A Mathematical Model for Predicting Throughput Capacity of a Cocoyam Chipper

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ABSTRACT

A mathematical expression for predicting the throughput capacity for chipping process of a mechanically operated cocoyam chipper is presented. The chipper throughput capacity model was developed by dimensional analysis, using the concept of Buckingham's Pi Theorem. The model was verified and validated by fitting it into experimental data from the developed mechanical cocoyam chipper. The results obtained revealed that the fitted model correlated well with the experimental data with R-square value of 0.91. Also, the difference between the means of the predicted and the measured throughput capacity was not statistically significant at 5% level of significance.

Key words: Modeling, Throughput capacity, Cocoyam, Chipper

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1 INTRODUCTION

Cocoyam (*Colocasia esculenta* and *Xanthosoma sagittifolium*) is a stem tuber that is widely cultivated in both the tropical and subtropical regions of the world (Onyeka, 2014). Cocoyam contains substantial amount of protein, vitamin C, thiamin, riboflavin, niacin, amino acids and significant amount of dietary fibre (Doku, 1980; Onayemi and Nwigwe, 1987), and other essential minerals such as vitamin B6, calcium, magnesium and phosphorus (Chukwu and Nwosu, 2008). Cocoyam corms when harvested are prone to deterioration if not properly stored. To ensure safe storage over a much longer period without the risk of losses from rotting, the cocoyam corms must be dried (Kay, 1987). Chipping operation is one of the most important post harvest processes for root and tuber crops which help to achieve the required size reduction for easy drying of the crops. Throughput capacity is an important criterion in evaluating the chipping performance (Ogundipe *et al*., 2011). Manual chipping operation is laborious and time consuming. It is necessary to mechanize the chipping process of cocoyam in order to overcome the difficulties of handling large scale processing with traditional method. Also, the major limiting factor in the utilization of cocoyam is the presence of oxalates which causes irritation when foods prepared from them are eaten (Sefa-Dedeh and Agyir-Sackey, 2004). Cocoyam processing helps to reduce the oxalate content and produce good quality flour (Onayemi and Nwigwe, 1987). Cocoyam flour can be used for the preparation of soup, biscuit, bread and beverages. Ejoh *et a*l. (2013) reported on the suitability of cocoyam flour for making biscuit. Sanful and Sophia (2010) reported on the use of cocoyam flour in preparation of cake. Ezeocha *et al*. (2010) also reported on the use of cocoyam flour in combination with trifoliate yam flour in *fufu* production. The effect of machine speed on chipping capacity was studied by Bolaji *et al*. (2008) and Ojomo *et al*. (2011). The results obtained showed that increase in machine speed results in increase in chipping capacity. Ogundipe *et al*. (2011) also studied the effect of feed rate on the chipping time. It was also observed that weight input and time taken to chip are directly proportional to one another. The study by Adejumo *et al*. (2011) reported that age, size, shape, orientation, feed rate and pressure of the tuber in contact with the chipping disc are factors affecting the chipping efficiency and the uniformity of the chips. Therefore, in modeling the throughput capacity of a cocoyam chipper, these parameters have to be studied in order to provide better understanding of the fundamental relationships of the different machine and crop variables. Ndirika (2005) developed a mathematical model for predicting the output capacity of selected stationary grain threshers. The model was verified and validated by fitting it into established experimental data from stationary grain threshers. There has been no model developed for predicting the performance of cocoyam processing machines. The purpose of this study was to develop and verify mathematical model for predicting the throughput capacity of a cocoyam chipper.

2 MATERIALS AND METHODS

2.1 Theoretical Development

The cocoyam chipper throughput capacity model was developed by dimensional analysis using the concept of Buckingham's Pi theorem (Ndirika, 1997; Walter, 1985). Assuming that the variables of importance are the cutting velocity (V_c), Chipping force (F), crop bulk density (β), feed rate (F_r) and chipping slot size (S_s).

