$\tau_1\tau_2$– g^*-closed sets

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Abstract:
This paper is to introduce a new class of sets called$\tau_1\tau_2$ -g-closed sets in bi topological spaces and to analyses the properties of this set.*

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1.Introduction

Levine [9] introduced semi open sets in 1963 and also Levine [10] introduced generalized closed sets in 1970. AbdEl – Monsef et al [1] introduced β-open sets. M.K.R.S.Veerakumar [12] introduced #g-closed sets in topological spaces. J.C.Kelley [7] initiated the study of bitopological spaces in 1963 . A nonempty set X equipped with two topological spaces τ_1 and $\tau_2$is called a bitopological space and is denoted by (X, $\tau_{1,}\tau_2$). Since then several topologists generalized many of the results in topological spaces to bitopological spaces. Fukutake [5] introduced generalized closed sets in bitopological spaces. Fukutake [6] introduced semi open sets in bitopologicalspaces .K.chandrasekharaRao and M.Mariasingam [3] defined and studied regular generalized closed sets in τbitopological settings. This paper is to introduce a new class of sets called$\tau_1\tau_2$– g*-closed sets in bitopological spaces and to study about their properties.

2. Preliminaries

DEFINITION 2.1

A subset A of a bitopological space (X,τ_1,τ_2) is called

1. $\tau_1\tau_2$-semi open if A $\subset\tau_2$cl($\tau_1$int(A)) and it is called $\tau_1\tau_2$-semi closed if $\tau_2$int($\tau_1$cl(A)) ⊂ A
2. $\tau_1\tau_2$-preopen if A $\subset\tau_2$ int(τ_1 cl(A)) and $\tau_1\tau_2$-pre closed if $\tau_2$cl($\tau_1$int(A)) ⊂ A.
3. $\tau_1\tau_2$-α-open if A ⊂$\tau_1$nt($\tau_2$cl($\tau_1$int(A))).

4. $\tau_1\tau_2$-semi preopen if A $\subset\tau_1$cl($\tau_2$int($\tau_1$cl(A))).
5. $\tau_1\tau_2$-regular open if A = $\tau_2$int($\tau_1$cl(A)).
6. $\tau_1\tau_2$-regular closed if A = $\tau_2$cl($\tau_1$int(A)).

DEFINITION 2.2:

A subset A of a bitopological space (X, τ_1,τ_2) is called a

1. $\tau_1\tau_2$-g-closed set$\tau_1\tau_2$-generalized closed set)if τ_2.cl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-open.
2. $\tau_1\tau_2$-sg-closed set($\tau_1\tau_2$semi generallized closed set)if $\tau_2$scl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-semi open.
3. $\tau_1\tau_2$gs-closed set ($\tau_1\tau_2$- generallized semi closed set)if τ_2.scl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-open.
4. $\tau_1\tau_2\alpha$g-closed set ($\tau_1\tau_2\alpha$- generallized closed set)if τ_2.αcl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-open.
5. $\tau_1\tau_2$-gα-closed set ($\tau_1\tau_2$- generallized α-closed set)if τ_2.αcl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-α-open.
6. $\tau_1\tau_2$-gp-closed set ($\tau_1\tau_2$generallized pre-closed set)if τ_2.pcl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-open.
7. $\tau_1\tau_2$-gsp-closed set ($\tau_1\tau_2$- generalized semi preclosed set)if τ_2.spcl(A) $\subset$ U , whenever A $\subset$ U,U is τ_1-open.
8. $\tau_1\tau_2$-gpr-closedset($\tau_1\tau_2$- generallized pre regular closed set)if τ_2.pcl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-regular open.
9. $\tau_1\tau_2$-μ-closed set if τ_2.cl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-gα*-open.
10. $\tau_1\tau_2$-ψ-closed set if τ_2.scl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-sg-open.
11. $\tau_1\tau_2$-pre semi closed set if τ_2.spcl(A) $\subset$ U , whenever A $\subset$ U, U is τ_1-g-open.

