONTINUOUS AND $**b$ -B-CONTINUOUS**

4.1 Definition A function $f : X \rightarrow Y$ is called $**b$ -pre continuous if $f^{-1}(V)$ is $**b$ -preopen in X for each open set V of Y .

4.2 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c\}$

$\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = c$; $f(b) = c$; $f(c) = a$; $f(d) = b$. Then f is $**b$ -pre continuous.

4.3 Definition A function $f : X \rightarrow Y$ is called $**b$ -semi continuous if $f^{-1}(V)$ is $**b$ -semi open in X for each open set V of Y .

4.4 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c, d\}$

$\sigma = \{Y, \phi, \{c\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = a$; $f(b) = d$; $f(c) = b$; $f(d) = c$

Then f is $**b$ -semi continuous.

4.5 Definition A function $f : X \rightarrow Y$ is called $**b$ -t-continuous if $f^{-1}(V)$ is $**b$ -t-set in X for each open set V of Y .

4.6 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c, d\}$

$\sigma = \{Y, \phi, \{b\}, \{b, c\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = a$; $f(b) = b$; $f(c) = d$; $f(d) = c$

Then f is $**b$ -t-continuous.

4.7 Definition A function $f : X \rightarrow Y$ is called $**b$ -t*-continuous if $f^{-1}(V)$ is $**b$ -t*-set in X for each open set V of Y .

4.8 Example Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$, $Y = \{a, b, c, d\}$

$\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Define a map $f : X \rightarrow Y$ by

$f(a) = c; f(b) = d; f(c) = a; f(d) = b$. Then f is $**b$ -t*-continuous.

4.9 Definition A function $f : X \rightarrow Y$ is called $**b$ -B-continuous if $f^{-1}(V)$ is $**b$ -B-set in X for each open set V of Y .

4.10 Example Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}, Y = \{a, b, c, d\}$

$\sigma = \{Y, \phi, \{b, c\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = b; f(b) = c; f(c) = d; f(d) = a$

Then f is $**b$ -B-continuous.

4.11 Definition A function $f : X \rightarrow Y$ is called $**b$ -B*-continuous if $f^{-1}(V)$ is $**b$ -B*-set in X for each open set V of Y .

4.12 Example Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}, Y = \{a, b, c\}$

$\sigma = \{Y, \phi, \{b\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = c; f(b) = a; f(c) = b; f(d) = b$

Then f is $**b$ -B*-continuous.

4.13 Definition A function $f : X \rightarrow Y$ is called Contra $**b$ -continuous if $f^{-1}(V)$ is $**b$ -closed in X for each open set V of Y .

4.14 Example Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define a map

$f : X \rightarrow X$ by $f(a) = d; f(b) = c; f(c) = a; f(d) = b$. Then f is Contra $**b$ -continuous.

4.15 Definition A function $f : X \rightarrow Y$ is called Contra g - $**b$ -continuous if $f^{-1}(V)$ is g - $**b$ -closed in X for each open set V of Y .

4.16 Example Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define a map

$f : X \rightarrow X$ by $f(a) = c; f(b) = d; f(c) = a; f(d) = b$. Then f is Contra g - $**b$ -continuous.

4.17 Definition A function $f : X \rightarrow Y$ is called Contra s - $**b$ -continuous if $f^{-1}(V)$ is s - $**b$ -generalized closed in X for each open set V of Y .

4.18 Example Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define a map $f : X \rightarrow X$ by $f(a) = c; f(b) = d; f(c) = a; f(d) = b$. Then f is Contra s - $**b$ -continuous.

4.19 Theorem

Let $f : X \rightarrow Y$ be a function. Then f is continuous if and only if it is $**b$ -pre continuous and $**b$ -B-continuous.