Throughput capacity, $T_c = f(V_c, F, \beta, F_r, S_s)$ (1)

Using the [M], [L], [T] system of dimension (Ndirika, 1997; Babashani, 2008), the dimensions of the variables identified in this study are presented in table 1, while the dimensional matrix is presented in table 2. The procedure for applying the Buckingham's Pi Theorem to identify the dimensionless group to be formed is as follows:

The total number of variables = 6
Number of fundamental dimension = 3
Number of dimensionless group to be formed = 6 – 3 = 3

Table 1: Dimensions of the variables influencing (T_c)

Variable	Symbol	Unit	Dimension [M], [L], [T]
Throughput capacity	T_c	kg/s	MT^{-1}
Cutting velocity	V_c	m/s	LT^{-1}
Chipping force	F	kgm/s^2	MLT^{-2}
Crop bulk density(wet basis)	β	kgm^3	ML^{-3}
Feed rate	Fr	kg/s	MT^{-1}
Chipping slot area	S_S	m^2	L^2

Table 2: Dimensional Matrix of the Variables

Parameters / Dimensions	T_c	V_c	F	β	F_r	S_S
M	1	0	1	1	1	0
L	0	1	1	-3	0	2
T	-1	-1	-2	0	-1	0

The required solution will be:

$$\pi_1 = K_c f\left[\pi_2, \pi_3\right] \qquad (2)$$

Where,
π_1, π_2 and π_3 = First, second and third pi terms respectively
K_c = Capacity constant
Using the Buckingham's Pi theorem, π_1, π_2 and π_3 were found to be:

$$\pi_1 = \frac{T_c}{F_r}, \quad \pi_2 = \frac{V_c \beta S_s}{F_r}, \quad \pi_3 = \frac{F \beta S_s}{F_r^{\,2}}$$

$$\therefore \frac{T_c}{F_r} = K_c\left[\frac{V_c \beta^2 S_s^{\,2} F}{F_r^{\,3}}\right] \qquad (3)$$

Rearranging equation 3, then

$$T_c = K_c\left[\frac{V_c\beta_d{}^2 S_s{}^2 F}{F_r{}^2}\right] \qquad (4)$$

Substituting value of β as $\frac{\beta_d}{1-M_c}$ into equation 4, then

$$T_c = K_c\left[\frac{V_c\beta_d{}^2 S_s{}^2 F}{(1-M_c)^2 F_r{}^2}\right] \qquad (5)$$

2.2 Value of Constant

The capacity constant, K_c was determined by linearizing π_1 and π_4 $\left[\frac{T_c}{F_r} and \frac{V_c\beta^2 S_s{}^2 F}{F_r{}^3}\right]$ from the developed model by method of least squares using data from available literatures (Awulu *et al*., 2015, Bolaji *et al*., 2008 and Aji *et al*., 2013). The slope of the line of best fit gives the value of the capacity constant which was found to be 2.3298×10^{-7} as presented in figure 1.

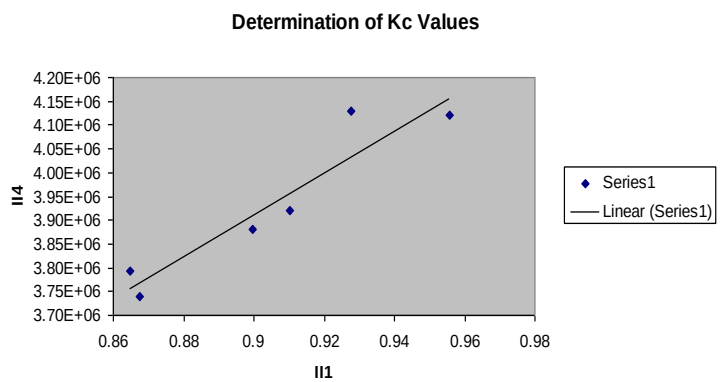

Fig. 1: Determination of capacity constant, K_c

2.3 Verification and Validation of the Model

The throughput capacity model was verified in order to confirm its consistency with established experimental results from a cocoyam chipper. The study was conducted using data from the developed cocoyam chipper, and the predicted model was compared with the experimental result. The crop and operating conditions for the cocoyam chipper used are presented in table 3. A model would have greater confidence if a good fit and a high significance level is attained. However, the method developed by Gregory and Fedler (1986) for calculating the coefficient of determination, R^2 statistically for non- linear as well as for linear function and with one or more independent variables was adopted. Thus,

$$R^2 = \frac{1-V_o}{V_t} \qquad (6)$$

Where,
R^2 = coefficient of determination
V_o = estimated variance about the mean from the measured data
V_t = estimated variance about the mean from the predicted model