3. Properties of $\tau_1\tau_2$-g*-closed

Definition: 3.1A subset A of (X, τ_1, τ_2) is said to be $\tau_1\tau_2$ -g*-closedif τ_2-$cl(A) \subset U$, whenever $A \subset U, U$ is τ_1-g-open.

The complement of $\tau_1\tau_2$ -g*-closedset is called $\tau_1\tau_2$ -g*-open set.

Example: 3.2 Let$X = \{a, b, c\}, \tau_1 = \{\varphi, X, \{a\}\}, \ \tau_2 = \{\varphi, X, \{b\}\}$.

$\tau_1\tau_2$-g*-closed sets = $\{\varphi, X, \{a, c\}, \{b, c\}\}$.

Theorem: 3.3 Every τ_2-closed set is$\tau_1\tau_2$-g*-closed.

Proof: Let $A \subset U, U$ is τ_1-g-open.

A is τ_2-closed $\Rightarrow \tau_2$-$cl(A) = A \Rightarrow \tau_2$-$clA \subset U \Rightarrow A$ is $\tau_1\tau_2$-g*-closed

Theorem: 3.4 Every$\tau_1\tau_2$-g*-closed sets in $\tau_1\tau_2$-g-closed.

Proof: Assume that A is $\tau_1\tau_2$-g*-closed.Let $A \subset U, U$ is τ_1-open.

U is τ_1-open $\Rightarrow U$ is τ_1-g-open$\Rightarrow \tau_2$- $cl(A) \subset U$ [by the assumption]

$\Rightarrow A$ is $\tau_1\tau_2$-g-closed.

Theorem: 3.5 Every $\tau_1\tau_2$-g*-closed sets in $\tau_1\tau_2$-gs-closed.

Proof: Assume that A is $\tau_1\tau_2$-g*-closed.Let $A \subset U$, U is τ_1-open,

then U is τ_1-g-open$\Rightarrow \tau_2$-$cl(A) \subset U$ [by the assumption]

But τ_2-$scl(A) \subset \tau_2$-$cl(A) \subset U \Rightarrow \tau_2$-$scl(A) \subset U$, whenever $A \subset U, U$ is τ_1-open

$\Rightarrow A$ is $\tau_1\tau_2$-gs-closed.

Theorem: 3.6 Every $\tau_1\tau_2$g*-closed sets in $\tau_1\tau_2$-αg-closed.

Proof: Assume that A is $\tau_1\tau_2$-g*-closed.Let $A \subset U$, U is τ_1-open, then U is τ_1-g-open$\Rightarrow \tau_2$-$cl(A) \subset U$ [by the assumption]

But τ_2-$\alpha cl(A) \subset \tau_2$-$cl(A) \subset U \Rightarrow A$ is $\tau_1\tau_2$-αg-closed.

Theorem: 3.7 Every $\tau_1\tau_2$-g*-closed sets in $\mathcal{J}_1\mathcal{J}_2$-gp-closed.

Proof: Assume that A is $\tau_1\tau_2$-g*-closed.

Let $A \subset U$, U is τ_1-open, then U is τ_1-g-open.

$\Rightarrow \tau_2$-$cl(A) \subset U$ [by the assumption].But τ_2-$pcl(A) \subset \tau_2$-$cl(A) \subset U$.

$\Rightarrow A$ is $\tau_1\tau_2$-gp-closed.

Theorem: 3.8 Every $\tau_1\tau_2$-g*-closed sets in $\tau_1\tau_2$gpr-closed.

Proof: Assume that A is $\tau_1\tau_2$-g*-closed.

Let $A \subset U$, U is τ_1- regular open, then U is τ_1-open and so it is τ_1-g-open $\Rightarrow \tau_2$-$cl(A) \subset U$ [by the assumption]

But τ_2-$pcl(A) \subset \tau_2$-$cl(A) \subset U \Rightarrow A$ is $\tau_1\tau_2$-gpr-closed.