Proof: Assume that f is continuous. Let V be open in Y . Since f is continuous. $\Rightarrow f^{-1}(V)$ is open in X . [Since open $\Rightarrow **b$ -preopen and a $**b$ -B-set [12]]. $\Rightarrow f^{-1}(V)$ is $**b$ -preopen and a $**b$ -B-

set in $X \Rightarrow f$ is $**b$ -pre continuous and $**b$ -B-continuous. Assume that f is $**b$ -pre continuous and $**b$ -B-continuous. Let V be open in Y . Since f is $**b$ -pre continuous and $**b$ -B-continuous, $\Rightarrow f^{-1}(V)$ is $**b$ -preopen and $**b$ -B-set in X . [Since $**b$ -preopen and a $**b$ -B-set \Rightarrow open[12]] $\Rightarrow f^{-1}(V)$ is open in X for each open set V of $Y \Rightarrow f$ is continuous

4.20 Theorem

Let $f : X \rightarrow Y$ be a function. Then f is completely continuous if and only if f is $**b$ -pre continuous and $**b$ -t-continuous.

Proof: Assume that f is completely continuous. Let V be open in Y . Since f is completely continuous, $\Rightarrow f^{-1}(V)$ is regular open in X . [Since regular open $\Rightarrow **b$ -preopen and $**b$ -t-set[12]]. $\Rightarrow f^{-1}(V)$ is $**b$ -preopen and $**b$ -t-set in $X \Rightarrow f$ is $**b$ -pre continuous and $**b$ -t-continuous. Assume that f is $**b$ -pre continuous and $**b$ -t-continuous. Let V be open in Y . Since f is $**b$ -pre continuous and $**b$ -t-continuous, $\Rightarrow f^{-1}(V)$ is $**b$ -preopen and $**b$ -t-set in X . [Since $**b$ -preopen and a $**b$ -t-set \Rightarrow regular open[12]] $\Rightarrow f^{-1}(V)$ is regular open in X for each open set V of $Y \Rightarrow f$ is completely continuous.

4.21 Theorem

Let $f : X \rightarrow Y$ be a function. Then f is completely continuous if and only if f is $**b$ -pre continuous and contra $s-**b$ -continuous.

Proof: Assume that f is completely continuous. Let V be open in Y . Since f is completely continuous, $\Rightarrow f^{-1}(V)$ is regular open. $\Rightarrow f^{-1}(V)$ is $**b$ -preopen and $s-**b$ -generalized closed [Since regular open $\Rightarrow **b$ -preopen and $s-**b$ -generalized closed[12]]. $\Rightarrow f$ is $**b$ -pre continuous and contra $s-**b$ -continuous. Assume that f is $**b$ -pre continuous and contra $s-**b$ -continuous. Let V be open in Y . Since f is $**b$ -pre continuous and contra $s-**b$ -continuous, $\Rightarrow f^{-1}(V)$ is $**b$ -preopen and $s-**b$ -generalized closed. [Since $**b$ -preopen and $s-**b$ -generalized closed \Rightarrow regular open[12]] $\Rightarrow f^{-1}(V)$ is regular open in X for each open set V of $Y \Rightarrow f$ is completely continuous.

4.22 Theorem

Let $f : X \rightarrow Y$ be a function. Then f is contra $**b$ -continuous if and only if it is locally $**b$ -closed continuous and contra $g-**b$ -continuous.

Proof: Assume that, f is contra $**b$ -continuous. Let V be open in Y . Since f is contra $**b$ -continuous, $\Rightarrow f^{-1}(V)$ is $**b$ -closed in X . [Since $**b$ -closed $\Rightarrow g-**b$ -closed and locally $**b$ -closed[12]] $\Rightarrow f^{-1}(V)$ is $g-**b$ -closed and locally $**b$ -closed in $X \Rightarrow f$ is contra- $g-**b$ -continuous and locally $**b$ -closed continuous. Conversely, Assume that f is contra $g-**b$ -continuous and locally $**b$ -closed continuous. Let V be open in Y . Since f is contra- $g-**b$ -

continuous and locally $**b$ -closed continuous. $\Rightarrow f^{-1}(V)$ is $g-^{**}b$ -closed and locally $**b$ -closed in X . $\Rightarrow f^{-1}(V)$ is $g-^{**}b$ -closed and locally $**b$ -closed for each open set V of Y . [Since $g-^{**}b$ -closed and locally $**b$ -closed $\Rightarrow **b$ -closed [12]] $\Rightarrow f^{-1}(V)$ is $**b$ -closed in X

$\Rightarrow f$ is contra $**b$ -continuous.

CONCLUSION

As an extension of this paper, $**b$ -compact and $**b$ -connected sets in topological spaces can be defined and can obtain theorems based on the above defined concepts.

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