Table 3: Crop and Operating Conditions for the Chipper

S/N	Parameters	Values / Levels 1	2	3
1	Cutting Velocity, V_c (rpm)	300	350	375
2	Cutting Velocity, V_c (m/s)	3.93	4.58	4.91
3	Chipping force, F (Kgm/s^2)	193.68	193.68	193.68
4	Feed Rate, Fr (kg/s)	0.056	0.058	0.06
5	Chipping Slot Area, Ss (m^2)	0.0014	0.0016	0.0018
6	Moisture Content, M_c (%)	52.39	52.39	52.39
7	Bulk density, β_d dry basis (kg/m^3)	231.28	231.28	231.28

Since the R^2 value from equation 6 must have a level of
Significance before the model is considered verified, the statistical significance test was done to ascertain how adequately the sample data test used for developing the model represents the whole population. The significance level for a given R^2 can be obtained by computing 't' as presented by (Snedecor and Cochron, 1980) in equation below, and then determining the significance level in a table of 't' values.

$$t = \frac{R(Df)^{\frac{1}{2}}}{(1-R^2)^{\frac{1}{2}}} \qquad (7)$$

Where t = student's t – value
R = coefficient of regression
Df = degrees of freedom (number of data points minus number of constant as defined in the model)

2.4 Instrumentation and Measurements

The data generated from the developed cocoyam chipper used in the verification of the output capacity model were evaluated by the following methods: *Capacity measurement:* The throughput capacity was measured by weighing the quantity of cocoyam tubers chipped by the machine per unit time, expressed in kg/hr.

Weight measurement: An electronic balance of sensitivity 0.01g and the range of 0.01g to 5000g was used for weighing.

Time measurement: Time was measured using a stop watch.

Moisture contents determination: Moisture content of the sliced cocoyam corms were determined by oven drying at a temperature of 75°C for 24 hours. The percentages of moisture content wet basis were calculated as according to Kajuna *et al*., (2001).

Bulk density determination: The bulk density was determined as the ratio between the mass of the cocoyam corms in a container to its volume (Kaleemullah and Kailappan, 2003).

3 RESULTS AND DISCUSSIONS

Model Validation

The throughput capacity model (equation 5) was validated with measured data from the developed cocoyam chipper. From the result, it was established that the model described well with the measured data obtained from the developed chipper with R^2 value of 0.9089. The data used for comparison between the values of throughput capacity obtained from the experimental result and that obtained using the formulated performance models are presented in table 4 and figure 2. When the means of the predicted and measured throughput capacities were compared statistically, it was revealed that there was no significant difference between the means at 5% level of significance, since the calculated 't' value (0.878) is less than the table 't' value (2.306). The result reveals a higher correlation of the model with data from the developed cocoyam chipper.

Table 4: Computed and Measured Throughput Capacity (T_c) for the Cocoyam Chipper

CuttingVelocity,V_c (m/s)	Throughput Capacity (kg/s)	
	Computed	Measured
3.60	0.03984	0.03992
3.93	0.04348	0.04356
4.25	0.04703	0.04795
4.58	0.05068	0.05086
4.91	0.05433	0.05448

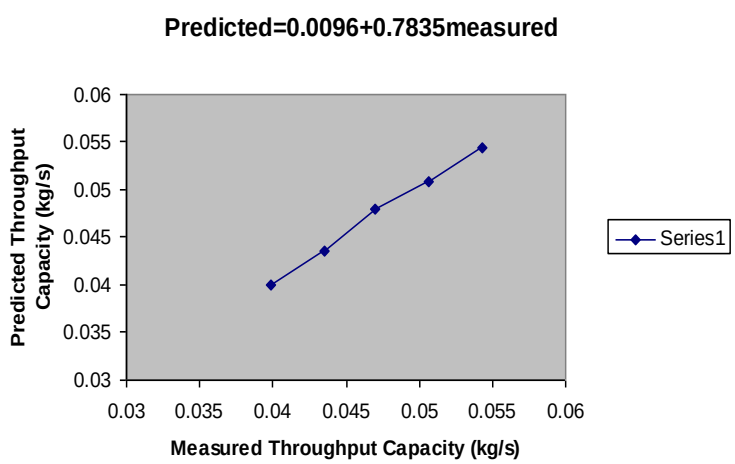

Figure 2: Predicted versus measured throughput capacity

4 CONCLUSIONS

Based on the results of this study, the following conclusions can be drawn: The throughput capacity of cocoyam chipper can be described using the mathematical model which include variables such as cutting velocity, feed rate, chipping force, chipping slot area, crop bulk density and moisture content. The difference between the means of the predicted and measured throughput capacities was not statistically significant at 5% level of significance. The predicted

throughput capacity model fitted well with experimental results from the developed cocoyam chipper.

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