Theorem: 3.9 Every $\tau_1\tau_2$-g*-closed sets in $\tau_1\tau_2$-rg-closed.

Proof: Assume that A is $\tau_1\tau_2$-g*-closed.

Let $A \subset U$, U is τ_1- regular open, then U is τ_1-open

$\Rightarrow \tau_2$-$cl(A) \subset U$ [by the assumption]$\Rightarrow A$ is $\tau_1\tau_2$rg-closed.

Theorem: 3.10 Every $\tau_1\tau_2$-g*-closed sets in $\tau_1\tau_2$-gsp-closed.

Proof:

The proof follows from the fact that τ_2-$spcl(A) \subset \tau_2$-$cl(A)$.

Remark: 3.11

The converses of the above theorems are need not be true. This can be seen from the following examples.

Example: 3.12

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{a\}, \{b\}\}$, $\tau_2 = \{\varphi, X, \{b\}, \{b, c\}\}$.

$\tau_1\tau_2$g*-closed sets $= \{\varphi, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$.

Here $\{c\}, \{b, c\}$ are $\tau_1\tau_2$-g*-closed sets. But they are not τ_2-closed.

Example: 3.13

Let

$X = \{a, b, c\}$,$\tau_1 = \{\varphi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$.

$\tau_1\tau_2$g*-closed sets $= \{\varphi, X, \{c\}, \{a, c\}, \{b, c\}\}$.

$\tau_1\tau_2$-g-closed sets $= \{\varphi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$.

$\tau_1\tau_2$-gs-closed sets $= \{\varphi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$.

$\tau_1\tau_2$gp-closed sets $= \{\varphi, X, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$.

$\tau_1\tau_2$-αg-closed sets $= \{\varphi, X, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$.

$\tau_1\tau_2$gsp-closed sets $= \{\varphi, X, \{a\}, \{b\}, \{c\}, \{a,b\}\{a,c\}, \{b,c\}\}$.

$\tau_1\tau_2$-rg-closed sets $= \{\varphi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$.

$\tau_1\tau_2$gpr-closed sets $= \{\varphi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$.

Here $\{b\}, \{a,b\}$ are $\tau_1\tau_2$g-closed,$\tau_1\tau_2$-gp-closed$\tau_1\tau_2$-αg-closed, $\tau_1\tau_2$rg-closed, $\tau_1\tau_2$-gpr-closed. But they are not $\tau_1\tau_2$-g*-closed.

Similarly$\{a\}, \{b\}, \{a,b\}$ are $\tau_1\tau_2$-gs-closed and $\tau_1\tau_2$gsp-closed but they are not $\tau_1\tau_2$g*-closed.

Example: 3.14

$\tau_1\tau_2$g*-closed sets in independent from $\tau_1\tau_2$-α-closed sets, $\tau_1\tau_2$-semi-closed sets, $\tau_1\tau_2$-pre-closed sets, $\tau_1\tau_2$-semi-pre-closedsets,

$\tau_1\tau_2$sg-closed sets and $\tau_1\tau_2$-gα-closed sets.This can be seen from
the following examples.

Example: 3.15

Let $X = \{a,b,c\}$,
$\tau_1 = \{\varphi, X, \{a\}, \{b,c\}\}, \tau_2 = \{\varphi, X, \{a\}, \{b\}, \{a,b\}\}$.

$\tau_1\tau_2$-g*-closed sets $= \{\varphi, X, \{c\}, \{a,c\}, \{b,c\}\}$.

$\tau_1\tau_2$-semi-closed sets $= \{\varphi, X, \{a\}, \{b\}, \{b,c\}\}$.

Here $\{a\}, \{b\}$ are $\tau_1\tau_2$-semi-closed sets,

but they are not $\tau_1\tau_2$-g*-closed.

Also $\{c\}, \{a,c\}$ are $\tau_1\tau_2$-g*-closed but they are not $\tau_1\tau_2$-semi-closed.

Example: 3.16

Let $X = \{a,b,c\}$,
$\tau_1 = \{\varphi, X, \{a\}, \{b,c\}\}, \tau_2 = \{\varphi, X, \{a\}, \{b\}, \{a,b\}\}$.

$\tau_1\tau_2$-g*-closed sets $= \{\varphi, X, \{c\}, \{a,c\}, \{b,c\}\}$.

$\tau_1\tau_2$-α-closed sets $= \{\varphi, X, \{a\}, \{b,c\}\}$.

$\tau_1\tau_2$-pre-closed sets $= \{\varphi, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.

$\tau_1\tau_2$-semi-pre-closed
sets $= \{\varphi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

Here $\{a\}$ is $\tau_1\tau_2$-α-closed but not $\tau_1\tau_2$-g*-closed and $\{c\}$ and $\{a, c\}$ are $\tau_1\tau_2$-g*-closed but not $\tau_1\tau_2$-α-closed.

Here $\{b\}$ is $\tau_1\tau_2$-pre-closed but not $\tau_1\tau_2$-g*-closed.

Here $\{a\}, \{b\}, \{a, b\}$ are $\tau_1\tau_2$-semi-pre-closed but they are not

$\tau_1\tau_2$-g*-closed.

$\tau_1\tau_2$-sg-closed sets $= \{\varphi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$.

Here $\{a\}, \{b\}, \{a, b\}$ are $\tau_1\tau_2$-sg-closed sets but they are not

$\tau_1\tau_{2-}$ g*-closed.

$\tau_1\tau_2 g\alpha$-closed sets $= \{\varphi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$.

$\Rightarrow \{b\}, \{a, b\}are\tau_1\tau_2$-g$\alpha$-closed but they are not $\tau_1\tau_2$ $-$ g*-closed.

Example: 3.17

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{a\}, \{a, b\}\}, \tau_2 = \{\varphi, X, \{a\},, \{a, c\}\}$.

$\tau_1\tau_2$-g*-closed sets $= \{\varphi, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$\tau_1\tau_2$-pre-closed sets $= \{\varphi, X, \{b\}, \{c\}, \{b, c\}\} =$ $\tau_1\tau_2$-semi-pre-closed sets=$\tau_1\tau_2$-sg-closed sets=$\tau_1\tau_2$-gα-closed sets. Here$\{a, c\}$ *is* $\tau_1\tau_2$-g*-closed set *but it is not*$\tau_1\tau_2$-pre-closed,$\tau_1\tau_2$-semi-pre-closed $\tau_1\tau_2$-sg-closed and $\tau_1\tau_2$-gα-closed.

Theorem: 3.18 IfA and B are $\tau_1\tau_2$-g*-closed sets then their union$A \cup B$ is $\tau_1\tau_2$-g*-closed set.

Proof: Let $A \cup B \subset U$, where U is τ_1-g-open.

$\Rightarrow A \subset U$and$B \subset U \Rightarrow \tau_2\text{-}cl(A) \subset U$ and $\tau_2\text{-}cl(B) \subset U$.

$\Rightarrow \tau_2\text{-}cl(A) \cup \tau_2\text{-}cl(B) \subset U$. (since A and B are $\tau_1\tau_2$-g*-closed). $\tau_{2-}(A \cup B) = \tau_2\text{-}cl(A) \cup \tau_2\text{-}cl(B) \subset U$.

$\Rightarrow$$A \cup B$ is $\tau_1\tau_2$-g*-closed**.**

Theorem: 3.19 If A is both τ_1-g-open and $\tau_1\tau_2$-g*-closed, and then$A$ is τ_2-closed.

Proof: Let $A \subset A = U$ and A is τ_1-g-open.

$\Rightarrow \tau_2\text{-}cl(A) \subset U = A$.......... (1) [since A is $\tau_1\tau_2$-g*-closed]

But always $A \subset \tau_2\text{-}cl(A)$ ……… (2)

From (1) and (2) $A = \tau_2\text{-}cl(A)$

⇒ A is τ_2-closed.

Theorem: 3.20 A subset A of (X, τ_1, τ_2) is $\tau_1\tau_2$-g*-closed then

$\tau_2\text{-}cl(A) - A$does not contain any non-empty τ_1-g-closed set.

Proof: Assume that A is $\tau_1\tau_2$-g*-closed .

To prove:τ_2- $cl(A) - A$ does not contain any non-empty τ_1-g-closed set.Suppose $F \subset \tau_2\text{-}cl(A) - A$, where F is a non-emptyτ_1-g-closedset⇒$F \subset \tau_2\text{-}cl(A)$ and $F \subset A^C$…………. (1)

$F \subset A^C \Rightarrow A \subset F^C$,$F^C$ is τ_1-g-open.

⇒ $\tau_2\text{-}cl(A) \subset F^C$ [since A is $\tau_1\tau_2$-g*-closed]

⇒ $F \subset (\tau_2\text{-}cl(A))^C$ …………….. (2)

⇒ $F \subset \tau_2 - cl(A) \cap (\tau_2\text{-}cl(A))^C = \varphi$ [by (1) and (2)]

⇒ $F = \varphi$.Which is a contradiction.

Therefore, $\tau_2\text{-}cl(A) - A$ will not contain any non-emptyτ_1-g-closed.

Theorem: 3.21 IfA is $\tau_1\tau_2$-g*-closed set and $A \subset B \subset \tau_2 - cl(A)$, B is also $\tau_1\tau_2$-g*-closed set.

Proof: Let $B \subset U$, where U is τ_1-g-open⇒ $A \subset B \subset U$

⇒ $\tau_2\text{-}cl(A) \subset U$ [since A is $\tau_1\tau_2$-g*-closed]

⇒ $B \subset \tau_2\text{-}cl(A) \subset U$ [given]

⇒ $B \subset \tau_2\text{-}cl(B) \subset \tau_2\text{-}cl(A) \subset U$. [since $\tau_2\text{-}cl(B)$ is the smallest closed set containing B]

⇒B is$\tau_1\tau_2$-g*-closed.

Theorem: 3.22 If $A \subset Y \subset (X, \tau_1, \tau_2)$ *and suppose that*, A is$\tau_1\tau_2 - g*-$closed in X then A is $\tau_1\tau_2$-g*-closed relative to Y.

Proof: Given that $A \subset Y \subset (X, \tau_1, \tau_2)anA$ is $\tau_1\tau_2 - g*$ $-$closed in X

To prove:A is $\tau_1\tau_2$-g*-closed relative to Y.

Let $A \subset Y \cap U$ *where* U *is*τ_1-g-open in X

$\Rightarrow A \subset Y \text{ and} A \subset U. \text{Since } A \text{ is} \tau_1\tau_2$-g*-closed,$\tau_2\text{-}cl(A) \subset U \Rightarrow Y \cap \tau_2\text{-}clA \subset Y \cap U.$

$\Rightarrow \tau_2\text{-}cl(A) \text{ with respect to } Y \subset Y \cap U \Rightarrow A$ is$\tau_1\tau_2$-g*-closed *with respect to Y.*

Theorem: 3.23 For each x in X, the set X-$\{x\}$ is a $\tau_1\tau_2$-g*-closed or τ_1-g-open.

Proof: Suppose X-$\{x\}$is not τ_1-g-open,then X is the only τ_1-g-open set containing X-$\{x\} \Rightarrow \tau_2\text{-}cl(X - \{x\}) \subset X \Rightarrow$X-$\{x\}$ is$\tau_1\tau_2$-g*-closed.

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